Coalitions in Cooperative Networks

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Cooperation and Coalitions

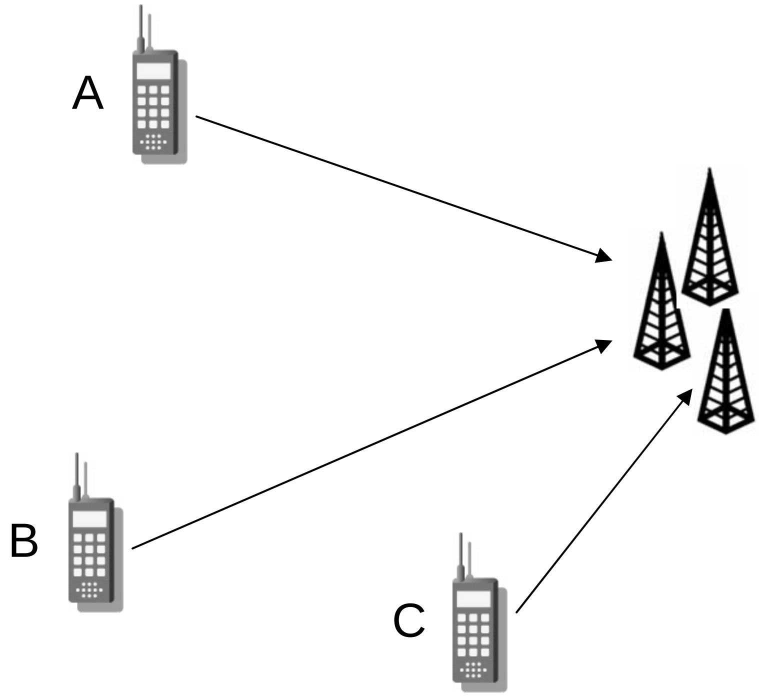
- Cooperative communications: users share power and bandwidth to mutually enhance transmissions
- Cooperation can achieve rate and diversity (spatial) gains
  - Rate gains from cooperative beamforming
  - When users cooperate, does it always result in gains for each cooperating user?

- Is there value in having all users cooperate?
  - Is it always possible?
  - Will users have incentives to break away (by themselves or as a coalition)?

- Can cooperative protocols induce the formation of disjoint groups of users (coalitions) that are stable?

Motivation: Example

- 3 users A, B and C communicating with their receivers (assume co-located)
- Receivers cooperate by jointly decoding their received signals.
- Within a coalition, sum-rate achieved is apportioned equally.
- What cooperative behavior emerges? (What coalitions form?)



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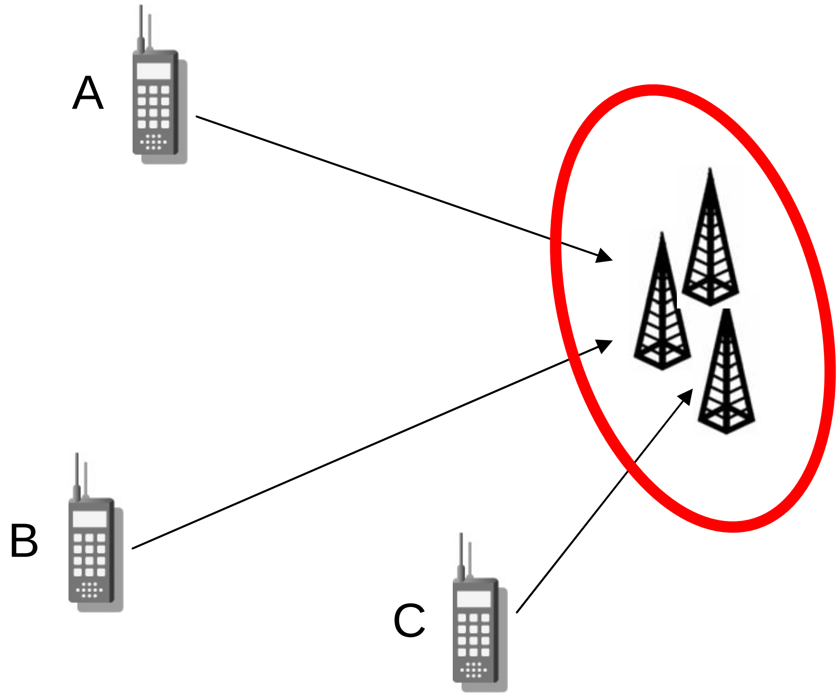
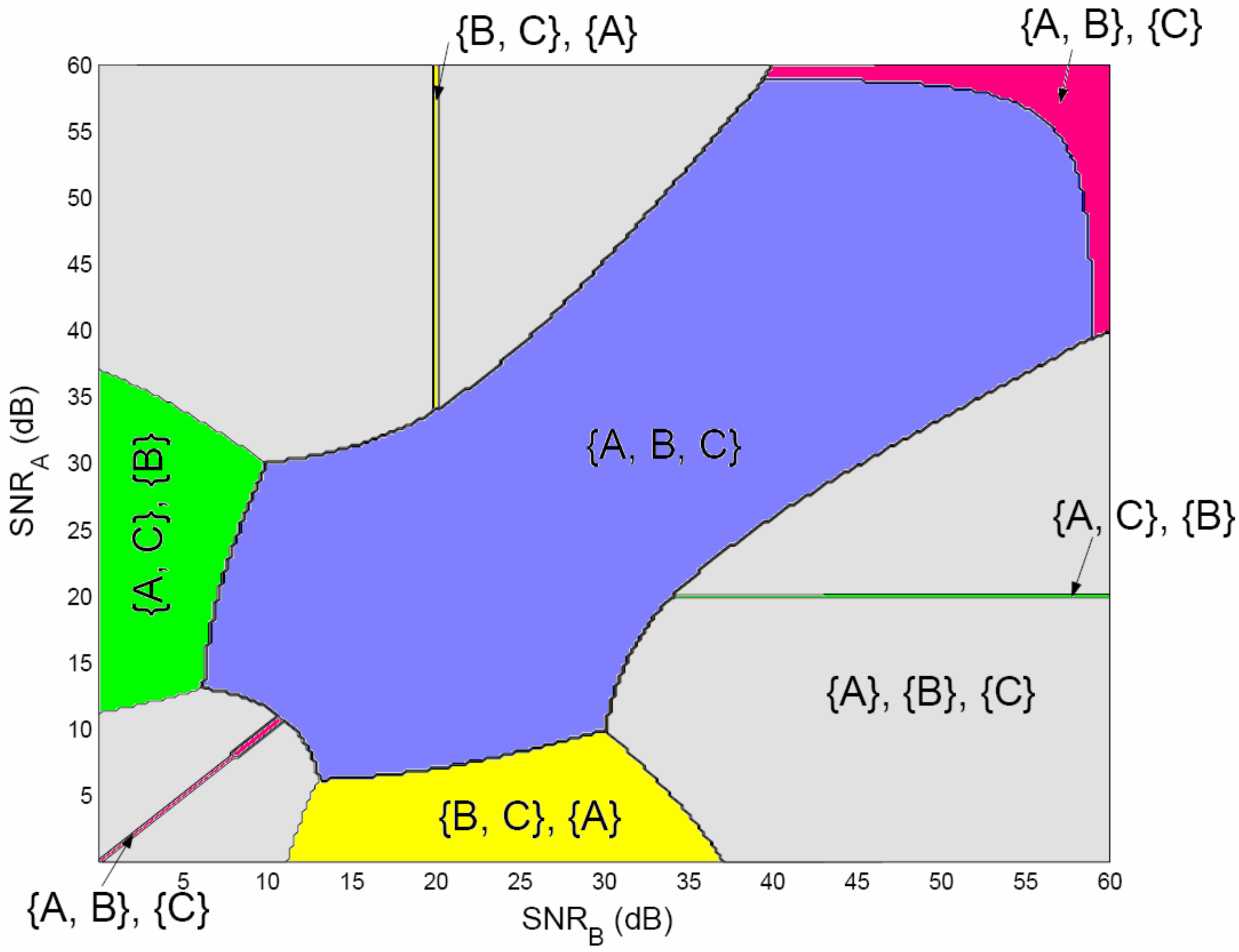


Example: Receiver Cooperation



- SNR of user C fixed at 20 dB
- Plot stable coalition structures.
- Equal rate splitting
- Grand coalition is not always stable
- Stable structure depends on apportioning scheme.

