“Space-Time Adaptive Acquisition and Demodulation in Mobile Cellular”

by

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SPACE-TIME ADAPTIVE ACQUISITION AND DEMODULATION IN MOBILE CELLULAR

by

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Contents

List Of Tables .................................................. v
List Of Figures ................................................ vi
Abstract .......................................................... x

1 Introduction ......................................................... 1
   1.1 Motivation ................................................. 1
   1.2 CDMA and TDMA: Recent Related Research ................. 3
       1.2.1 CDMA Algorithms .................................... 3
       1.2.2 TDMA Algorithms .................................... 4
   1.3 Organization of the Dissertation ............................ 6

2 Preliminaries .................................................... 7
   2.1 Elementary Hypothesis Testing .............................. 7
   2.2 Likelihood Ratio Test ...................................... 9
   2.3 Criteria for Hypothesis Testing ........................... 10
       2.3.1 Bayes Criterion ...................................... 10
       2.3.2 Neyman-Pearson Criterion ........................... 12
   2.4 M Hypotheses (M-ary) ..................................... 14
   2.5 Composite Hypotheses ..................................... 15
       2.5.1 Random Parameter Vector ............................. 15
       2.5.2 Nonrandom Parameters ............................... 16
   2.6 The Wiener Filter ......................................... 18

3 An Asynchronous CDMA Receiver with Adaptive Beamforming
   and Nulling ..................................................... 21
   3.1 Introduction ................................................ 21
   3.2 Asynchronous DS-CDMA System Model ........................ 23
   3.3 Receiver Structure ......................................... 28
       3.3.1 Synchronization System ............................... 30
       3.3.2 DOA Acquisition System ............................. 43
4 Space-Time Adaptive Multistage Wiener Filtering for Asynchronous CDMA

4.1 Introduction .................................................. 60
4.2 Asynchronous DS-CDMA System Model .......................... 62
4.3 Receiver Structure ............................................. 63
  4.3.1 Test Statistic for Code-Timing Acquisition ............... 66
  4.3.2 Demodulator .................................................. 76
    4.3.2.1 Minimum Mean-Squared Error (MMSE) Detector .......... 76
    4.3.2.2 Maximum-Likelihood (ML) Detector ...................... 77
  4.3.3 Lower-Complexity Multistage Representation of The Test .... 80
  4.3.4 Parameter Estimation ...................................... 83
4.4 Adaptive Multistage Realization of The Test .................... 86
4.5 An Iterative Version of the Multistage Filter .................. 87
4.6 Numerical Results ............................................ 89
  4.6.1 DS-CDMA Scenario I and II .............................. 89
  4.6.2 DS-CDMA Scenario III .................................... 95
4.7 Conclusions .................................................. 96

5 A Cochannel TDMA Array Receiver

5.1 Introduction .................................................. 111
5.2 TDMA System Properties ...................................... 112
5.3 Cochannel TDMA System Model ................................ 113
5.4 Receiver Structure ............................................ 114
  5.4.1 Synchronization System .................................. 114
  5.4.2 DOAs Acquisition System ................................. 120
    5.4.2.1 A Constrained MLE of DOAs ............................ 120
    5.4.2.2 A Constrained MLE of DOAs with the ESPRIT Algorithm ... 121
  5.4.3 Beamforming and Equalization System ..................... 122
5.5 Numerical Results ............................................ 126
  5.5.1 IS-136 Scenario I ......................................... 127
  5.5.2 IS-136 Scenario II ....................................... 130
5.6 Conclusions ......................................................... 131

6 Conclusions and Future Studies ................................. 137
  6.1 Conclusions ..................................................... 137
  6.2 Future Studies .................................................. 139

Bibliography .......................................................... 139

Appendix A
  Gold Sequence Generator ........................................ 145

Appendix B
  ESPRIT ............................................................. 146

Appendix C
  Analysis of the Parameter ($\kappa^{-1}[\eta] \Delta_1[\eta]$) ............. 150
List Of Tables

3.1 Simulated channel parameters for DS-CDMA Scenarios I and II. 50
3.2 Simulated channel parameters for DS-CDMA Scenario III. 51
4.1 Training-Based Multistage Decomposition Algorithm. 90
4.2 Iterative Multistage Decomposition Algorithm. 91
4.3 Simulated channel parameters for DS-CDMA Scenarios I and II. 92
# List Of Figures

2.1 Components of a hypothesis testing problem ............................... 8
2.2 The classical Wiener filter .................................................. 18

