CSI Internal Seminar

Correlation Coefficients among Some Statistics Used for Signal Detection

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3:30PM – 4:30PM
EEB248

REFERENCES

This talk is based on the following two papers:

[1] J. Bae et al., "Explicit correlation coefficients among random variables, ranks, and magnitude ranks," *IEEE Trans. Inf. Theory*, Vol. 52, No. 5, pp. 2233-2240, May 2006.

[2] J. Bae et al., "An asymptotic relative performance measure for signal detectors based on the correlation information of statistics," *IEICE Trans. Commun.*, Vol. E88-B, No. 5, pp. 4643-4646, Dec. 2005.

CONTENTS OF THIS TALK

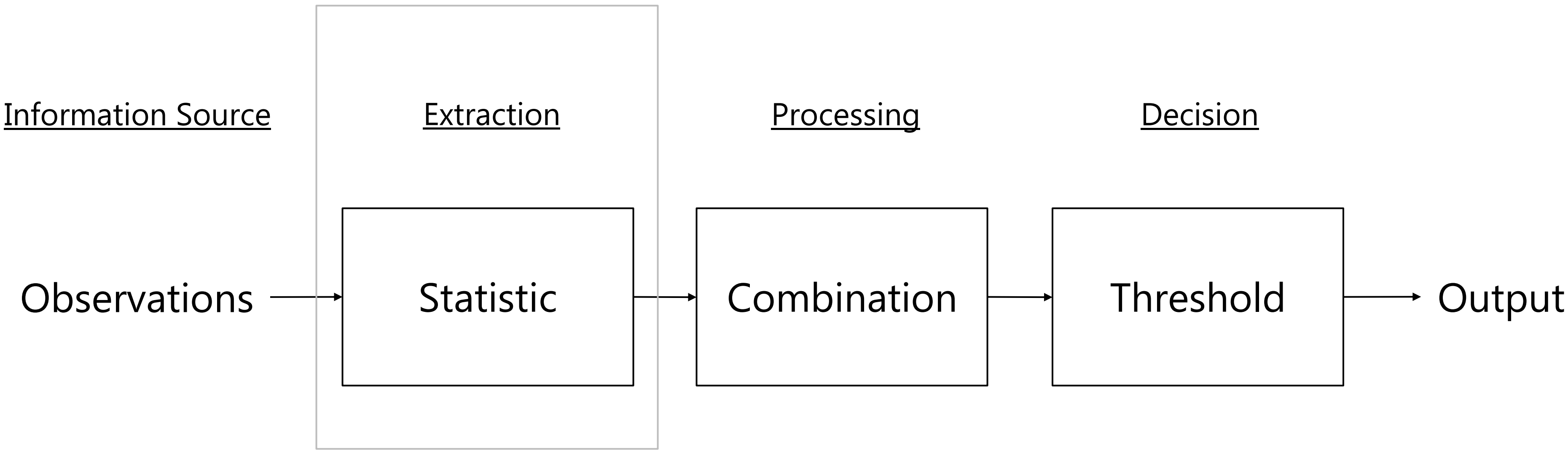
Motivation, Results, and Approach

Why? What? How?

60% 30% 10%

DETECTOR STRUCTURE
Statistic , combination, and threshold

Information is extracted from observations and conveyed to the output in a detector:
Detection is about information.



Motivation

RAW OBSERVATIONS vs. STATISTICS
Negative information coming from noise

Using only partial information via statistics is sometimes beneficial for signal detection.

Laplace noise

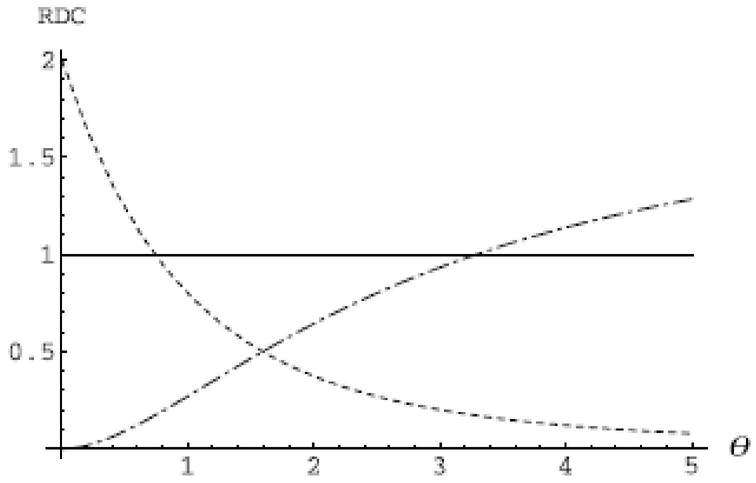


Fig. 2 Asymptotic performance comparisons in Laplace noise. (dashed: $RDC_{S,L}(\theta)$, dot-dashed: $RDC_{M,L}(\theta)$)