Outline of Talk

- Use coalitional game theory to determine the stable sum-rate optimal coalition structure
  - Receiver cooperation (joint decoding) in interference channel
  - Rx. cooperation with multiuser detection (MUD) in a MAC.
  - Ideal (noiseless) Tx. cooperation (and perfect Rx. cooperation) in an IC
  - Tx. cooperation via partial decode-and-forward in a MAC.
- Stable coalition: users have no incentives to leave the coalition
- Model utility of a coalition as its information-theoretic achievable sum-rate.
- Focus on the grand coalition (GC) of all users
  - stability depends on cooperative scheme, channel gains, and transmit power

Gaussian MAC [La-Anantharam,IT'2003]; Ad-hoc nets. [Zhu-Poor,2007]

Coalitional Games - Overview

- Coalitional games – users cooperate to form coalitions
- Characteristic function form (CFF) games
  - Utility achieved by any coalition is unaffected by the strategy of users outside the coalition
  - Allow tractable analysis of coalitions
  - Receiver coop.: CFF ; transmitter coop: not CFF
- CFF games are of two types
  - *Transferable utility (TU)*: total utility is arbitrarily apportioned between the coalition members subject to feasibility
  - *Non-transferable utility (NTU)*: limited set of allocations
  - TU vs. NTU: single value vs. a set of tuples required to express utility of coalition members

TU Games - Overview

- A coalitional game with transferable utility $\langle \mathcal{K}, v \rangle$
  - Finite set of players $\mathcal{K}$
  - Value function: $v : \mathcal{S} \rightarrow \mathcal{R}^{+} \ \forall \mathcal{S} \subseteq \mathcal{K}$
- Payoff: share of the value $v(\mathcal{S})$ to each player.
- Payoff vector: vector of values to players in a coalition
- Number of coalitional structures (set partitions) grow exponentially with number of players
  - Finding the stable structure NP-hard
- Games where grand coalition is stable are tractable and interesting
  - Two properties for GC stability – cohesive and superadditive

TU Games – Cohesive Property

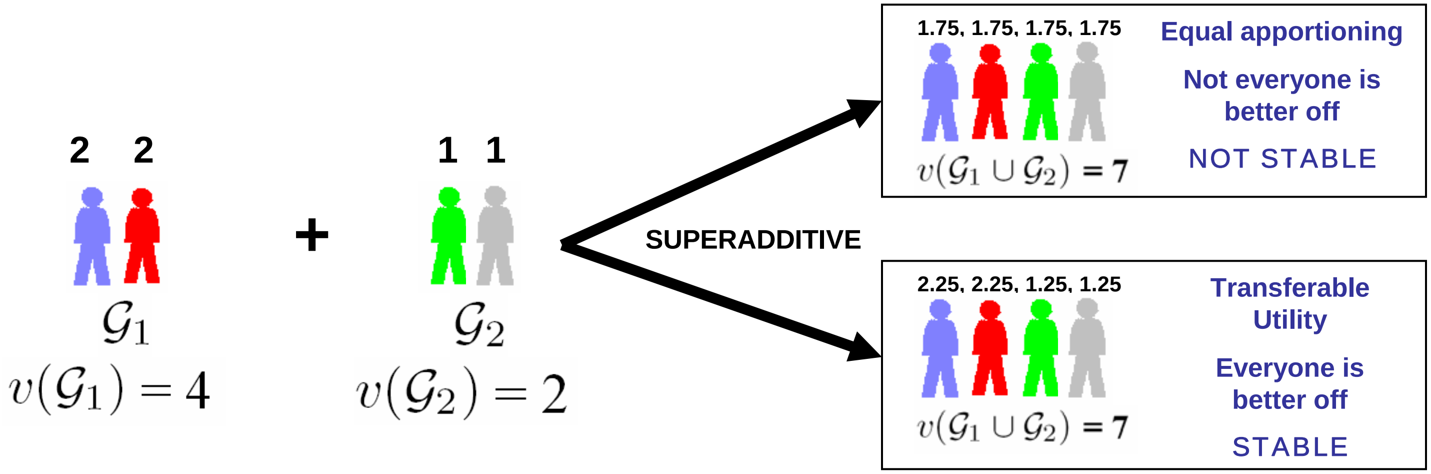
- A TU game is cohesive if for any partition $\{\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_N\}$ of $\mathcal{K}$

$$v(\mathcal{K}) \geq \sum_{n=1}^{N} v(\mathcal{S}_n)$$

- A TU game is superadditive if for disjoint subsets $\mathcal{G}_1, \mathcal{G}_2$:

$$v(\mathcal{G}_1 \cup \mathcal{G}_2) \geq v(\mathcal{G}_1) + v(\mathcal{G}_2)$$

- Superadditivity is a stronger condition



$\mathcal{G}_1$

$v(\mathcal{G}_1) = 4$

$\mathcal{G}_2$

$v(\mathcal{G}_2) = 2$

$v(\mathcal{G}_1 \cup \mathcal{G}_2) = 7$

Equal apportioning

Not everyone is better off

NOT STABLE

$v(\mathcal{G}_1 \cup \mathcal{G}_2) = 7$

Transferable Utility

Everyone is better off

STABLE

TU Games - Core

- Core: payoff vectors for coalition structures whose users have no incentives to leave
  - In general, NP-hard to find stable structures
  - Cohesive: grand coalition is the only candidate for the core

- For a cohesive TU game:

$$\mathcal{C}(v) = \left\{ \underset{\downarrow}{\underline{x}_{\mathcal{K}}} : \sum_{m\in\mathcal{S}} x_m \geq v(\mathcal{S}),\ \forall \mathcal{S}\subset\mathcal{K} \right\}$$

$|\mathcal{K}|$-length payoff (utility) vector achieved by the GC

- Existence of empty core ≡ feasibility of linear program

- Core can be empty → No stable form of cooperation

Core - Example

$$\mathcal{S} = \{1,2,3\}$$

$$v(\mathcal{S}) = 1$$

$$v(\{i\}) = 0, \; \forall i = 1,2,3.$$

$$v(\mathcal{G}) = \alpha, \; \forall |\mathcal{G}| = 2$$

$$0 < \alpha < 1$$

$$\begin{aligned}
R_1 &\geq v(\{1\}) = 0 \\
R_2 &\geq v(\{2\}) = 0 \\
R_3 &\geq v(\{3\}) = 0 \\
R_1 + R_2 &\geq v(\{1,2\}) = \alpha \\
R_2 + R_3 &\geq v(\{2,3\}) = \alpha \\
R_3 + R_1 &\geq v(\{3,1\}) = \alpha \\
R_1 + R_2 + R_3 &= v(\mathcal{S}) = 1
\end{aligned}$$

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Existence of a non-empty core ≡ *Feasibility of a linear program*

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R_1 + R_2 + R_3 &= v(\mathcal{S}) = 1
\end{aligned}$$

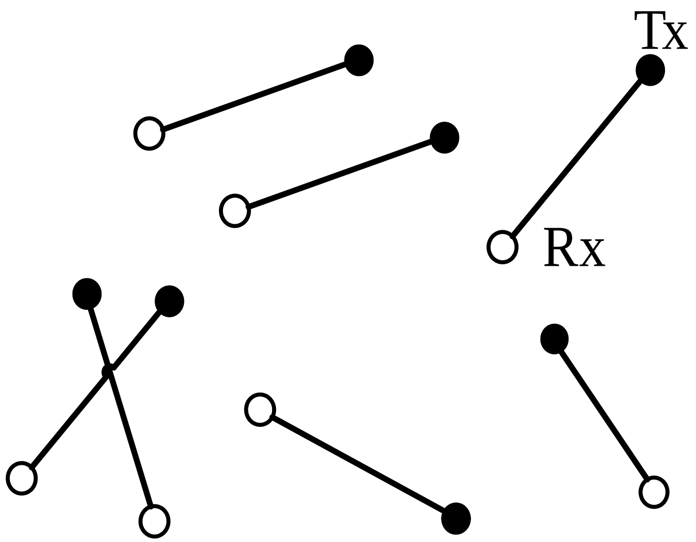
Existence of a non-empty core $\equiv$ *Feasibility of a linear program*

game is superadditive.