3.1 Illustration of near-far problem .............................................. 23
3.2 A multipath channel model .................................................. 25
3.3 Construction of a typical DS signal, Illustrations (a) and (b) are parts of two information sequences, (c) and (d) are two 31-bit spreading waveforms, (e) and (f) are the two modulated waveforms to be transmitted, one for the desired user 1 and the other for the interfering user (user 2). Illustration (g) is the combined waveform of (e) and (f). Note that user 2 is assumed to be delayed by $\tau_2 - \tau_1 = 4T_c$ seconds with respect to user $1$ ........................................... 26
3.4 Conceptual block diagram of synchronization detector .................. 48
3.5 Synchronization results for DS-CDMA Scenario I, (Upper): odd-number of antenna elements, (Lower): even-number of antenna elements .................................................. 55
3.6 Synchronization results for DS-CDMA Scenario II, (Upper): odd-number of antenna elements, (Lower): even-number of antenna elements .................................................. 56
3.7 Demodulation results for (Upper): DS-CDMA Scenario I and (Lower): DS-CDMA Scenario II .................................................. 57
3.8 Synchronization results using even-number of antenna elements for DS-CDMA Scenario III under the condition of (Upper): stringent power control, (Lower): poor power control .................................................. 58
3.9 Synchronization results of the sample cross-correlation algorithm (Upper) and the proposed algorithm (Lower). (K=8, J=12, SNR=6dB, and MAI=14dB in a Near-Far Interference Environment, i.e., DS-CDMA Scenario III) .................................................. 59

4.1 Values of parameter $(\kappa_1^{-1}|i\Delta_1[i])$ as a function of the time phase $i$ for NFR = 3dB under various SNR values .................................................. 73
4.2 The structure of the test statistic .............................................. 75
4.3 Conceptual block diagram of the proposed receiver.  

4.4 Block diagram of the multistage decomposition of the likelihood ratio test for \( M = 4 \).  

4.5 The structure of the proposed MMSE receiver.  

4.6 Convergence dynamics of the steering vector of the proposed receiver implementation with system parameters \( J = 2, \ K = 6, \ M = 2, \ N = 31, \ \text{SNR} = 10\text{dB}, \ \text{and} \ \text{NFR} = 10^{F_{10}}, \end{array} \) where \( \Gamma_i \sim N(4,16) \).  

4.7 (Upper): The bit-error-rate performance of full rank vs. SNR parameterized by \( J \), for \( K = 6, \ N = 31, \ \text{and} \ \text{NFR is 0dB} \). (Lower): The bit-error-rate performance of full rank vs. SNR parameterized by \( J \), for \( K = 6, \ N = 31, \ \text{and} \ \text{NFR is 3dB} \), when the channel parameters of all users are known.  

4.8 The acquisition and BER performance vs. stages \( M \) parameterized by \( J \) for \( K = 6, \ L = 6JN, \ N = 31, \ \text{SNR} = 7\text{dB}, \ \text{and} \ \text{NFR is 3dB} \).  

4.9 Training-based algorithm in Table 4.1 with the parameters \( J = 2, \ K = 6, \ N = 31, \ \text{SNR} = 14\text{dB}, \ \text{and} \ \text{NFR is 0dB} \). (Upper): The probability of correct acquisition vs. the number of training data samples \( L \) parameterized by \( M \). (Lower): The probability of correct demodulation vs. the number of training data samples \( L \) parameterized by \( M \).  

4.10 Training-based algorithm in Table 4.1 with the parameters \( J = 2, \ K = 6, \ N = 31, \ \text{and} \ \text{SNR} = 14\text{dB} \). (Upper): The probability of correct demodulation vs. the number of training data samples \( L \) parameterized by \( M \) for NFR is 3dB. (Lower): The probability of correct demodulation vs. the number of training data samples \( L \) parameterized by \( M \) for NFR is 4.  

4.11 Training-based algorithm in Table 4.1 with the parameters \( J = 2, \ K = 6, \ M = 2, \ N = 31, \ \text{and} \ \text{NFR is 3dB} \). (Upper): The probability of correct acquisition vs. SNR parameterized by \( L \). (Lower): The bit-error-rate performance vs. SNR parameterized by \( J \).  

4.12 Training-based algorithm in Table 4.1 with the parameters \( K = 6, \ L = 6JN, \ M = 2, \ N = 31, \ \text{and} \ \text{NFR is 3dB} \). (Upper): The probability of correct acquisition vs. SNR parameterized by \( J \). (Lower): The bit-error-rate performance vs. SNR parameterized by \( J \).
4.13 Training-based algorithm in Table 4.1 with the parameters $L = 6(JN)$, $M = 2$, $N = 31$, SNR = 14dB, and NFR is 0dB, (Upper): The acquisition-error-rate performance vs. the number of users $K$ parameterized by $J$. (Lower): The bit-error-rate performance vs. the number of users $K$ parameterized by $J$.

4.14 Iterative algorithm in Table 4.2 with the parameters $J = 2$, $K = 6$, $M = 2$, $N = 31$, and NFR is 3dB, (Upper): The probability of correct acquisition vs. SNR parameterized by $\alpha$. (Lower): The bit-error-rate performance vs. SNR parameterized by $\alpha$.