Gaussian noise

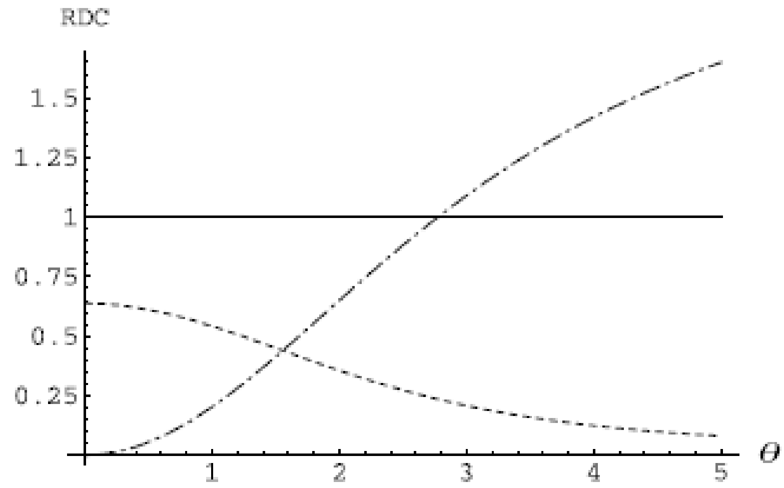
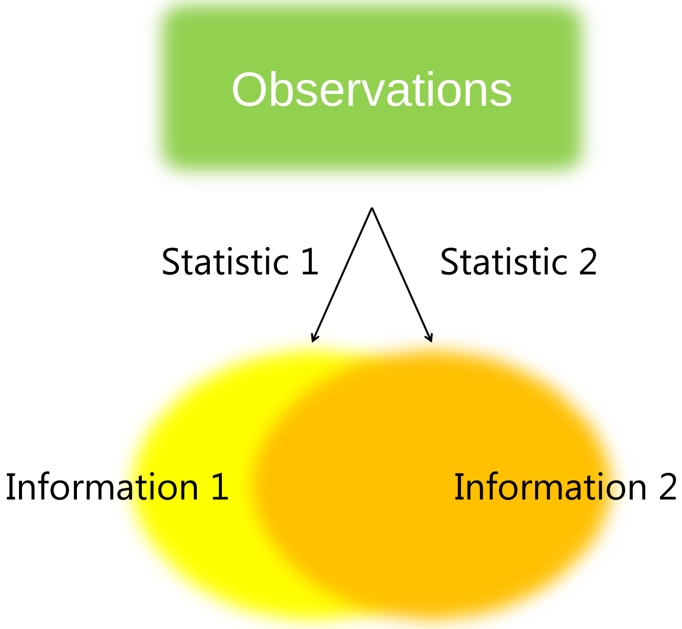


Fig. 3 Asymptotic performance comparisons in Gaussian noise. (dashed: $RDC_{S,L}(\theta)$, dot-dashed: $RDC_{M,L}(\theta)$)

CORRELATION AND INFORMATION

Physical meaning of the correlation of statistics

Our Assumption: Information extracted via two statistic is similar (different) if the correlation between the statistics are large (small) enough.



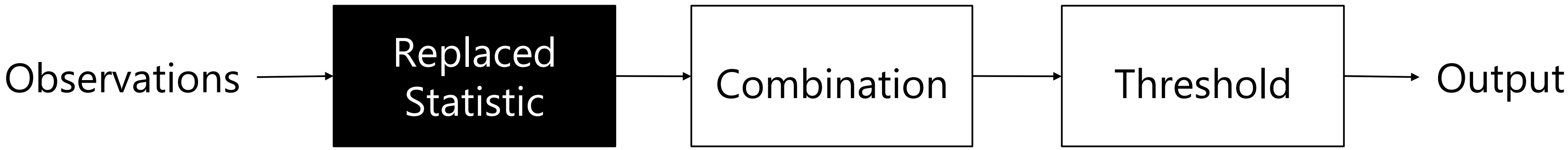
Correlation between statistics is large (small)

(It can be said that) information 1 and 2 are (hardly) overlapping

A DETECTOR USING A NEW STATISTIC

Need for substitute

We *might* keep the detector performance almost the same if we employ another statistic with large correlation with the original statistic: Let it provide similar information.

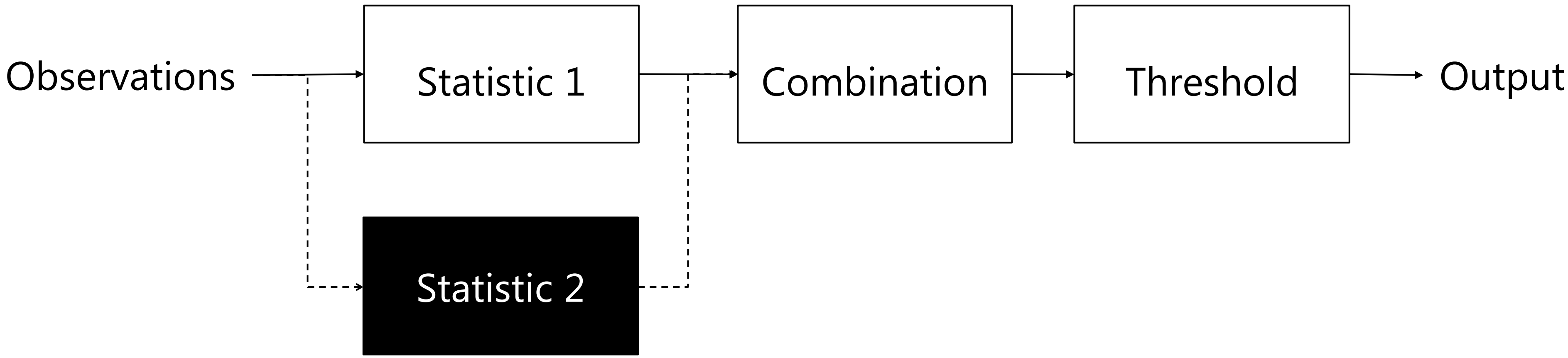


Add another statistic ?

