Problem I

We have a system with four filters, $G_1(s), G_2(s), G_3(s), G_4(s)$. The system functions, in terms of these filters is

$$Y(s) = (X(s) - Y(s) \cdot G_3(s)) \cdot G_4(s) \cdot (G_2(s) - G_1(s))$$

Draw a block diagram of this system and label it figure 1.

Problem II

In figure 1,

Let $G_1(j\omega) = 0$,

Let $G_2(j\omega) = 1$ if $1 \leq |\omega| \leq 2$,

Let $G_3(j\omega) = 0$,

Let $g_4(t) = e^{-2t} U(t)$
Question 1 What is the impulse response, $G_4$?

Question 2 What is the frequency transfer function from $x$ to $y$?

**Problem III**

a) The Laplace transform of a causal filter response is

$$H_f(s) = \frac{s}{(s + 2)^2 + 4}$$

Compute the impulse response, $h_f(t)$.

b) Do the same for

$$H_f(s) = \frac{1}{(s + 2)^2 + 4}$$

**Problem IV**

Let $G_1(s) = 1$,

Let $g_2(t) = e^{-t}U(t)$,

Let $G_3(s) = 0$,

Let $G_4(s) = s$

Question 1 What is the System Function from $x$ to $y$?
Question 2 Write the differential equation which has this System function.

**Problem V**

Let \( G_1(s) = \frac{1}{s-2} \),
Let \( g_2(t) = e^{-t}U(t) \),
Let \( G_3(s) = s + 1 \).
Let \( G_4(s) = K \)

Question 1 For what values of K is the System Function from \( x \) to \( y \) stable?

Question 2 Answer the similar question when \( G_1(s) = \frac{-1}{s-2} \)

**Problem VI**

A system has input, \( x(t) \) and output \( y(t) \) related by

\[
\frac{d^2}{dt^2}y(t) + 4\frac{d}{dt}y(t) + 4y(t) = \frac{d}{dt}x(t)
\]

Question 1 Find the impulse response of this system

Question 2 Find the response when \( x(t) = e^{-t}U(t) \)