4.15 Iterative algorithm in Table 4.2 with the parameters $\alpha = 1/(6JN)$, $K = 6$, $M = 2$, $N = 31$, and NFR is 3dB, (Upper): The probability of correct acquisition vs. SNR parameterized by $J$. (Lower): The bit-error-rate performance vs. SNR parameterized by $J$.

4.16 The acquisition and BER performance vs. stages $M$ parameterized by $J$ for $K = 6$, $L = 6JN$, $N = 31$, and SNR = 8dB.

4.17 Training-based algorithm in Table 4.1 with the parameters $J = 2$, $K = 6$, $M = 4$, and $N = 31$, (Upper): The acquisition-error-rate vs. SNR parameterized by $L$. (Lower): The bit-error-rate performance vs. SNR parameterized by $L$.

4.18 Training-based algorithm in Table 4.1 with the parameters $K = 6$, $L = 6JN$, $M = 4$, and $N = 31$, (Upper): The acquisition-error-rate vs. SNR parameterized by $J$. (Lower): The bit-error-rate performance vs. SNR parameterized by $J$.

5.1 Frame synchronizer, beamformer, and equalizer model.

5.2 A non-constrained MLE of DOA.

5.3 A constrained MLE of DOA with/without the ESPRIT algorithm.

5.4 IS-136 Scenario I with channel parameters.

5.5 IS-136 Scenario II with channel Parameters.

5.6 Frame synchronization results of (a): the sample cross-correlation algorithm, (b): the SQ algorithm, and (c): the adaptive GLRT algorithm for IS-136 Scenario I.

5.7 Signal constellation results of the adaptive GLRT algorithm using (a-c): the non-constrained MLE of the DOA, (d-f): the two MLEs of the DOA, and (g-i): the constrained MLE of the DOA along with the ESPRIT for bursts 1, 5, and 7 in IS-136 Scenario I.
5.8 The acquisition results of the impinging angles of (a): burst 1, (b): burst 5, and (c): burst 7 acquired by the constrained MLE of the DOA for IS-136 Scenario I. (d): Beampatterns (Gain in dB) of the two MLEs of the DOA for burst 5 over the interval of \([t_a, t_b]\) in IS-136 Scenario I. Burst 5 is the SOI whereas bursts 3, 4, and 6 are the cochannel interferers. ................................. 135

5.9 (a): Frame synchronization results of the adaptive GLRT algorithm for IS-136 Scenario II. (b): Zoom-in of (a): x-axis(10 190) and y-axis(0 250). ................................. 136
Abstract

A “smart” antenna receiver (receiving antenna array) that uses adaptive spatial-temporal signal processing is being made available to a future generation of mobile radio and phone systems. The spatial locations of the system users can be exploited by an adaptive array receiver that is capable of spatially suppressing multiple-access interference (MAI) that occurs in mobile communications. As a consequence adaptive antenna arrays can be made to achieve a better quality of service and a significant increase of capacity.

In this dissertation adaptive self-synchronizing receivers are developed for asynchronous DS-CDMA (direct-sequence code-division multiple-access) systems and cochannel TDMA (time-division multiple-access) systems with such a smart J-element antenna array. In this new approach a generalized maximum likelihood ratio test (GLRT) is formulated to automatically adaptively acquire both synchronization and direction-of-arrival (DOA) in both asynchronous DS-CDMA systems and cochannel TDMA systems. The primary requirement is knowledge of the desired signature code sequences. However, its full direct realization and implementation is not feasible at the present due to both a computationally intensive matrix-inversion and an eigen-decomposition operation.

A considerably lower complexity version of such an asynchronous DS-CDMA receiver is developed here that utilizes the concept of the multistage reduced-rank Wiener filter system, introduced recently by Goldstein and Reed. This new technique results in a self-synchronizing detection criterion that requires no matrix
inversions or eigen-decomposition of a covariance matrix. Also this multistage adaptive filtering scheme achieves a rapid adaptive convergence under limited observation-data support. These important features contribute significantly to a reduction of the computational cost and the amount of data sample support needed to accurately obtain the required estimates of the problem.
Chapter 1

Introduction

1.1 Motivation

Wireless communications for mobile cellular is currently undergoing a very rapid development. Many of the emerging wireless systems incorporate a considerable number of signal processing techniques in order to provide wireless transmission service. To make optimal use of available bandwidth and to provide maximal flexibility many wireless systems operate in a multiple-access mode in which the channel bandwidth is shared by a system of users on a random-access basis. The most widely used multiple-access schemes in current mobile communications are code-division multiple-access (CDMA) and time-division multiple-access (TDMA).

