Computing the Shannon Information Capacity of an Optical Fiber

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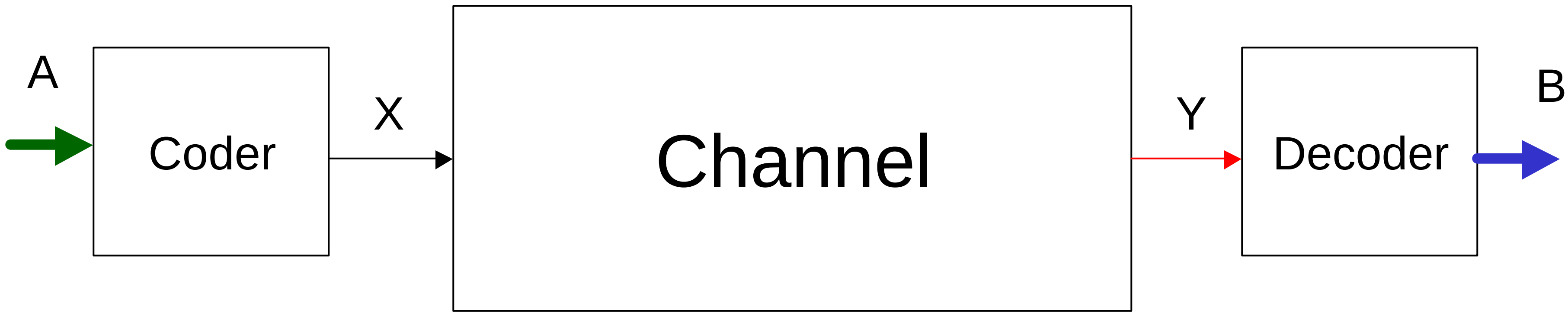
December 4, 2000

Outline of this Presentation

- Quick review of Shannon’s formula for the capacity of of a communications channel
- Applying Shannon’s formula to an optical fiber link with specified types of physical transmitters and receivers
- Extending Shannon’s formula to an optical fiber link viewed from the perspective of quantum theory
- Really strange stuff