A DETECTOR USING MORE THAN TWO STATISTICS

Need for more information

We *might* enhance the detector performance if we employ additional statistic with little correlation with the existing statistic: Let it provide additional new information.



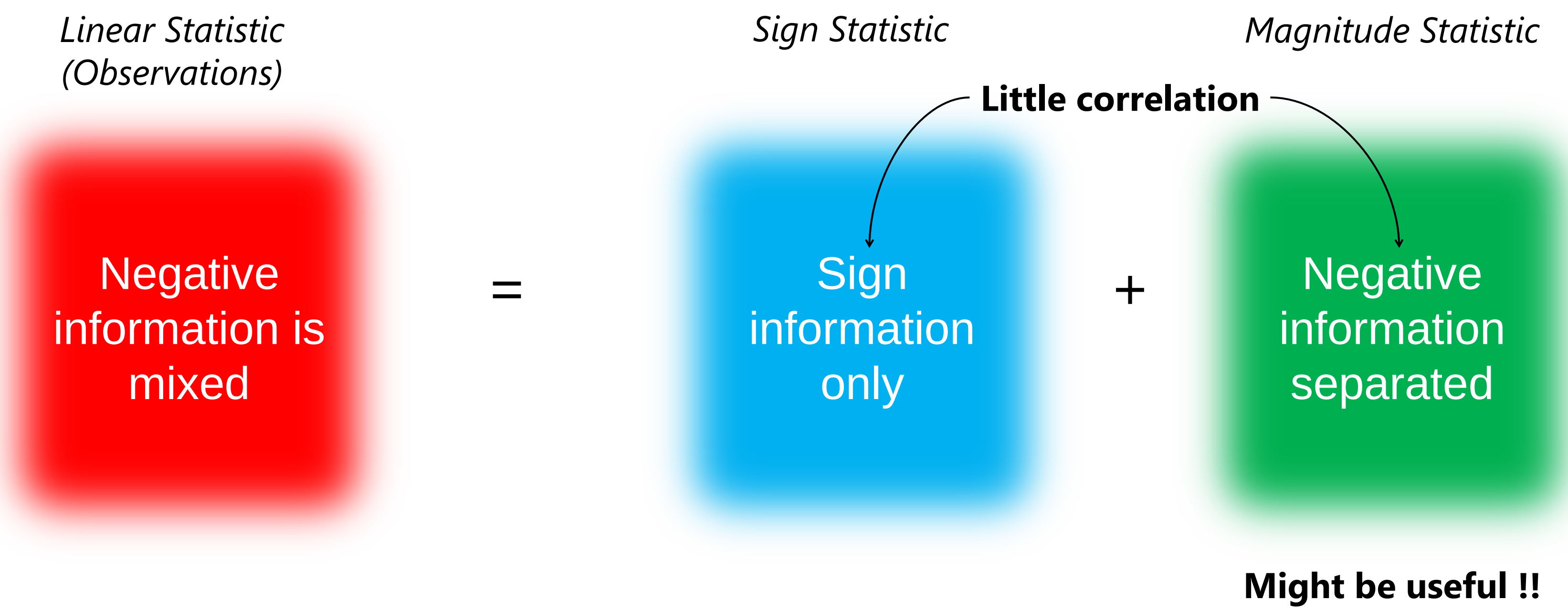
What if the new statistic has a negative information ?

Motivation

NEGATIVE INFORMATION
No useless information

Negative information can be beneficial if it is separated and processed individually.

<u>Laplace Noise Case</u>



Sign and magnitude has little correlation.

CORRELATION OF SIGN AND MAGNITUDE STATISTICS

Laplace noise

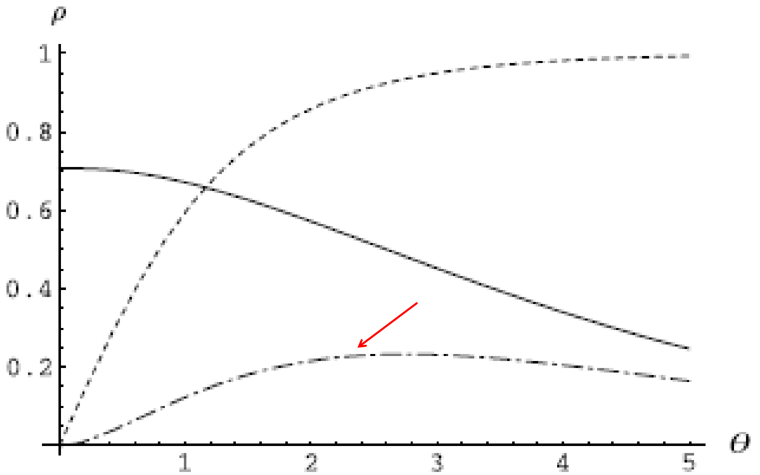


Fig. 4 Correlation coefficients for Laplace noise. (solid: $\rho_{Z_i X_i}(\theta)$, dashed: $\rho_{|X_i| X_i}(\theta)$, dot-dashed: $\rho_{Z_i |X_i|}(\theta)$)