Recently, direct-sequence (DS) CDMA has become one of the most popular multiple-access techniques for mobile radio and phone systems. In DS-CDMA communications, users are multiplexed by distinct code waveforms, rather than by the orthogonal frequency bands that are used in frequency-division multiple-access (FDMA), or by orthogonal time slots in TDMA. In DS-CDMA all system users are allowed to transmit information simultaneously and to occupy the same broad-band frequency spectrum. However, DS-CDMA systems often suffer from
multiple-access interference (MAI) that is associated with the near-far problem. One way to alleviate the near-far problem is to use power control schemes. However, power control is not easy to accomplish, especially in a wireless environment, where the power levels often vary dramatically as a function of time. Moreover, the need for power control may incur an excessive complexity to the system. Even if perfect power control is used, the system still needs to be implemented with a powerful forward error correction (FEC) code to mitigate this problem. Another way of handling the near-far problem is to use near-far resistant receivers which, in principle, can yield a substantial improvement over the receiver with perfect power control.

Code-timing synchronization of multiple-access signals is another important issue that deserves increased attention. Most previous receivers for mobile radio systems use detection systems that require precise knowledge of the propagation delays of all users that are usually not known \textit{a priori}. To use such algorithms the propagation delays must be estimated, and as a consequence such receivers suffer from complexity and errors that occur with the estimation of the propagation delays [30]. These errors reduce the capability of such a receiver to adequately establish code acquisition and demodulation.

The incorporation of adaptive-array antennas in cellular systems to mitigate MAI, time dispersion, and multipath fading that occur in mobile communications presently attracts considerable attention. This is due to the fact that the base stations are being equipped with a number of antenna elements, i.e., an array of antenna elements. A \( J \)-element array antenna is known to be able to perform beamforming with \( J - 1 \) degrees of freedom to control the directions of \( J - 1 \) nulls of the antenna. As a consequence an adaptive array receiver can adaptively provide a substantial increase in the output signal-to-interference-plus-noise ratio (SINR) and system capacity by nulling a possible \( J - 1 \) co-existing (interfering) users. However, most array-receiver algorithms suffer from a computational
burden which is caused by the requirement of a covariance-matrix inversion or an eigen-decomposition. To remedy this situation the reduced-rank multistage Wiener filter of Goldstein and Reed [11] is utilized to reduce this requirement. This new technique obviates the necessity of either a covariance matrix inversion or an eigen-decomposition. This makes practical the realization of an adaptive array receiver to mitigate MAI due to both the reduction of the computational cost and the amount of data samples needed to estimate a covariance matrix. Hence, a primary objective of this thesis is to develop a \( J \)-element self-synchronizing near-far-resistant array-antenna receiver that utilizes the concept of the multistage reduced-rank Wiener filter for use in mobile communications.

\section*{1.2 CDMA and TDMA: Recent Related Research}

\subsection*{1.2.1 CDMA Algorithms}

Some receiver algorithms for DS-CDMA systems are analyzed briefly in what follows:

In 1986 Verdú [52] proposed the optimal multiuser detection criterion and maximum-likelihood sequence detection for DS-CDMA signals. Unfortunately, Verdú's methods are generally too complicated to realize in a practical DS-CDMA system. As a consequence a number of simplified detector-demodulation schemes have been proposed to obtain a near optimum performance with a reduced computational complexity [24, 25, 54, 26, 14, 21]. These receivers, however, deal primarily only with detectors that required a precise knowledge of the propagation delays of all users, which is usually unknown \textit{a priori}.

Conventional methods of code acquisition [35, 32] often utilize a tapped delay line with its weights being the elements of the spreading code sequence, i.e., a
matched filter. However, such a receiver can suffer severely from near-far problems. To alleviate this some near-far-resistant timing-acquisition algorithms have been considered. In particular, proposed in [47] is an important code timing acquisition algorithm, which is based on the minimum-mean-squared-error (MMSE) criterion. However, an all-ones training sequence is required for it to work. In [3] a maximum-likelihood (ML) synchronization for a single user is developed. But the method presented in [3] again requires a training period. Subspace-based code-timing estimators that use a single antenna element are presented in [2, 48, 49]. However, these timing estimators involve extensive computations due to the need for an eigen-decomposition and the knowledge of the number of active users. In [23, 43, 44] multiple-element antenna algorithms that utilize large-sample maximum-likelihood (LSML) estimation and multiple signal classification (MUSIC) in order to perform code-timing acquisition over a fading channel. These two systems are very computational expensive and thus of limited practical use.

In [21, 16, 15] an adaptive training-based or blind multistage Wiener filtering algorithm [11] with the requirement of a priori synchronization (i.e., a priori knowledge of timing) is applied to implement the linear MMSE receiver equipped with a single antenna element for asynchronous DS-CDMA systems. More recently, an adaptive receiver equipped with a single antenna for an asynchronous CDMA system is proposed in [50] that does not require prior synchronization. Also a lower-complexity version of this receiver is proposed in [50] that is based on the concept of the reduced-rank multistage decomposition.