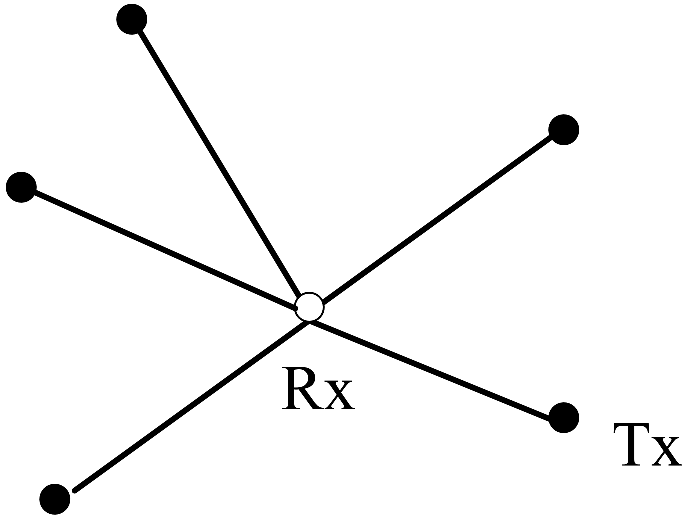
core will be non-empty only if $\alpha \leq \frac{2}{3}$

Cooperative Coalitions in IC & MAC

Channel Models : IC & MAC



Interference Channel



Multiaccess Channel

Channel Models : IC & MAC

- Additive white Gaussian noise and multiplicative gain
- IC: K transmit-receive links $\mathcal{K} = \{1, 2, \ldots, K\}$
- k^{th} link input/output: X_k, Y_k

$$Y_m = \sum_{k=1}^{K} h_{m,k} X_k + Z_m; \quad Z_m \sim \mathcal{CN}(0,1)$$

- $h_{m,k}$: fading gain between m^{th} xmitter and k^{th} receiver.
- Power constraint at each transmitter:

$$E\left|X_k\right|^2 \leq P_k$$

- MAC: All K transmitters transmit to one receiver.
- Transmitters use Gaussian codebooks
  - Codebook design depends on the cooperation (tx. or rx.) model

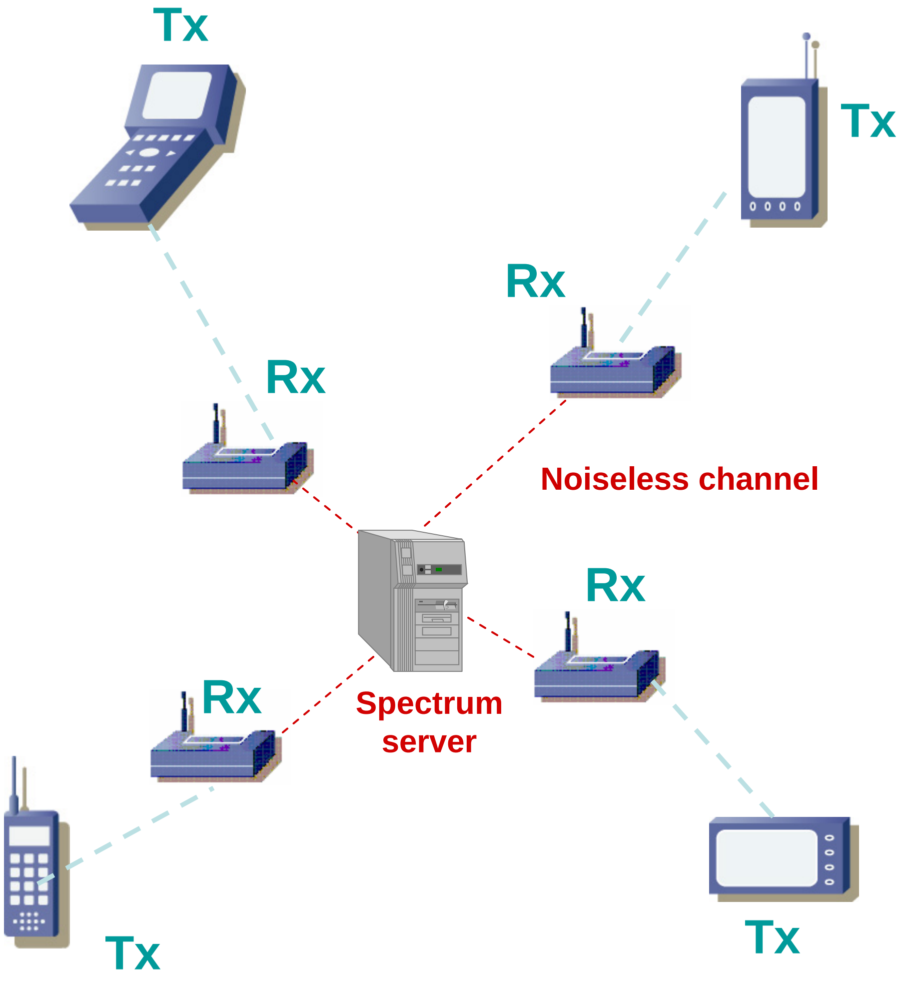
Receiver Cooperation in an IC

IC: Receiver Cooperation

- Interference Channel (IC): Network of K transmit-receive links [Carleial, IT-1978]

- When "different" types of devices/networks coexist, Tx cooperation may not always be possible.
  - Rx cooperation more desirable

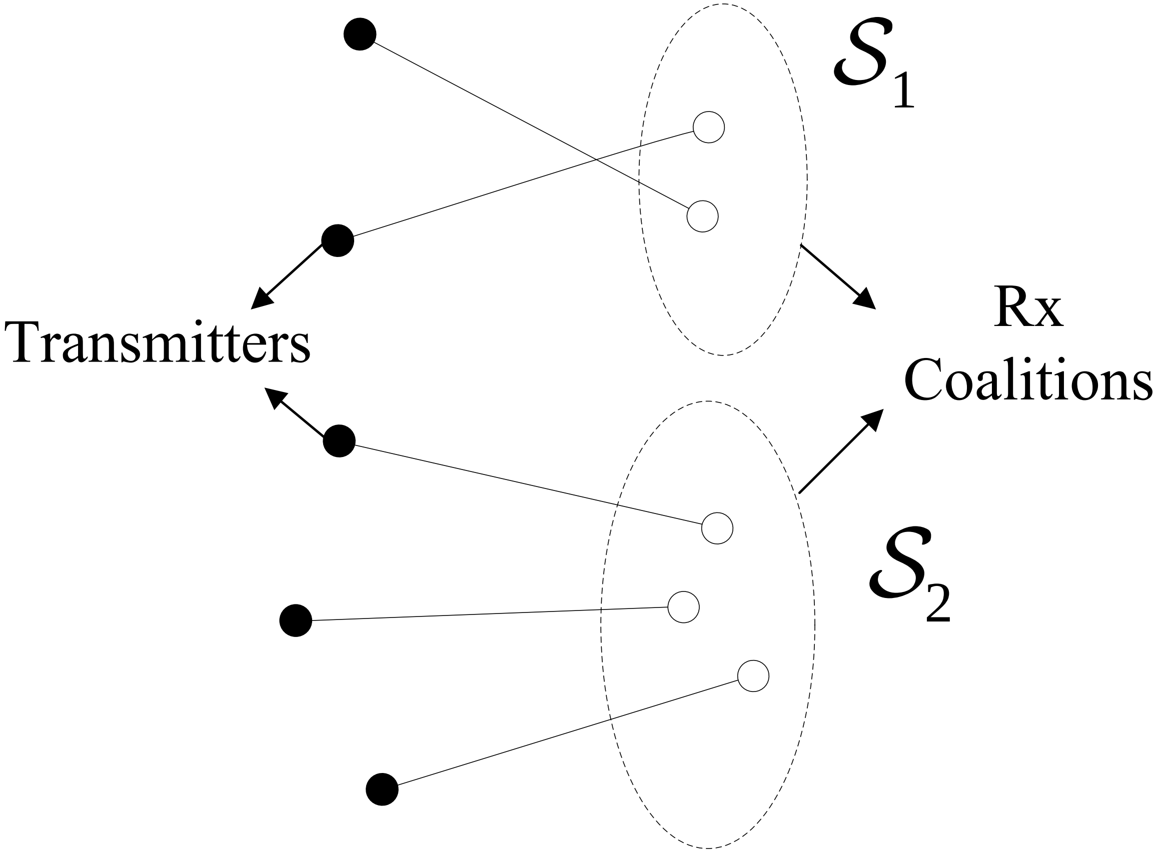
- Noise-free links with central entity/processor assumed (e.g. spectrum server)