Quick Review of Shannon's Formula



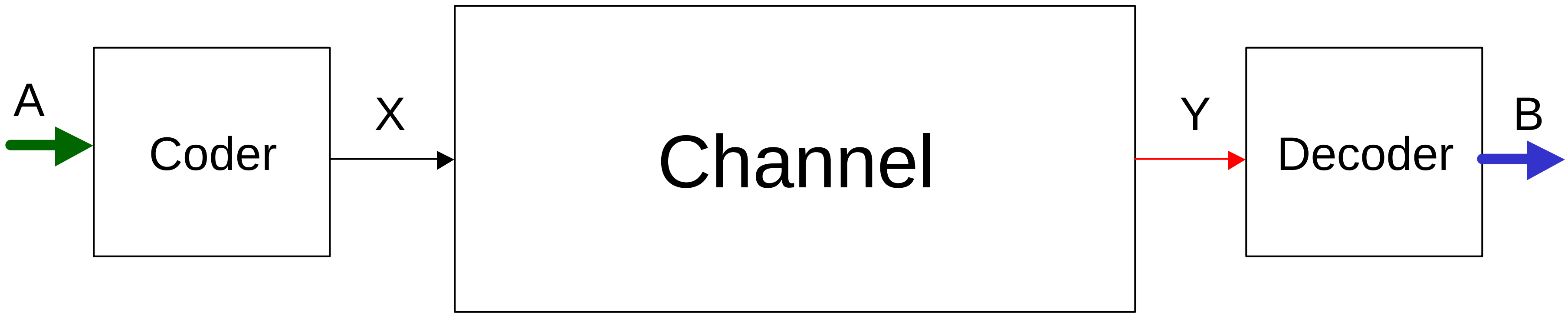
Symbols from information source {a}

Channel input symbols {x}

Channel output symbols {y}

Transported symbols from information source {b}

Quick Review of Shannon's Formula

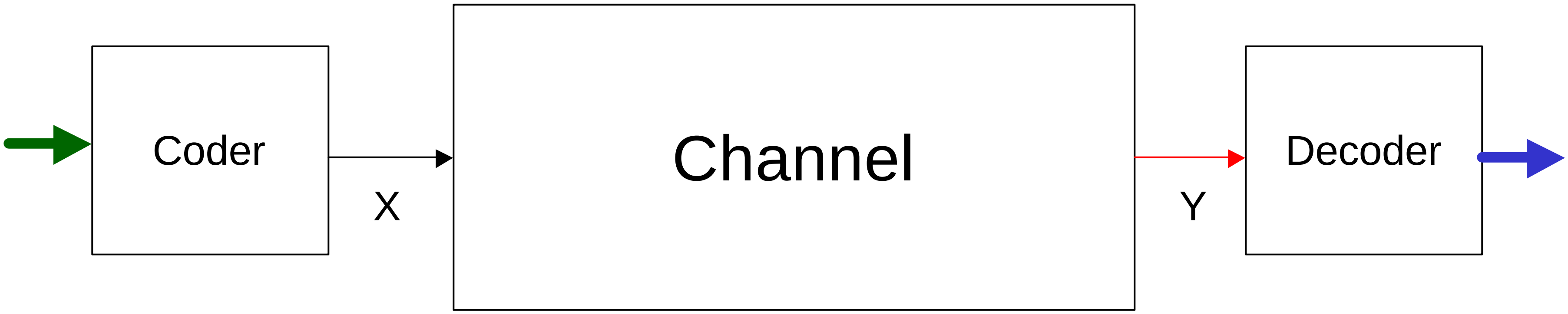


Source information rate = H(A)

I(X:Y)= Mutual information between X and Y=
H(X) - H(X|Y)

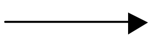
where: h(u) = -p(u) log p(u); H (U)=sum {h(u)}

Quick Review of Shannon's Formula



C (channel capacity) = H(X) - H(X|Y)

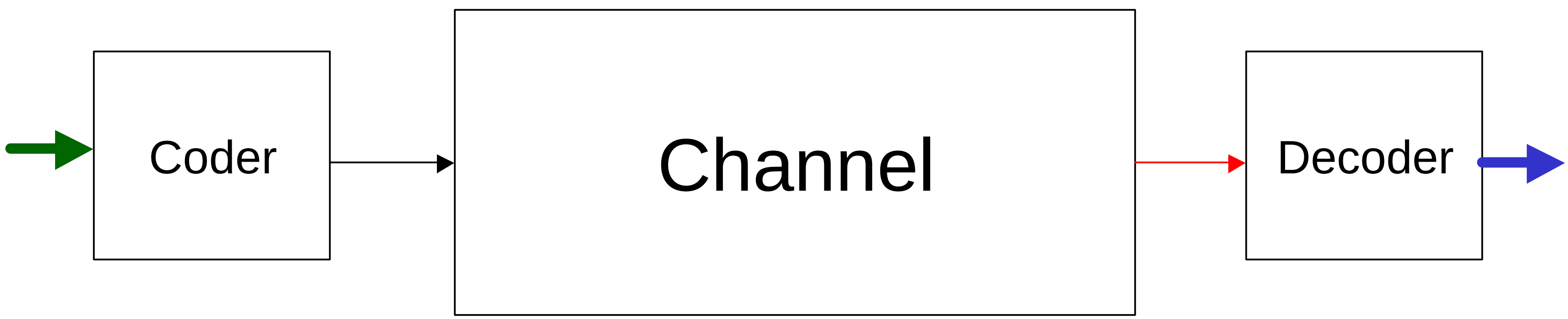
maximized over all apriori probability distributions of X