Gaussian noise

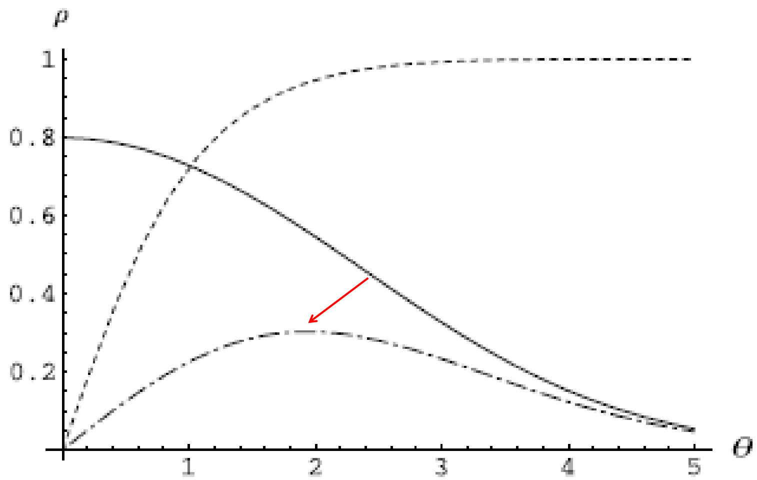


Fig. 5 Correlation coefficients for Gaussian noise. (solid: $\rho_{Z_i X_i}(\theta)$, dashed: $\rho_{|X_i| X_i}(\theta)$, dot-dashed: $\rho_{Z_i |X_i|}(\theta)$)

(INTERIM) SUMMARY OF MOTIVATIONS

¶ Performance of a detector, a linear/nonlinear combination of statistics, is affected by both the way of combination and the statistics used for detection.

¶ Little attention has been paid to possible detection methods using two different statistics simultaneously and those using another statistic instead of the statistic employed originally.

¶ Among the first steps is to address <u>the bivariate correlation of statistics</u> to answer questions such as:

> "What would be the detector performance if we replace a statistic with another one, fixing the way of combination?"
> "How can we predict the detector performance with a new statistic?"
> "Which pair of statistics should be taken to get a better performance if more than two statistics can be used together in a detector?"

¶ **For low power communications, efficient detection is necessary. (Why?)**

Motivation

SOME THOUGHTS OF MINE ABOUT DETECTION
Detection is not popular

¶ Is the linear detector enough for every applications ? Has anyone seen the rank (or another advanced detector in commercial communication systems?

¶ I could not find any detector block in the block diagrams of modern commercial communication systems: There is no advanced detector used but the linear detector (or matched filter).

¶ Detection is just a theory or mathematics, not engineering any more.

¶ Good for dissertation theme, because it is easy to publish: Most engineers are not interested in this topic any more; no competition.

¶ I had to ask to myself when I was a graduate student. "Can I get a detection job in the future?"

¶ Am I wasting your precious time ?

¶ Why am I stick to detection ? (I am proud of being a detectionist.)

Chapter 1

Introduction

1.1 An Overview

Although *signal detection* has very much practical aspects and been of great interest so far, in fact, it is very classical, theoretical, and of a long history. It is based on the hypothesis testing of statistics, therefore not easy to understand and develop. Most of communication engineers in earlier ages were involved in researches related to the detection. The fact that it has been survived in this highly-speeded developing era proves its importance in communication theory. Whenever a new disturbance model is introduced to meet highly precise and complicated circumstance description, a new detection scheme becomes necessary. As the disturbance model gets descriptive, it becomes complex necessarily. It is natural that a novel detection scheme is necessary for a much more intricate disturbance.

Earlier detectionists were interested in the optimum detection based on maximum likelihood (ML) ratio [1]. It has the maximum power function over all the signal strength. Because it is not available all the way, however, the application area cannot help being so limited. Actually, it is very difficult to obtain the optimum scheme for the given disturbance model in most cases, since mathematical manipulations are intangible.

Detection doesn’t matter at high SNR.