IC: Rx Cooperation Game

- Receivers in a coalition jointly decode received signals.
- Model a cooperating coalition of links as a SIMO-MAC
  - Receivers in a coalition behave like a multi-antenna receiver
  - Independent transmitters using Gaussian codebooks
- Signals from links not in coalition treated as interference (noise) by the coalition
  - CFF game as transmitters independent
  - coalition rates independent of transmissions of links outside
- Model as coalitional game with transferable utility.
  - links in a coalition can share the sum rate flexibly.
- What are the optimal & stable coalition structures?

IC: Rx Cooperation Game



IC: Rx Cooperation Game

- A coalition $\mathcal{S}$ forms a Gaussian SIMO-MAC with $|\mathcal{S}|$-transmitters and a $|\mathcal{S}|$-antenna receiver
- $\underline{R}_{\mathcal{S}} = (R_k)_{k\in\mathcal{S}}$ is a vector of rates for links in $\mathcal{S}$.
- Value $v(\mathcal{S})$: maximum sum-rate achieved by links in $\mathcal{S}$

$$\boxed{v(\mathcal{S}) = \max_{\underline{R}_{\mathcal{S}}\in\mathcal{C}_{\mathcal{S}}} \sum\nolimits_{k\in\mathcal{S}} R_k = I(X_{\mathcal{S}};Y_{\mathcal{S}})}$$

- Capacity region $\mathcal{C}_{\mathcal{S}}$ of a $|\mathcal{S}|$-link Gaussian SIMO-MAC

$$\boxed{\mathcal{C}_{\mathcal{S}} = \left\{\underline{R}_{\mathcal{S}} : \sum\nolimits_{k\in\mathcal{A}} R_k \le I\left(X_{\mathcal{A}};Y_{\mathcal{S}} \mid X_{\mathcal{S}\backslash\mathcal{A}}\right);\ \ \forall\mathcal{A}\subseteq\mathcal{S}\right\}}$$

- Dominant face of $\mathcal{C}_{\mathcal{S}}$: $\boxed{D(\mathcal{S}) = \left\{\underline{R}_{\mathcal{S}} : \sum\nolimits_{k\in\mathcal{S}} R_k = v(\mathcal{S})\right\}}$

IC: Rx Cooperation Game

- Rx. cooperation game is superadditive
  - – follows from properties of mutual information
- TU game – all rate tuples on $D(\mathcal{S})$ feasible

Theorem : The grand coalition (coalition of all links) maximizes spectrum utilization for the receiver cooperation IC coalitional game.

Theorem : The core of the receiver cooperation IC coalitional game is non-empty. In fact, every point on the dominant $D(\mathcal{S})$ of the capacity region $\mathcal{C}_{\mathcal{K}}$ of the grand coalition belongs to the core.

- Core is non unique
  - –fairness of rate vector in the core?

Fair Rate Allocations

- With transferable utility, what is a fair allocation of rates to the links?
  - Can we attribute fairness criteria to points on the dominant face?

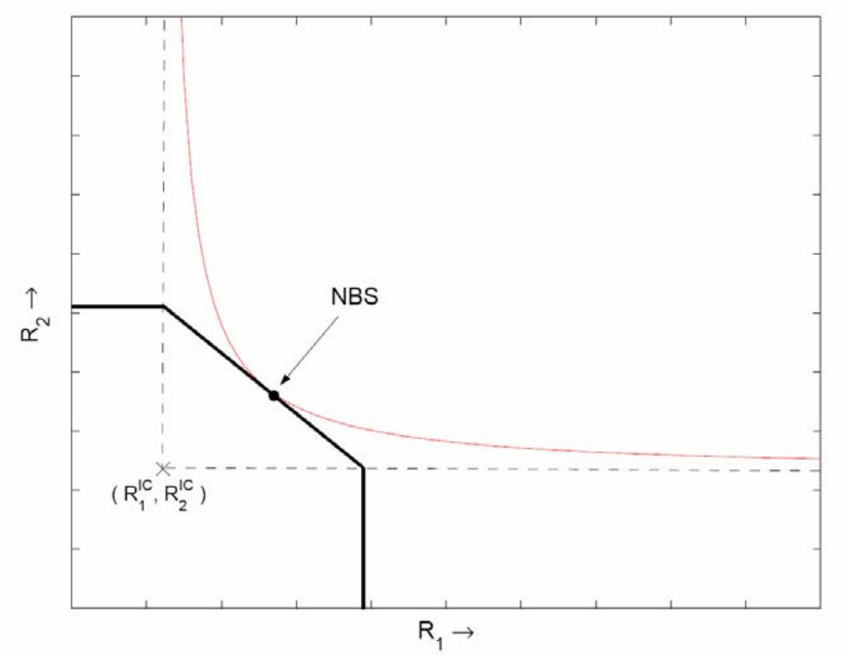
- Treat as a bargaining problem: two bargaining solutions proposed:
  - Nash bargaining solution (NBS) – gains over direct IC rates
  - Proportional Fairness (PF) solution

Nash Bargaining Solution (NBS)

- NBS: Maximizes the product of rate gains achieved by each link over its IC rates

$$\underline{R}_{\mathcal{K}}^{NBS} = \arg \max_{\{R_{\mathcal{S}}:R_k>R_k^{IC}\}} \prod_{k=1}^{K} (R_k - R_k^{IC}) \; ; \quad R_k^{IC} = I(X_k;Y_k)$$

- Properties of NBS:
  - Pareto optimal (max. sum-rate)
  - Symmetric (link label independent)
- Pareto optimality of NBS $\Rightarrow$ $\underline{R}_{\mathcal{K}}^{NBS} \in D(\mathcal{K})$
- Suffices to search for NBS on $D(\mathcal{K})$



Proportional Fairness Solution

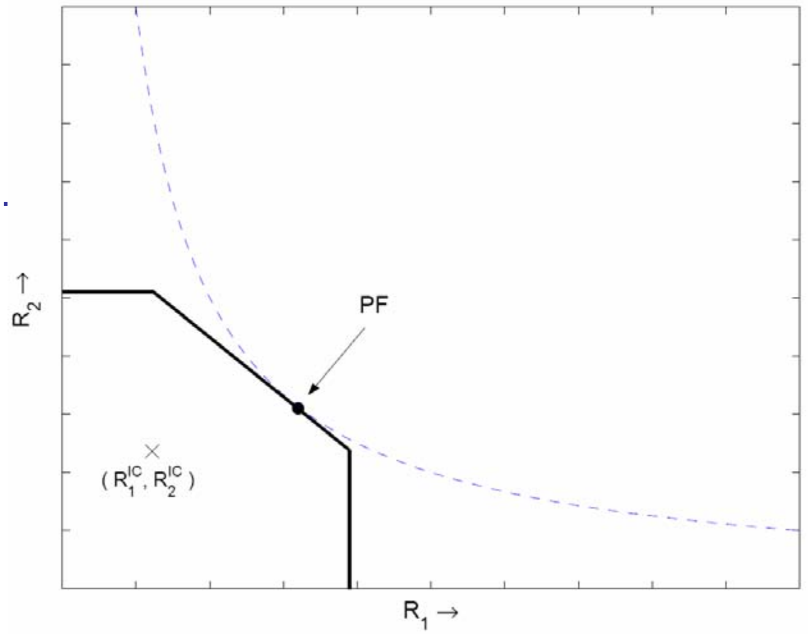
- An allocation of rates is a proportional fair solution *iff*

$$\sum_{k=1}^{K}(R_k - R_k^{PF})/R_k^{PF} \le 0 \Leftrightarrow \arg\max \sum_{k=1}^{K}\log R_k$$