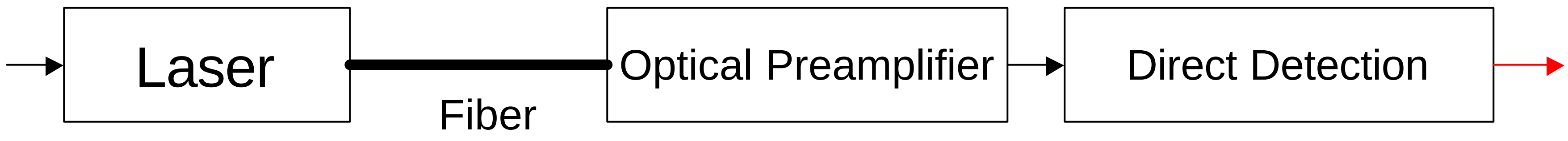


Channel input symbols {x}

Channel output symbols {y}

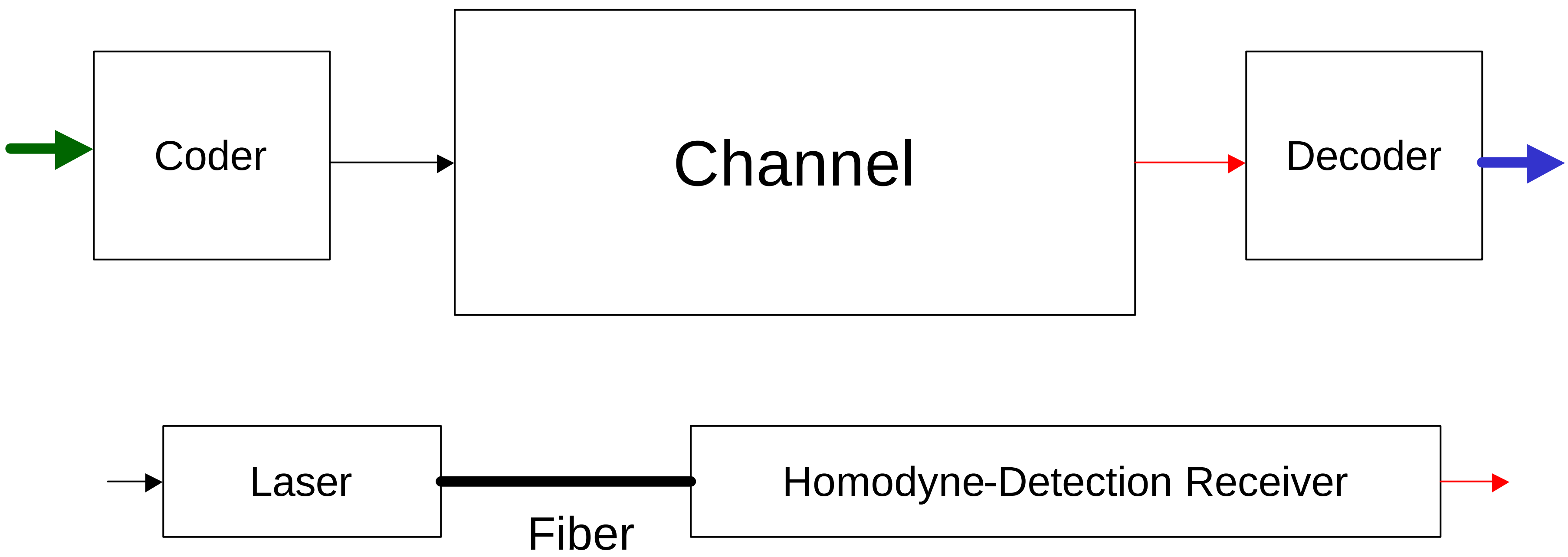
Applying Shannon's formula to an optical fiber link with specific types of physical transmitters and receivers