1.2.2 TDMA Algorithms

Some receiver algorithms for TDMA systems are described briefly as follows:
A sample cross-correlation approach can fail to establish reliable synchronization with code sequences in cochannel TDMA when severe cochannel interference (CCI) occurs. To improve this situation an algorithm that uses a recursive least squares (RLS) channel estimator and the Viterbi algorithm (VA) to jointly recover the two cochannel symbol streams is proposed in [57]. However, how to deal with asynchronous cochannel bursts is not described explicitly in [57]. The problem of asynchronous TDMA cochannel bursts is dealt with in [34]; however, the cochannel bursts are assumed to be precisely time aligned in both their analysis and simulations. In [22] the asynchronous nature of cochannel TDMA signals is described again, but the algorithm of how to acquire frame synchronization is neglected. The receivers in [29] employ a joint maximum-likelihood sequence estimation (JMLSE) of the cochannel signals. However, the computational complexity of such a system is quite intensive.

The algorithm in [20] is one of the first attempts to employ an adaptive array beamformer for the separation of asynchronous TDMA bursts. In this case additional synchronizing code sequences are used to achieve frame synchronization using a least-squares (LS) algorithm. This approach is developed further in [8, 18]. The modified sequential beamforming (MSB) algorithm presented in [8] and the sequential separation (SQ) algorithm presented in [18] also employ the LS algorithm to achieve the frame synchronization needed to find the weights for partial beamforming. To achieve complete beamforming two passes are required due to the lack of information about the direction-of-arrivals (DOAs). Such a two-pass beamforming criterion is adopted again in [9] with a subspace method that replaces the LS method. Both approaches process data in a non-causal manner that complicates the problems of data management and processing period.
1.3 Organization of the Dissertation

This thesis is organized as follows: Chapter 2 provides a brief review of the fundamental concepts of hypothesis testing theory and the classical Wiener filter. In Chapter 3 a full-rank DS-CDMA array receiver is developed which assumes no prior knowledge of synchronization. Chapter 4 provides a low complexity version of an adaptive self-synchronizing DS-CDMA receiver with a \( J \)-element antenna array that utilizes the concept of the multistage reduced-rank Wiener filter. A cochannel TDMA receiver that uses an antenna array is proposed in Chapter 5. Finally, conclusions and future possible studies are presented in Chapter 6.
Chapter 2

Preliminaries

In this chapter some technical background and preliminary work are presented and discussed. Briefly, this dissertation formulates the detection of the target signal as the hypothesis testing problem needed to distinguish signal from noise, i.e., whether there is a target signal present or not. In Sections 2.1 to 2.5 of this chapter the likelihood ratio test is shown to be the optimal decision rule according to both the Bayes and Neyman-Pearson criteria. In Section 2.6 the optimal linear filter is developed in terms of minimizing the mean-square value of the error between correlated random processes.

2.1 Elementary Hypothesis Testing

In digital binary communications the received signal is processed in order to determine whether a “0” or “1” is present. In radar or sonar the presence or absence of a target is determined from measurements made in the field of propagation. For example in the seismic problem the presence of an oil deposits is inferred from measurements of very-low-frequency sound propagated in the earth. In this thesis detection theory and signal processing algorithms are developed to explore the answers to similar questions when the information-bearing signals are corrupted by interference and other noise signals. The first part of this chapter
Figure 2.1: Components of a hypothesis testing problem.

serves mainly as a review of that part of statistical-decision theory [17, 51] that
can be used to solve the signal detection problems that arise in mobile radio.

The foundations of statistical-decision theory rest on statistical hypothesis
testing. The basic elements of the hypothesis testing problem include: (1) a set
of hypotheses that characterize the possible outputs of a source, (2) a probabilis-
tic mapping from the hypothesis space (signal space) to the observation space,
(3) a decision rule to assign each point in the observation space to one of the
hypotheses in an optimal fashion. The purpose of hypothesis testing is to deter-
determine which of hypotheses is the most “consistent” with a set of the observed
data. Hence, decision criteria need to be found to minimize the probability of
making an erroneous decision. The components of a simple hypothesis testing
problem are shown in Figure 2.1.

In the simple binary hypothesis problem each of two source outputs corre-
sponds to one of two hypotheses, denoted by \( H_0 \) and \( H_1 \). Each time an experi-
ment is conducted only one of four possible outcomes can happen:

1. \( H_0 \) true; choose \( H_0 \).
2. \( H_0 \) true; choose \( H_1 \).
3. \( H_1 \) true; choose \( H_1 \).
4. $H_1$ true; choose $H_0$.

The first and third alternatives correspond to correct choices; the second and fourth alternatives correspond to errors. The decision procedure operates by segmenting the observation space, denoted by $Z$, into two disjoint decision regions $Z_0$ and $Z_1$, i.e., $Z = Z_0 + Z_1$, where “+” denote the direct sum or union of $Z_0$ and $Z_1$. That is, all values of the observed data $x$ fall into either $Z_0$ or $Z_1$. If a given $x$ lies in $Z_0$, $H_0$ is announced, otherwise $H_1$ is proclaimed. To determine which hypothesis best describes the observations an optimal decision criterion is desirable.