- For the IC coalitional game, $\underline{R}_{\mathcal{K}}^{PF}$ simplifies as:

$$\underline{R}_{\mathcal{K}}^{PF} = \arg\max_{\underline{R}_{\mathcal{K}} \in \mathcal{C}_{\mathcal{K}}} \prod_{k=1}^{K} R_k$$

- PF solution is a special case of
  NBS
- Suffices to find PF solutions on $D(\mathcal{K})$



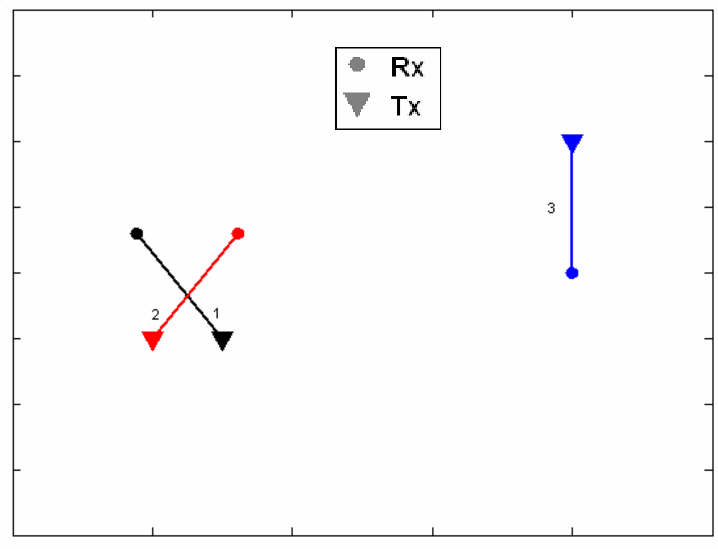
Rx Game – Illustration of Results

- Three-link IC with channel gains

$$h_{m,k} = \frac{A_{m,k}}{d_{m,k}^{\alpha/2}}, \quad \forall m,k \in \mathcal{K} = \{1,2,3\}, m \neq k$$

- path-loss exponent $\alpha = 3$
- Consider two network topologies
- For each topology, the transferable utility allocations of NBS and PF presented (GC sum-rate optimal)
- Also consider an equal rate (ER) strategy
  - Non-transferable utility strategy where value $v(\mathcal{S})$ split equally among the members of $\mathcal{S}$.

Topology 1

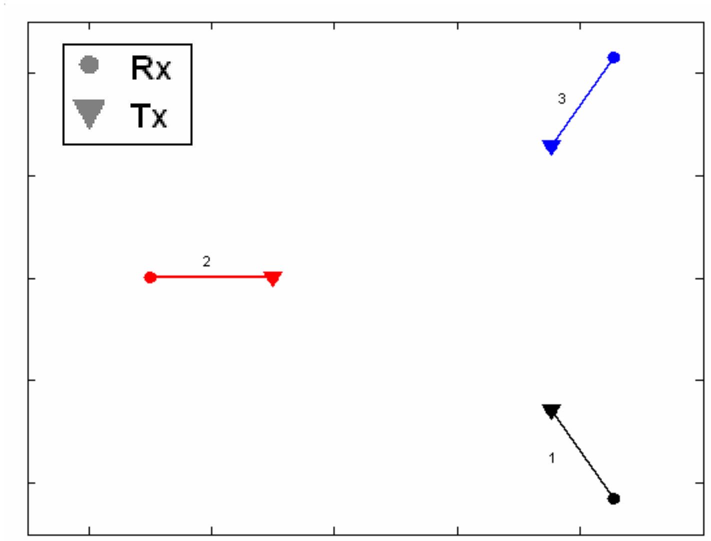


Coalition	R_1	R_2	R_3	Sum-rate
Transferable Utility Allocation Strategies				
$\{1,2,3\}_{NBS}$	1.4391	1.4346	1.0671	3.9408
$\{1,2,3\}_{PF}$	1.4372	1.4365	1.0671	3.9408
Non-transferable Utility Allocation Strategy (Equal Rate)				
{1,2,3}	1.3136	1.3136	1.3136	3.9408
{1,2},{3}	1.4174	1.4174	0.9355	3.7703
{2,3}, {1}	0.4170	0.2055	0.2055	0.8280
{3,1}, {2}	0.2115	0.4129	0.2115	0.8359
{1},{2},{3}	0.4170	0.4129	0.9355	1.7654
Stable ER Coalition: {1,2},{3}				

Topology 1

- NBS and PF : different sum-rate maximizing GC allocations
- Equal rate (ER) allocation:
  - GC NOT stable: 1 and 2 achieve better rates via {1,2} coalition though 3 prefers the GC (weak interference case)
  - ER tuple does not lie on $D(\mathcal{K})$ $\Rightarrow$ PF is not the equal rate tuple

Topology 2 (Perfect Symmetry)

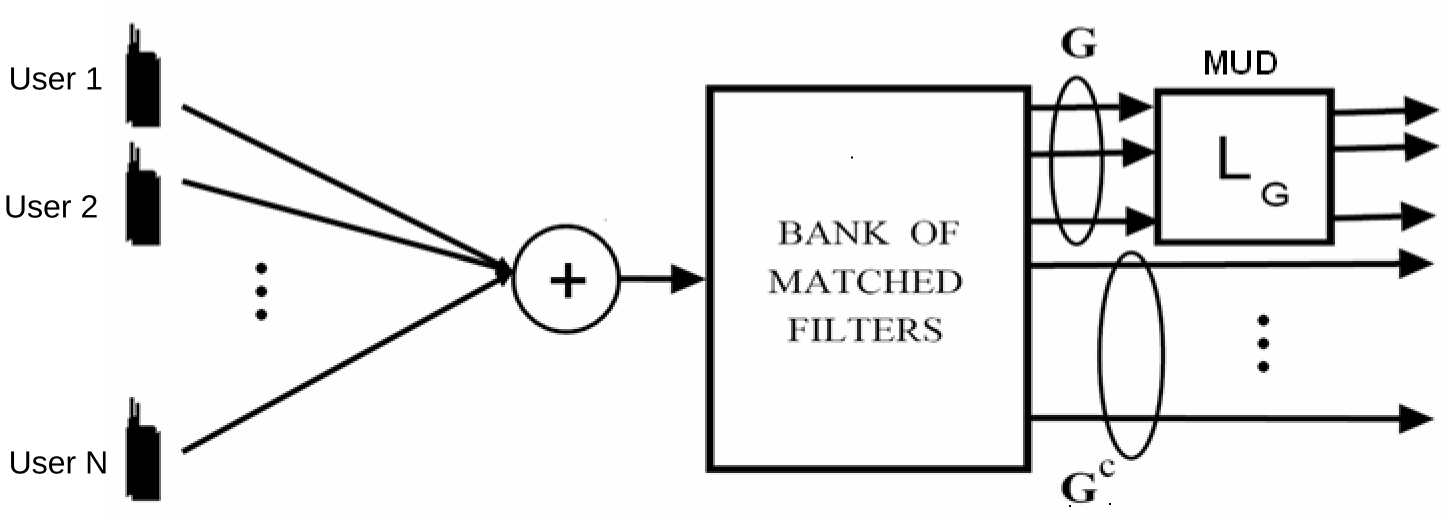


Coalition	R_1	R_2	R_3	Sum-rate
Transferable Utility Allocation Strategies				
$\{1,2,3\}_{NBS}$	0.9988	0.9988	0.9988	2.9964
$\{1,2,3\}_{PF}$	0.9988	0.9988	0.9988	2.9964
Non-transferable Utility Allocation Strategy (Equal Rate)				
{1,2,3}	0.9988	0.9988	0.9988	2.9964
{1,2},{3}	0.9671	0.9671	0.9673	2.9015
{2,3}, {1}	0.9673	0.9671	0.9671	2.9015
{3,1}, {2}	0.9671	0.9673	0.9671	2.9015
{1},{2},{3}	0.9673	0.9673	0.9673	2.9019
Stable ER Coalition: {1,2,3}				