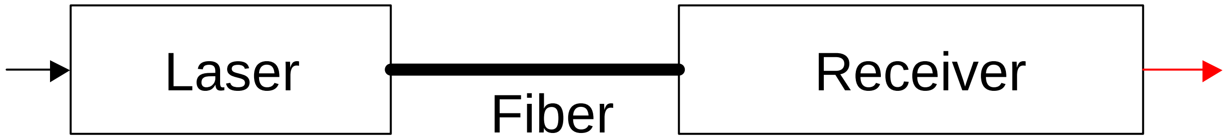
Example 1

Applying Shannon's formula to an optical fiber link with specific types physical transmitters and receivers



Example 2

Applying Shannon's formula to an optical fiber link with specific types physical transmitters and receivers



Classical channel* capacity: $C = B \log[1 + E/N]$

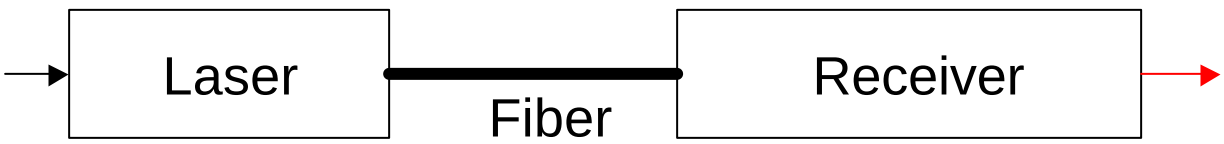
where B= channel bandwidth, E=average energy per received symbol, N=equivalent noise spectral density (watts/Hz) at the receiver

* linear, flat frequency response, time invariant, additive white Gaussian noise

Homodyne receiver case: N= hf Heterodyne receiver case: N=2hf

Optical preamplifier: N=2hf

Applying Shannon's formula to an optical fiber link with specific types physical transmitters and receivers