2.2 Likelihood Ratio Test

**Definition 1:** The likelihood function $L(H_i|x)$ of the hypothesis $H_i$ given the observed data $x$ and a specific probability model is defined to be $cP(x|H_i)$, the constant $c$ of proportionality is a fixed arbitrary constant; i.e.,

$$L(H_i|x) = cP(x|H_i).$$

**Definition 2:** The likelihood ratio of two hypotheses on the same observed data $x$ is defined to be the ratio of the likelihoods on the observed data. Let $\Lambda_i(x)$ denote likelihood ratio, then

$$\Lambda_i(x) = \frac{L(H_i|x)}{L(H_0|x)} = \frac{P(x|H_i)}{P(x|H_0)}.$$  

(2.1)
2.3 Criteria for Hypothesis Testing

2.3.1 Bayes Criterion

A Bayes test is based on two assumptions. The first assumption is that the source outputs are governed by probability assignments, called a priori probabilities and denoted by $P_i$, where the subscript "i" corresponds to the i-th hypothesis. The second assumption is that a cost function is assigned to each possible action. Let $C_{i,j} \triangleq C(\hat{H}_i|H_j)$ denote the cost of making the decision $\hat{H}_i$ when $H_j$ is true. Typically, the cost of mistaking hypothesis $\hat{H}_i$ for $H_j$, namely, $C_{i,j}$ for $i \neq j$ is presumed to be greater than that of the correct decision $C_{i,i}$. Then the average risk, known as the Bayes risk $R$, is defined to be the expected value of the cost. This is given by

$$R \triangleq E \{C_{i,j}\} = \sum_{i,j} C_{i,j} P_j Pr[x \in Z_i|H_j]$$

$$= \sum_{i,j} C_{i,j} P_j \int_{Z_i} P(x|H_j)dx.$$

Here $P(x|H_j)$ is the conditional probability density function of the observed data $x$, given that hypothesis $H_j$ is true. Hence, the Bayes optimum test is based on the split of the decision regions in such a manner as to minimize $R$.

In binary hypothesis testing the risk $R$ can be expressed by

$$R = C_{0,0}P_0 \int_{Z_0} P(x|H_0)dx + C_{1,0}P_0 \int_{Z_1} P(x|H_0)dx$$

$$+ C_{1,1}P_1 \int_{Z_1} P(x|H_1)dx + C_{0,1}P_1 \int_{Z_0} P(x|H_1)dx. \hspace{1cm} (2.2)$$
Note that $Z_0$ and $Z_1$ denote two disjoint regions such that $Z = Z_0 \cup Z_1$, where "\cup" denotes set union. Thus,

$$
\int_{Z_1} P(x|H_1)dx = \int_{Z-Z_0} P(x|H_1)dx = 1 - \int_{Z_0} P(x|H_1)dx.
$$

(2.3)

By the use of (2.3), Eq.(2.2) can be rewritten in the form,

$$
R = P_0C_{1,0} + P_1C_{1,1}
+ \int_{Z_0} [P_1(C_{0,1} - C_{1,1})P(x|H_1) - P_0(C_{1,0} - C_{0,0})P(x|H_0)]dx.
$$

(2.4)

To minimize $R$, all points of the observed data $x \in Z$ that make the integrand negative are assigned to $Z_0$, otherwise they are put into $Z_1$. The values of $x$ for which the integrand precisely equals zero can be put anywhere. Hence, to minimize $R$ in (2.4) one must assign all of the points $x$ in $Z$ in such a manner that the lower inequality in

$$
P_1(C_{0,1} - C_{1,1})P(x|H_1) \geq \frac{H_1}{H_0} P_0(C_{1,0} - C_{0,0})P(x|H_0)
$$

is true, into set $Z_0$ and vise versa. Therefore, this decision rule is characterized by the likelihood ratio test (LRT), given by

$$
\Lambda_1(x) = \frac{P(x|H_1)}{P(x|H_0)} \geq \frac{P_0(C_{1,0} - C_{0,0})}{P_1(C_{0,1} - C_{1,1})}.
$$

(2.5)

(2.5) can be expressed more succinctly as

$$
\Lambda_1(x) \geq \frac{H_1}{H_0} \eta, \quad \text{where} \quad \eta \triangleq \frac{P_0(C_{1,0} - C_{0,0})}{P_1(C_{0,1} - C_{1,1})}.
$$

(2.6)

In (2.6) the data processing operations are captured entirely by the LR, i.e., $[P(x|H_1)/P(x|H_0)]$. Moreover, only the value of the likelihood ratio relative to
the threshold matters. To simplify the computation of the likelihood ratio, any positive monotonic operation can be performed on the likelihood ratio and the threshold simultaneously without affecting the comparison.