- NBS, PF and ER lead to identical allocations
  - GC is sum rate maximizing and stable in all three cases

MAC: Receiver Cooperation via Multiuser Detectors

MAC : Coalitional Games for Linear MUD



- Linear multiuser detectors for receiver coalitions
- SINR achieved by a user in a coalition is its payoff
- Non-transferable utility game – fixed rates achieved
- Users within a coalition benefit from the interference suppression offered by their MUD

MMSE and Decorrelating Detector

- Decorrelating Detector:

Theorem: The grand coalition is stable and sum-rate maximizing in the high SNR regime for a decorrelating detector game.

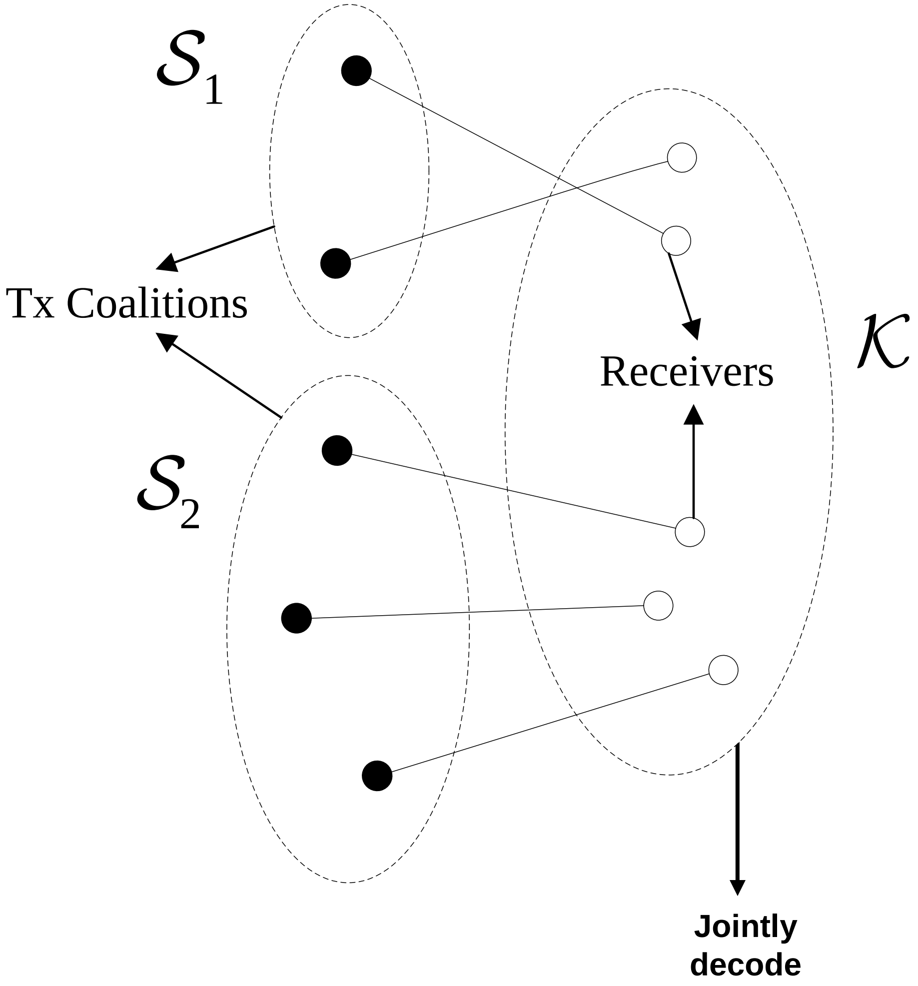
- MMSE:

Theorem: The grand coalition is always stable and sum-rate maximizing for a MMSE MAC coalitional game.

IC: Ideal Transmitter Cooperation

IC: Ideal Transmitter Cooperation

- Transmitter cooperation:
  - Through ideal noise-free inter-user links.
  - Cooperating transmitters encode jointly by optimally choosing their transmit covariance matrices.
- All K receivers jointly decode their recd. signals
  - K-antenna receiver
- Results in a MIMO MAC model



IC: Ideal Transmitter Cooperation

- Collection of N coalitions modeled as a MIMO-MAC.

$$\underline{Y}_{\mathcal{K}} = \sum_{n=1}^{N} H_{\mathcal{S}_n} \underline{X}_{\mathcal{S}_n} + Z_{\mathcal{K}}$$

$\searrow$ $K \times |\mathcal{S}_n|$ matrix

- MIMO-MAC with individual transmit power constraints.
  - Users in coalition $\mathcal{S}_n$ choose their covariance matrix $Q_{\mathcal{S}_n}$ subject to power constraints.
- Determine:
  - Coalitions that optimize spectrum use (maximize sum-rate)
  - Stable coalitions (belong to the core)

Transmitter Cooperation: Value

- The value of a coalition $\mathcal{S}$ given by the MIMO capacity

$$v(\mathcal{S}) = \max_{Q_{\mathcal{S}}:EX_k^2 \le P_k} I(X_{\mathcal{S}};Y_{\mathcal{K}})$$

- Value of coalition $\mathcal{S}$ depends on the actions of players outside the coalition (interference)
  - Tx cooperation game not of characteristic function form.
  - Difficult to analyze the game in the present form.

- Consider a jamming game
  - A coalition that breaks away experiences worse-case jamming interference from the remaining transmitters
  - Assumption of worst case signaling by non-coalition members
  - Quantifiable lower bound on the payoff of a break-away coalition

How convert to characteristic form?

- Model $v(\mathcal{S})$ to account for jamming from users in $\mathcal{S}^c$ as

$$v(\mathcal{S}) = \min_{Q_{\mathcal{S}^c}} \max_{Q_{\mathcal{S}}} \log\left(\frac{\left| I + H_{\mathcal{S}} Q_{\mathcal{S}} H_{\mathcal{S}}^{\dagger} + H_{\mathcal{S}^c} Q_{\mathcal{S}^c} H_{\mathcal{S}^c}^{\dagger} \right|}{\left| I + H_{\mathcal{S}^c} Q_{\mathcal{S}^c} H_{\mathcal{S}^c}^{\dagger} \right|} \right)$$

$$s.t. \qquad EX_k^2 \leq P_k$$

- A min-max optimization problem.
- Models a two-player mutual information game between coalitions $\mathcal{S}$ and $\mathcal{S}^c$.
- Saddle point in the mutual-information game between $\mathcal{S}$ and $\mathcal{S}^c$ [Diggavi & Cover, IT-2001]
  - $v(\mathcal{S})$ concave in $Q_{\mathcal{S}}$ and convex in $Q_{\mathcal{S}^c}$

IC: Tx. Cooperation Game

- Tx. Coop. game is cohesive (saddle point property).
- Game has TU (sum-rate = virtual MIMO capacity)
  - grand coalition is the only candidate for the core.
- Existence of a non-empty core?
  - Show existence of stable rate tuples
  - Analytical proof of stability difficult as stability requires comparing high-dimensional rate regions
- Counter examples show that the grand coalition cannot always guaranteed to be stable.
  - Indicates channel and power ranges for which a non-empty core can result