DETECTION vs. POWER

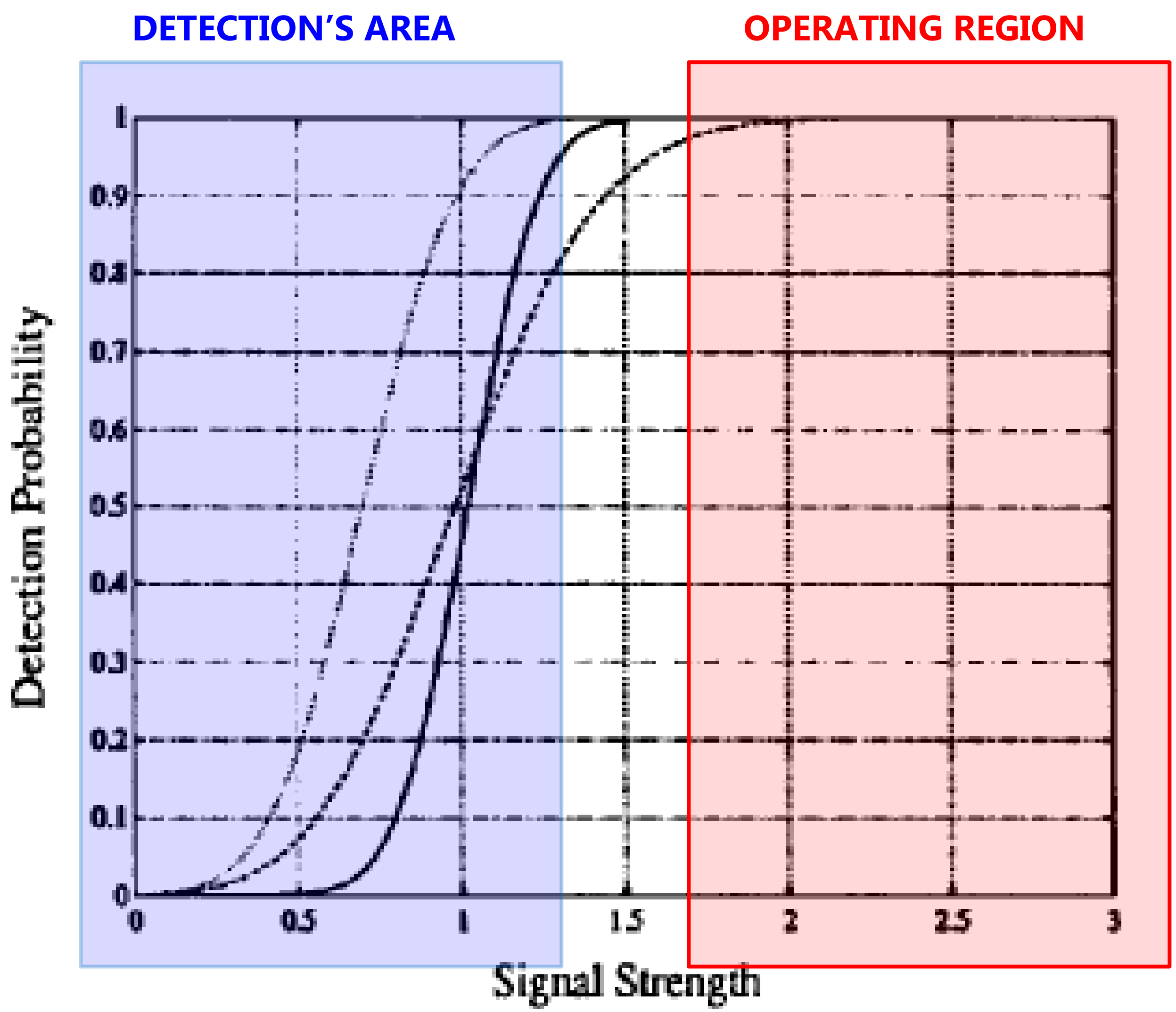


Fig. 1. Detection probability of known signal under Gaussian noise.

APPLICATION AREAS OF DETECTION
Low (unfair) power situation

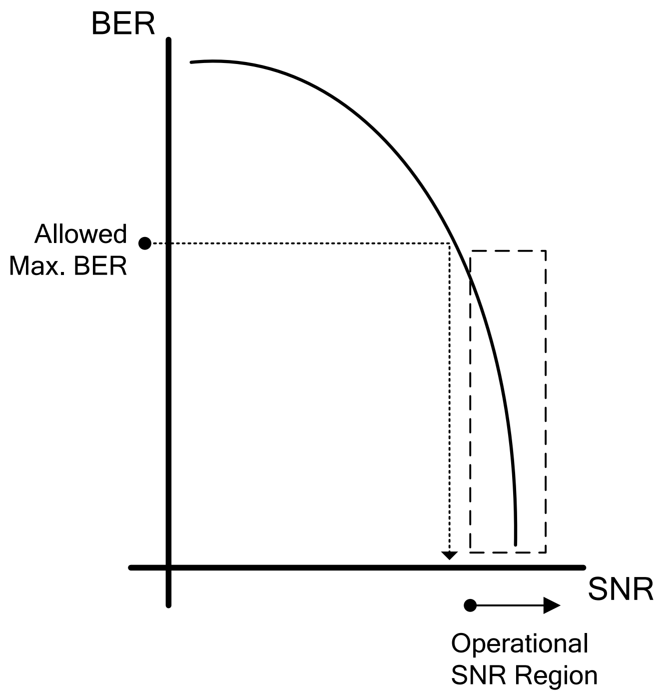
Raising (or controlling) power was easier and economic.