If \( C_{0,0} = C_{1,1} = 0 \) and \( C_{0,1} = C_{1,0} = 1 \) are assumed, the expression for \( R \) in (2.2) reduces to

\[
R = P_0 \int_{Z_1} P(x|H_0) \, dx + P_1 \int_{Z_0} P(x|H_1) \, dx. \tag{2.7}
\]

For this choice of cost function it is evident that (2.7) expresses the total probability of making an error. Thus, for this cost assignment the Bayes test minimizes the probability of making an erroneous decision. Note next that if the natural logarithm function (a monotonically increasing function) is applied to (2.7), the log likelihood test becomes

\[
\ln[\Lambda_1(x)] \underset{H_0}{\overset{H_1}{\gtrless}} \ln \eta, \quad \text{where} \quad \eta = \frac{P_0}{P_1}. \tag{2.8}
\]

This processor is commonly called a minimum probability of error receiver. When equal priori probabilities are assumed for each hypothesis (i.e., \( P_0 = P_1 = 1/2 \)) the test in (2.6) and (2.8) leads to the threshold \( \ln(P_0/P_1) = 0 \), so that the resulting maximum likelihood test reduces to

\[
P(x|H_1) \underset{H_0}{\overset{H_1}{\gtrless}} P(x|H_0).
\]

### 2.3.2 Neyman-Pearson Criterion

In many physical applications, it is difficult to assign a priori probabilities \( P_i \). Hence, a hypothesis testing procedure called the Neyman-Pearson criterion, that functions without priori probabilities, is used to derive the decision rule. Consider the binary hypothesis problem of radar, in which \( H_1 \) represents the presence of
a target signal and $H_0$ represents its absence. Four possible outcomes can result and the conditional probabilities are defined as follows:

Probability of Detection: $P_D = Pr[\hat{H}_1|H_1] = \int_{Z_1} P(x|H_1)dx$.
Probability of False Alarm: $P_{FA} = Pr[\hat{H}_1|H_0] = \int_{Z_0} P(x|H_0)dx$.
Miss Detection Probability: $P_M = Pr[\hat{H}_0|H_1] = \int_{Z_0} P(x|H_1)dx = 1 - P_D$.
The remaining probability $Pr[\hat{H}_0|H_0] = \int_{Z_1} P(x|H_0)dx$ is left nameless and equals $1 - P_{FA}$. In general, one would like to make $P_{FA}$ as small as possible and $P_D$ as large as possible. But in practical problems, these are conflicting objectives. One way of treating this is to constrain $P_{FA} = \alpha \leq \beta$ and to design a test to maximize $P_D$ under this constraint. That is,

$$\max_{Z_1} P_D \quad \text{subject to} \quad P_{FA} = \alpha.$$  

The solution is obtained by the use of Lagrange multipliers. First, the maximization of $P_D$ is equivalent to the minimization of $P_M = 1 - P_D$. Then by the Lagrange method one needs to minimize

$$F = P_M + \lambda[P_{FA} - \alpha]$$

or its equivalent expression,

$$F = \int_{Z_0} P(x|H_1)dx + \lambda \left[ \int_{Z_1} P(x|H_0)dx - \alpha \right]$$

$$= \int_{Z_0} [P(x|H_1) - \lambda P(x|H_0)]dx + \lambda (1 - \alpha),$$
where $\lambda$ is the Lagrange multiplier. Clearly, if $P_{FA} = \alpha$, then minimizing $F$ minimizes $P_M$ (i.e., $P_D$ is maximized). To minimize $F$ point $x$ is assigned to $Z_0$ when the term in the bracket is negative. This implies the following test:

$$\frac{P(x|H_1)}{P(x|H_0)} < \lambda, \text{ assign point to } Z_0 \text{ or say } H_0.$$

Thus, $F$ is minimized by the LRT given as follows:

$$\Lambda_1(x) = \frac{P(x|H_1)}{P(x|H_0)} \overset{H_1}{\underset{H_0}{\sim}} \lambda.$$

To satisfy the constraint $\lambda$ is chosen so that $P_{FA} = \alpha$. If the density of $\Lambda_1$ when $H_0$ is true is denoted by $P(\Lambda_1|H_0)$, then the following integral must be satisfied

$$P_{FA} = \int_\lambda^\infty P(\Lambda_1|H_0)d\Lambda_1 = \alpha. \quad (2.9)$$

The threshold is then given by solving (2.9) for $\lambda$. Observe that decreasing $\lambda$ is equivalent to increasing $Z_1$, the region where $H_1$ is assigned. Thus $P_D$ increases as $\lambda$ decreases. In most cases of interest, $P_{FA}$ is a continuous function of $\lambda$. Hence one can decrease $\lambda$ until $P_{FA} = \alpha$ is achieved.