Classical channel capacity: $C= B \log[1 + E/N]$

Homodyne receiver case: N= hf   Heterodyne receiver case: N=2hf

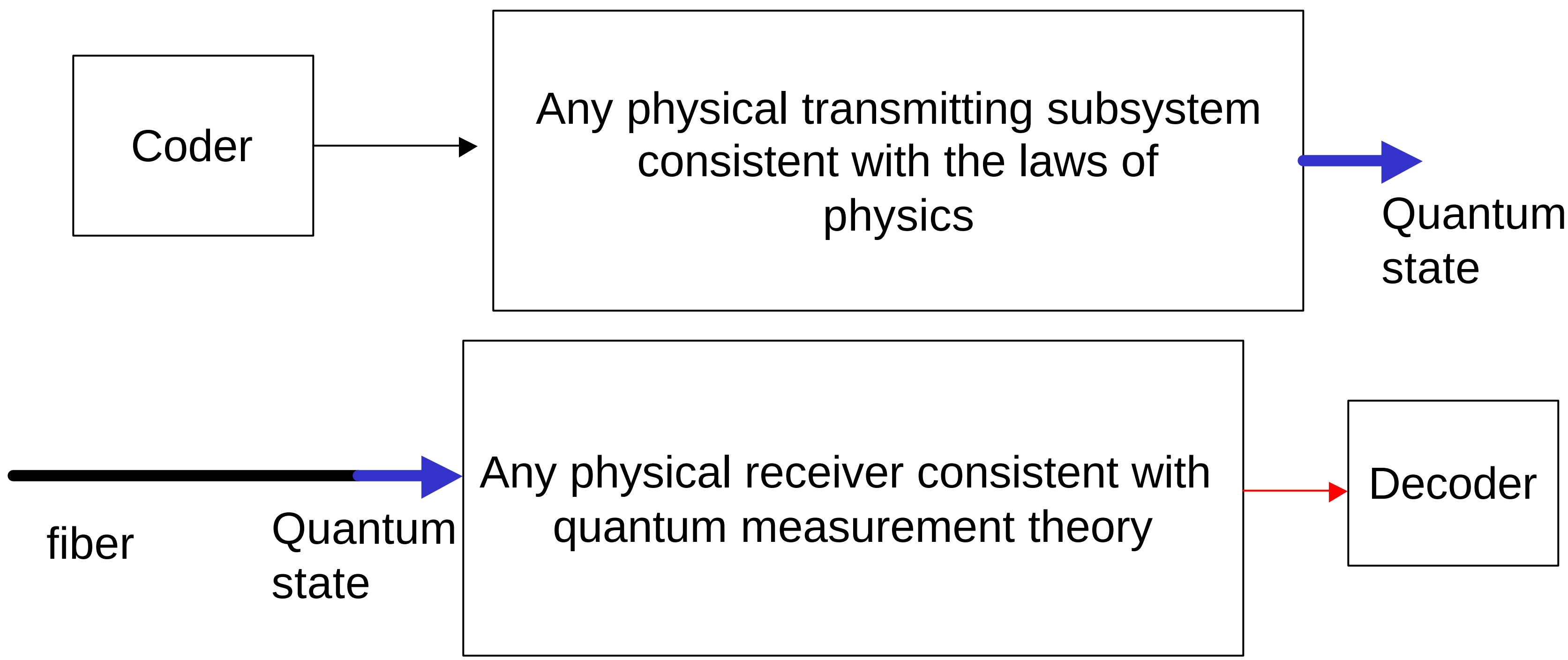
Optical preamplifier: N=2hf

E/N ~P/hfB; for B=20 GHz, hfB ~3 x 10**-9 watts ~ -55 dBm

C (20 GHz, -25dBm) ~ 20 GHz x 10 bps/Hz = 200 Gbps

C (20 GHz, -8 dBm) ~ 20 GHz x 16 bps/Hz = 320 Gbps

Extending Shannon's formula to an optical fiber link viewed from the perspective of quantum theory



Extending Shannon’s formula to an optical fiber link viewed from the perspective of quantum theory

Ref: Nielsen and Chuang: Quantum Computation and Quantum Information

Special case: multiple uses of the channel create a composite density operator that is the tensor product of the density operators associated with each use*:

Product state capacity

Special case: multiple uses of the channel create a composite density operator that is the tensor product of the density operators associated with each use*:

Product state capacity

*If you think you understand this, read it again!

Special Case Ref: Yuen and Ozawa Physical Review Letters Jan 25, 1993

Assumption: the received signal is in a single electromagnetic field mode, subject to a constraint that the average received number of photons (energy constraint), as defined by a measurement of the “number operator”, is less than N photons

c (capacity per use of the channel)=

$$(n+1) \log (n+1) - n \log n$$

where n= E/hf, and E is the average energy per received symbol

Special Case: Yuen and Ozawa Physical Review Letters Jan 25 1993

c (capacity per use of the channel)=

$(n+1) \log (n+1) - n \log n =$

$\log (n+1) + n \log(1 + 1/n)$

where n= E/hf, and E is the average energy per received symbol.

n	.01	0.1	0.5	1.0	2.0	5.0	10.0	infinity
log (1+n)	.014	0.14	0.58	1.0	1.58	2.58	3.45	infinity
n log (1+ 1/n)	.067	0.35	0.79	1.0	1.17	1.32	1.38	1.4427

(bits)

Special Case: Yuen and Ozawa Physical Review Letters Jan 25 1993

c (capacity per use of the channel)=
(n+1) log (n+1) - n log n =log (1+n) + n log(1 + 1/n)

For a bandwidth B << f (optical),

C = B {log (1 + P/hfB) + (P/hfB) log (1 + hfB/P };

C (20GHz, -25dBm) = 20 GHz x 10 bps/Hz x 1.14

for B => infinity,

C=> pi { 2P/3h} ** 0.5; C (-25dBm) ~ 100,000 Gbps

where P = average power constraint

More generally…. Bennett and Shor IEEE Transactions on Information Theory, October 1998

Quantum entanglement….