Tx. Cooperation: Example 1

- IC with 3 Tx – Rx links and $P_k = 1$ for all k

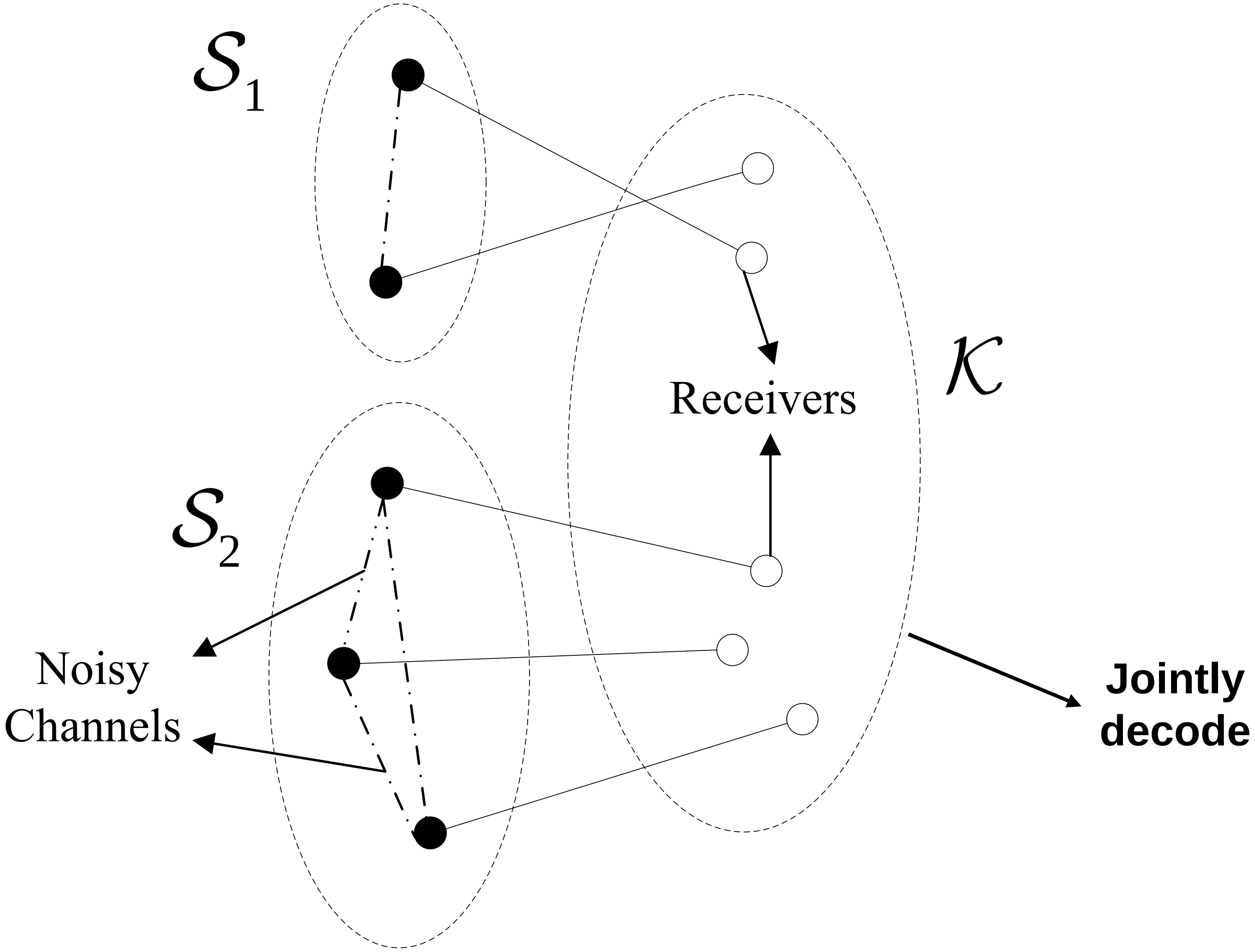
$$\mathbf{H} = \begin{array}{cl} \left(\begin{array}{ccc} 0.3019 & 0.3772 & 1.8021 \times 10^{-2} \\ 2.6256 \times 10^{-8} & 3.1413 \times 10^{-5} & 2.5662 \times 10^{-5} \\ 2.6893 \times 10^{-6} & 1.9941 \times 10^{-3} & 0.8502 \end{array}\right) & \begin{array}{l} \leftarrow \textbf{Rx 1} \\ \leftarrow \textbf{Rx 2} \\ \leftarrow \textbf{Rx 3} \end{array} \\ \begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ \textbf{Tx 1} & \textbf{Tx 2} & \textbf{Tx 3} \end{array} & \end{array}$$

  - All direct channels are strongest except for link 2
  - Link 3 has very little interference from links 1 and 2
  - Link 1 has significant interference from user 2

- Recall: Existence of a non-empty core ≡ feasibility of a LP.
- Infeasibility ⇒ Empty core ⇒ grand coalition not stable.
- No stable coalition exists.
  - Conjecture: non-empty core when all transmitters have comparable channel gains and transmit powers

MAC: Transmitter Cooperation
via
Partial Decode-and-Forward (PDF)

MAC: Partial Decode-and-Forward



MAC: Noisy Inter-User Links

- IC with perfect inter-user links: core is not guaranteed to be non-empty
  - Despite game being cohesive (GC is sum-rate optimal)

- Will relaxing the assumption of noise-free inter-user links change the stability of the core?
  - Consider a K-user MAC
  - Users decode-and-forward messages over noisy inter-user links
  - Model as a MAC with generalized feedback (MAC-GF) [Willems, IT'82] (full-duplex model)
  - K-user generalization assuming all users cooperate [Sankar, Kramer, Mandayam, ISIT'05]

[Sendonaris, Erkip, Aazhang, 2002]

MAC: Tx. Coop. Channel Model

- Consider a K-user clustered model
  - Inter-user channels stronger than user-destination channel
  - Simplest relaxation of the noise-free inter-user links assumption
  - Users more likely to cooperate to overcome poor direct channel

- Channel model for a coalition structure $\{\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_N\}$

$$\boxed{Y_d = \sum_{n=1}^{N} \underline{h}_{\mathcal{S}_n} \underline{X}_{\mathcal{S}_n} + Z_d}$$

- Clustered model:

$$\boxed{\left|h_{m,k}\right| > \left|h_{d,k}\right| \quad \forall m,k \in \mathcal{K}}$$

MAC: Partial Decode-and-Forward

- Transmitter cooperation:
  - Cooperating transmitter k splits power P_k between private, common, and cooperative messages as $(P_{k,d}, P_{k,c}, P_{k,u})$
  - Cooperating transmitters decode only common message
  - Destination decodes all messages

- PDF transmitter cooperation is not in characteristic function form (CFF)
  - rates achieved by a coalition are not independent of transmit strategies of users outside
  - Assume worst case jamming interference for a coalition that breaks away
  - Simplifies game to CFF form and allows tractable analysis

MAC: Partial Decode-and-Forward

- PDF Rate region for coalition $\mathcal{S}$:

$$\boxed{\mathcal{R}_{\mathcal{S}}^{PDF} = co\left(\bigcup_{\underline{P}} \mathcal{R}_{\mathcal{S}}(\underline{P})\right) \;\; ; \underline{P} = (P_{k,d}, P_{k,c}, P_{k,u})_{k\in\mathcal{S}}}$$

  - $\mathcal{R}_{\mathcal{S}}(\underline{P})$ is the PDF rate region achieved by a specific allocation vector $\underline{P}$ at all users.

- Each rate tuple on the hull achieved with different $\underline{P}$
  - Non-transferrable utility game – use a set of rate tuples (region) to characterize the value set $\mathcal{V}(\mathcal{S})$ of a coalition $\mathcal{S}$
  - To evaluate value need to determine the boundary of PDF region

PDF: Optimal Power Allocation

- What power allocations maximize regions $\mathcal{R}_{\mathcal{S}}^{PDF}$ for all $\mathcal{S}$?
- Two-user MAC-GF: [Kaya, Ulukus, TW-2007]
  - For a fixed $P_{k,u}$ (power for cooperation), user k (=$1,2$) allocates the remaining power to the better channel (direct or inter-user)
  - sets either $P_{k,d}$ or $P_{k,c}$ to zero, i.e., either common or private message
- For a K-user clustered MAC-GF:

Theorem: The optimal power allocation for K clustered users is $P_{k,d}$ = 0 and $P_{k,c} = P_k - P_{k,u}$.