Decades ago	Years ago	Near future
Radar / Sonar	Multiuser detection	Low power communication
The power level of reflected signal is low	Competing with the power control technique	Limitation and difficulty in raising power

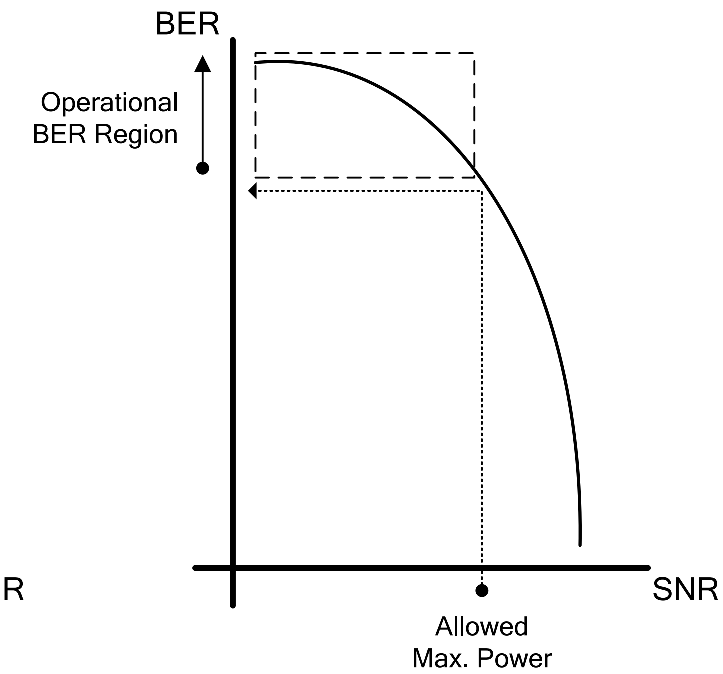
HIGH SPEED vs. LOW POWER SYSTEMS

The design approaches may be different

High Speed System Design

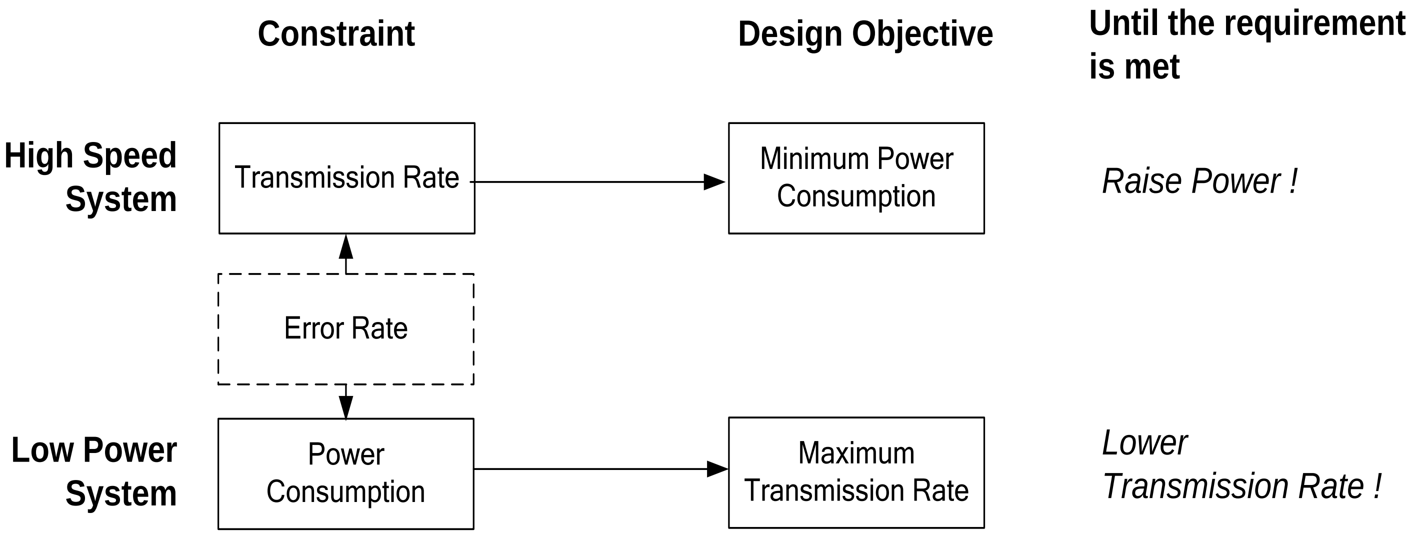


Low Power System Design



EXAMPLE: DESIGN FLOW

Which is the main constraint, power or rate ?



TWO PARADIGMS
Power saving vs. Simple detector

Save the power

Achieved low power consumption even with a complicated detector

↓

Maximum utilization of information is required

↓

Among the first steps, the correlation properties of statistics might be investigated.

Use more power

Use more power, instead keep the detector simple

↓

Might have limitation in low power communication systems

What statistic used for detection ?

Results

FIVE STATISTICS COMMONLY USED FOR SIGNAL DETECTION

There are five statistics commonly used for signal detection: Linear, sign, magnitude, rank, and magnitude rank statistics

	Linear	Sign	Magnitude	Rank	Magnitude Rank
Notation	$\vec{X}$	$\vec{Z}$	$\overrightarrow{\lvert X \rvert}$	$\vec{R}$	$\vec{Q}$
N - tuple form	(X_1, X_2, X_3)	(Z_1, Z_2, Z_3)	$(\lvert X_1 \rvert, \lvert X_2 \rvert, \lvert X_3 \rvert)$	(R_1, R_2, R_3)	(Q_1, Q_2, Q_3)
Example	(-1,-4,5)	(-1,-1,1)	(1,4,5)	(2,1,3)	(1,2,3)
Information Extracted	All	Information 1	Information 2	Information 3	Information 4
Correlation	?	?	?	?	?

SOME NOTATIONS

$f(x)$: probability density function (pdf) of X

$F(x)$: cumulative distribution function (cdf) of X

$u(x)$: unit step function

σ_x^2 : Variance of X

$$m_x^{\pm} = \int_{-\infty}^{\infty} xf(x)u(\pm x)\,dx$$

$$G(y) = F(y) - F(-y)$$

$$g(y) = f(y) - f(-y)$$

Results

INTEGRAL FORMULAS AND LIMITS

	Integral formula	Property
$\rho_{X_iR_i}$	$\frac{\sqrt{3}}{\sigma_x}\int_{-\infty}^{\infty} x\{2F(x)-1\} f(x)\ dx$	$0 \le \rho_{X_iR_i} \le 1$
$\rho_{X_iQ_i}$	$\frac{\sqrt{3}}{\sigma_x}\int_{-\infty}^{\infty} x\{2G(x)-1\} f(x)\ dx$	$\rho_{X_iQ_i} \le \rho_{X_iR_i}$
$\rho_{\lvert X_i\rvert R_i}$	$\frac{\sqrt{3}}{\sqrt{\sigma_x^2+4m_x^+m_x^-}}\int_{-\infty}^{\infty} \lvert x\rvert\{2F(x)-1\} f(x)\ dx$	$\rho_{\lvert X_i\rvert R_i} \le \rho_{\lvert X_i\rvert Q_i}$
$\rho_{\lvert X_i\rvert Q_i}$	$\frac{\sqrt{3}}{\sqrt{\sigma_x^2+4m_x^+m_x^-}}\int_{-\infty}^{\infty} \lvert x\rvert\{2G(x)-1\} f(x)\ dx$	$0 \le \rho_{\lvert X_i\rvert Q_i} \le 1$
$\rho_{Z_iR_i}$	$\sqrt{3F(0)\{1-F(0)\}}$	$0 \le \rho_{Z_iR_i} \le \frac{\sqrt{3}}{2}$
$\rho_{Z_iQ_i}$	$\frac{\sqrt{3}}{\sqrt{F(0)\{1-F(0)\}}}\left\{F(0)-\int_{-\infty}^{\infty} F(-x)f(x)\ dx\right\}$	$\lvert\rho_{Z_iQ_i}\rvert \le \frac{\sqrt{3}}{2},\ \rho_{Z_iQ_i} \le \rho_{Z_iR_i}$
$\rho_{R_iQ_i}$	$\begin{cases} 1-2F(0), & F(0)=0,1, \\ 12\frac{n-1}{n+1}\left\{\int_{-\infty}^{\infty} G(\lvert x\rvert)F(x)f(x)\ dx-\frac{1}{4}\right\}, & 0<F(0)<1 \end{cases}$	$-1 \le \rho_{R_iQ_i} \le 1$

What distribution considered ?

Results

NUMERICAL EXAMPLES
From the integral formulas, the seven correlation coefficients have been obtained for the following distributions:

Uniform distribution:

$$f_U(x) = \frac{1}{b-a}\{u(x-a) - u(x-b)\}, \quad b > a$$

A zero-mean asymmetric distribution:

$$f_a(x) = \begin{cases} \frac{a}{a+1}, & -1 \le x < 0 \\ \frac{1}{a(a+1)}, & 0 \le x < a \\ 0, & \text{elsewhere} \end{cases}$$

Even Distributions: Gaussian, Laplace, and logistic distributions

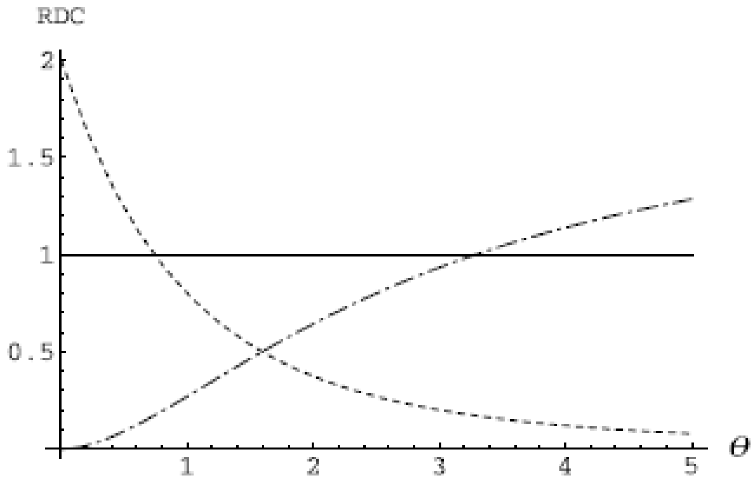
Unilateral distributions: Exponential, Rayleigh, gamma, and F distributions

Can you find any clue ?