  - Every rate tuple on the boundary is maximized when no private messages are sent (cooperating users decode all messages).

PDF Game: GC and Stability

- Is the NTU PDF game cohesive – are the rate regions of all other coalitions a subset of the GC region $\mathcal{R}_{\mathcal{K}}^{PDF}$?
- NTU game: value $\mathcal{V}(\mathcal{S})$ is the set of all rate vectors achieved by $\mathcal{S}$
- NTU game is cohesive when

$$\boxed{\bigcap_{n=1}^{N} \mathcal{V}(\mathcal{S}_n) \subseteq \mathcal{V}(\mathcal{K})}$$

- Core: no coalition $\mathcal{S} \subset \mathcal{K}$ achieves rate tuples that lie on or outside the GC rate region
- Proving cohesiveness not straightforward
  - Provide counter-example

PDF Game: Example

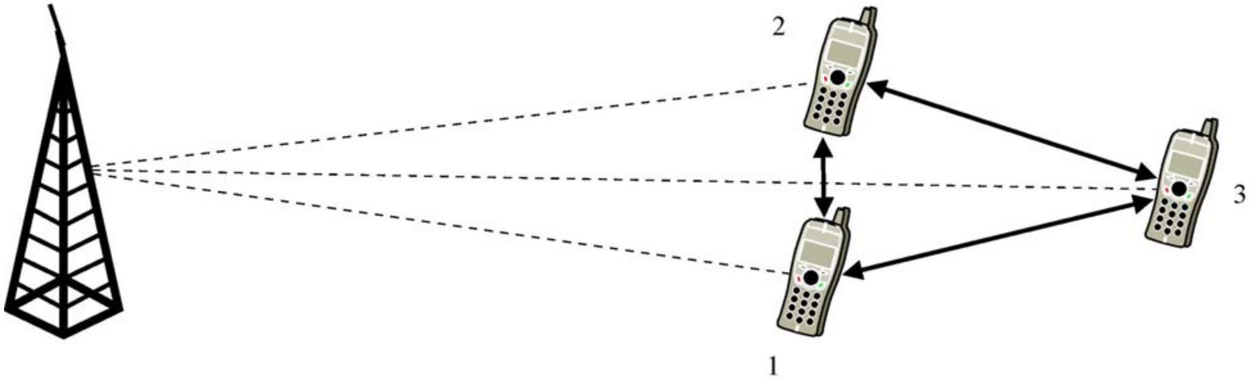
- Three-user MAC with

$$h_{d,1} = h_{d,2} = 0.05 \qquad h_{d,3} = 0.025$$

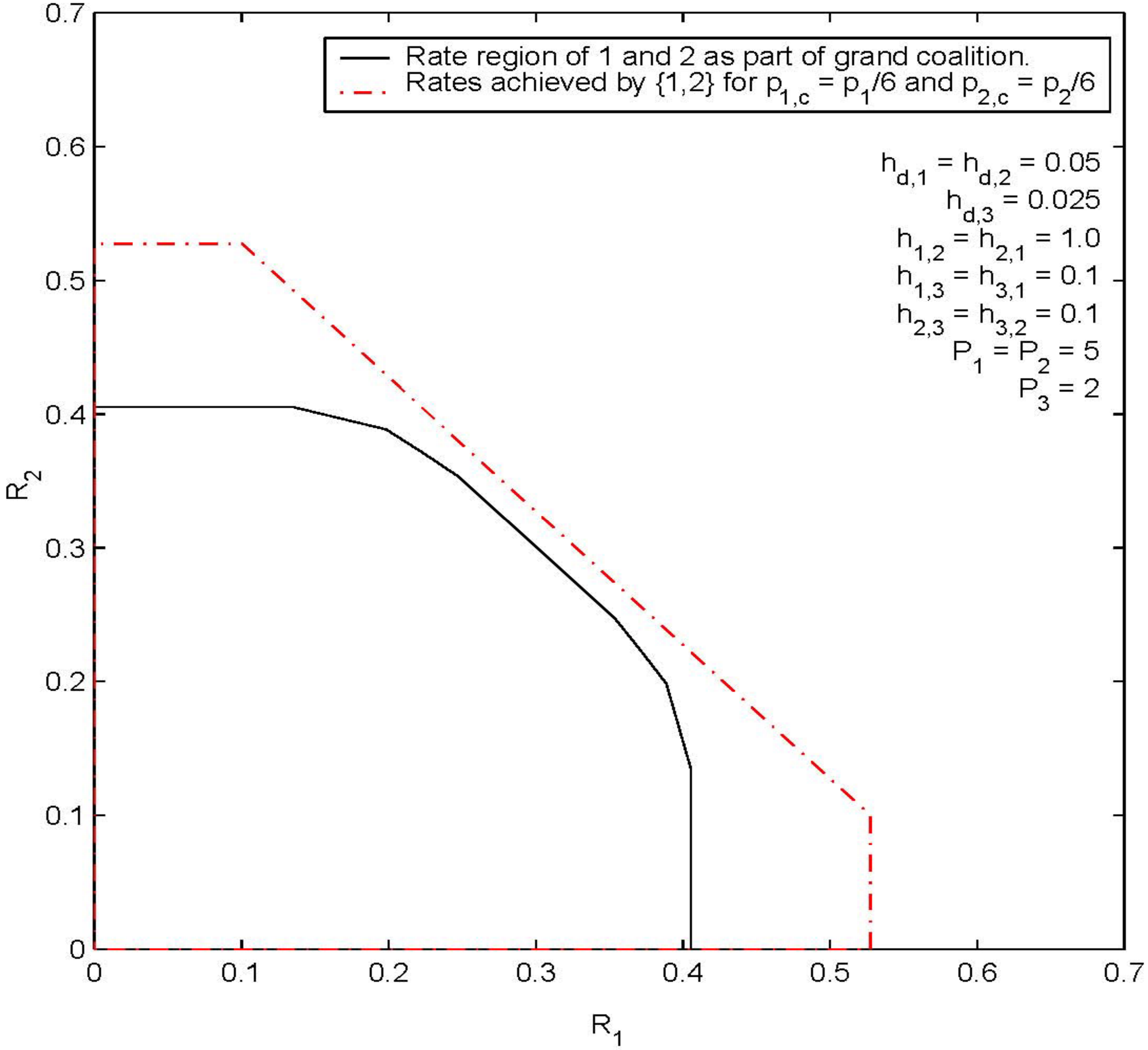
$$h_{1,2} = h_{2,1} = 1 \qquad h_{1,3} = h_{3,1} = h_{2,3} = h_{3,2} = 0.1$$

and $P_1 = P_2 = 5$ and $P_3 = 2$.

- Users 1,2 : strong inter-user channel and user 3 smaller transmit power



PDF Game: Example



Conclusions

- Cooperation in wireless networks can be studied using coalitional game theory.
- GC is the only candidate for the core in cohesive games.

- Ideal receiver cooperation is guaranteed to have a non-empty core.
  - bargaining theory can be used for ‘fair’ allocations.

- A non-empty core cannot be guaranteed for transmitter cooperation in general.
- Cohesiveness and stability depends on incentives and disincentives for cooperation
  - Noise enhancement in decorrelating detectors in low SNR regime
  - Channel gains and weak jammers in ideal tx. cooperation game
  - And noisy inter-user links in tx. PDF game

Related Papers

- S. Mathur, L. Sankar, N. Mandayam, “Coalitions in Cooperative Wireless Networks”, submitted to IEEE JSAC.
- S. Mathur, L. Sankar., N. Mandayam, “Coalitional Games in Gaussian Interference Channels”, Proc. IEEE ISIT, 2006.
- S. Mathur, L. Sankar., N. Mandayam, “Coalitional Games in Receiver Cooperation for Spectrum Sharing”, Proc. CISS 2006.
- S. Mathur, “Coalitional Games in Cooperative Networks”, MS Thesis. http://www.winlab.rutgers/edu/~suhas

Thank you