APPLICATION EXAMPLE

Correlation between statistics and performance of the detectors using the statistics

Results



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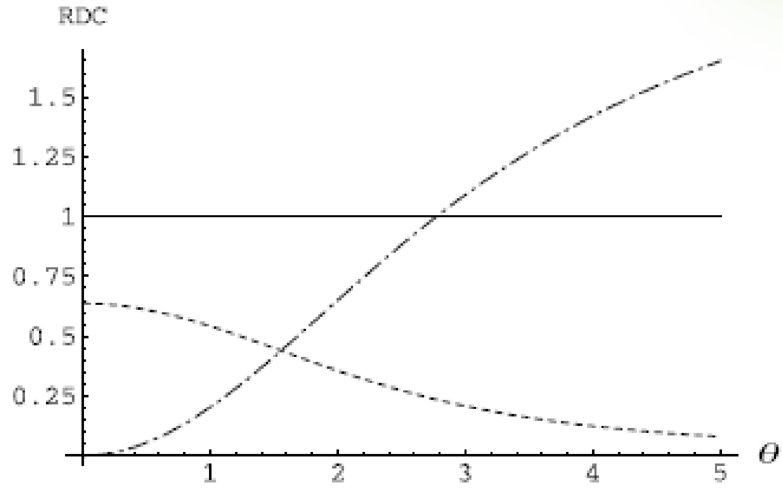

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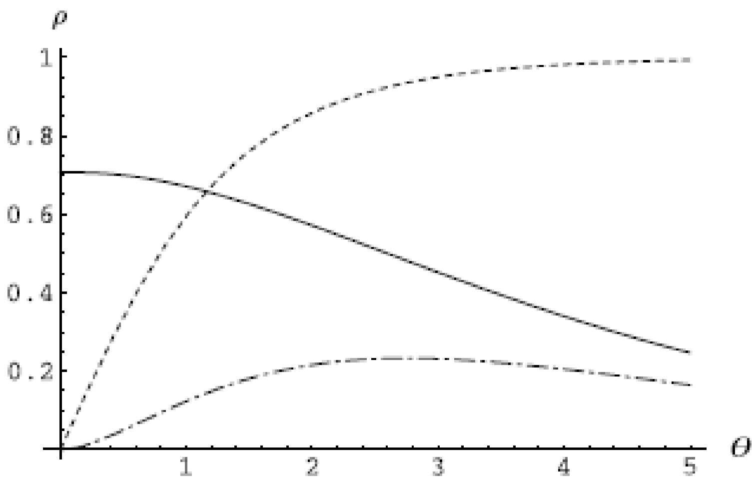

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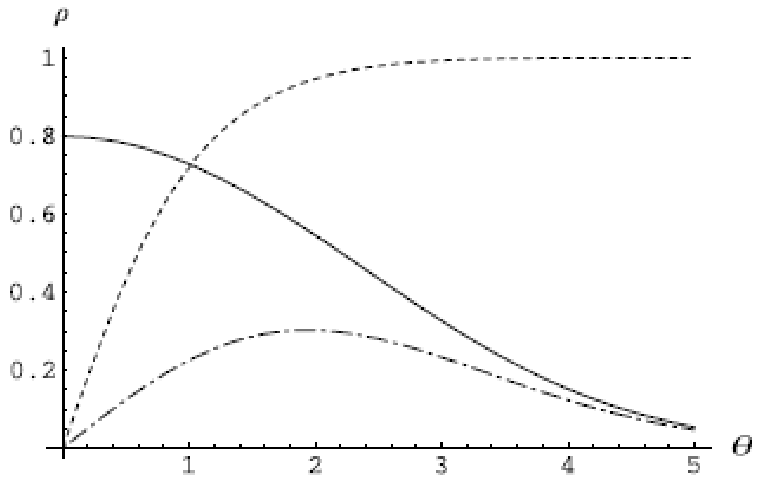

Fig. 5 Correlation coefficients for Gaussian noise. (solid: $\rho_{Z_iX_i}(\theta)$, dashed: $\rho_{|X_i|X_i}(\theta)$, dot-dashed: $\rho_{Z_i|X_i|}(\theta)$)

A PERFORMANCE MEASURE BASED ON CORRELATION PERFORMANCE

Shows relative performance of linear, sign, and magnitude detectors using correlation coefficients among the linear, sign, and magnitude statistics

$$M(\theta) = \frac{\theta^{-p} \rho_{Z_i|X_i|}(\theta)}{\rho_{Z_i X_i}(\theta) - \rho_{X_i|X_i|}(\theta)}, \qquad p \geq 0$$

SUMMARY OF RESULTS

¶ Closed formulas for correlation coefficients

We have 10 pairs of correlation coefficients in total. Three of them were thought to be trivial. So we concentrated on the rest of seven pairs. They are derived through the joint probability distributions among the statistics in i.i.d. random vectors, which are potentially applicable in detection theory.

¶ Lower and upper limits of the correlation coefficients

¶ Examples of distributions having the limits

¶ An outgrowth

The concept of independent and semi-identically distributed (i.s.i.d.) is introduced; All the elements have the identical distribution f(x) while the first element has another distribution h(x). It is useful to derive some probability functions related to ranks..

HOW TO HAVE THE CORRELATION COEFFICIENTS

Joint distribution, correlation coefficients, and limits

SUMMARY OF THE PRESENTATION

¶ Maximum utilization of information is necessary to enhance detector performance in a low power communication system.

¶ Among the first steps is to address the bivariate correlation of statistics to investigate the information characteristics of the statistics

¶ Explicit integral formulas for the correlation coefficients of seven pairs among five statistics commonly used in signal detection are derived and upper and lower limits are also evaluated.