### 2.4 M Hypotheses (M-ary)

Frequently, more than two viable models for data generation are defined for a given situation. The detection problem is to determine which of those models best “fits” a set of measurements. In the simple $M$-ary test there are $M$ source outputs, each of which corresponds to one of $M$ hypotheses. Thus, there are $M^2$ alternatives that may occur each time while the experiment is conducted. Typically, only the quantities $P_f[\hat{H}_i|H_0]$ for $i = 1, 2, \ldots, M - 1$ are required to
make the decision, particularly when the hypothesis $H_0$ represents the situation when no signal is present.

2.5 Composite Hypotheses

In the previous sections the likelihood ratio test ($\Lambda_i(x)$) is derived and determined in terms of conditional probability densities. However, in many circumstances, the exact nature of those conditional densities are not known. More generally, the conditional probability densities for $H_i$ may rely on a set of uncertain parameters which can be grouped into a parameter set or vector denoted by $\zeta_i$, i.e., $P(x|H_i, \zeta_i)$. This situation is said to be a composite hypothesis. This parameter vector can be categorized further in accordance with whether or not the parameters are random or nonrandom.

2.5.1 Random Parameter Vector

When the probability density function of a random parameter vector $\zeta_i$ is known, the likelihood function under $H_i$ is expressed as

$$P(x|H_i) = \int P(x|H_i, \zeta_i)P_{\zeta_i}(\zeta_i)d\zeta_i.$$  

The likelihood ratio in this random parameter case becomes

$$\Lambda_i(x) = \frac{P(x|H_i)}{P(x|H_0)} = \frac{\int P(x|H_i, \zeta_i)P_{\zeta_i}(\zeta_i)d\zeta_i}{\int P(x|H_0, \zeta_0)P_{\zeta_0}(\zeta_0)d\zeta_0}.$$  

If the random parameter vectors $\zeta_i$ have unknown densities, the best test procedure is not specified.
2.5.2 Nonrandom Parameters

The second case of interest is the case in which the parameter $\zeta_i$ is an unknown nonrandom variable. This requires one to consider the problem of estimating nonrandom variables. The available observed data sometimes can be used to "guess" the value of the parameter. If a reasonable guess is obtained, then it can be used in the hypothesis test. It should be noted however that the data used for estimating unknown parameters is often the same data used in the decision rule. Procedures intended to yield "good" guesses of the value of a parameter are said to be parameter estimates. A logical procedure is to estimate $\zeta_i$ on the assumption that $H_i$ is true, then to estimate $\zeta_0$ on the assumption that $H_0$ is true. Finally, these estimates are applied to the LRTs.

One parameter estimation procedure that fits nicely into the problem of composite hypothesis testing is the maximum likelihood estimate. Let $\zeta_i$ denote a vector of parameters for $H_i$. The maximum likelihood (ML) estimate $\hat{\zeta}_i$ of $\zeta_i$ is the value that maximizes the conditional probability density (likelihood) $P(x|H_i, \zeta_i)$. To use $\hat{\zeta}_i$ in a decision rule, the parameter vector needs to be estimated separately for each hypothesis. Then the estimated parameter values are used in their corresponding individual conditional densities to obtain the LRTs. The procedure is called a generalized likelihood ratio test (GLRT) for the unknown parameter problem in hypothesis testing, and (2.1) can be expressed as

$$
\Lambda_i(x) = \frac{\text{max}_{\zeta_i} P(x|H_i, \zeta_i)}{\text{max}_{\zeta_0} P(x|H_0, \zeta_0)} = \frac{P(x|H_i, \hat{\zeta}_i)}{P(x|H_0, \hat{\zeta}_0)},
$$

where $\hat{\zeta}_i$ and $\hat{\zeta}_0$ represent the ML estimates of $\zeta_i$ in $H_1$ and $\zeta_0$ in $H_0$, respectively.

Example [17]
Let the observed data \( N \)-vector \( \mathbf{x} \) be a Gaussian random vector under each hypothesis. Suppose that both given hypotheses correspond to Gaussian random vectors having different mean values but sharing the same covariance. Under hypothesis \( H_1 \) the nonzero mean is assumed to be unknown while zero mean is assigned to hypothesis \( H_0 \) as follows:

\[
H_0 : \quad \mathbf{x} \sim G(\mathbf{0}, \sigma^2 \mathbf{I}) \\
H_1 : \quad \mathbf{x} \sim G(\mathbf{m}, \sigma^2 \mathbf{I}), \quad \mathbf{m} = [m, m, \ldots, m]^T, \quad m : \text{unknown},
\]

where \( G(\mathbf{m}, \sigma^2 \mathbf{I}) \) denotes a Gaussian random vector with mean vector \( \mathbf{m} \) and covariance matrix \( \sigma^2 \mathbf{I} \) and the symbol \( ^T \) denotes matrix transpose operator. The unknown quantity \( \mathbf{m} \) occurs only in the exponent of the conditional density under \( H_1 \). Hence, the maximization of this density is equivalent to the maximization of the exponent. Thus to find this maximization, the derivative of the exponent is performed with respect to \( m \) as follows:

\[
\frac{\partial}{\partial m} \left[ -\frac{1}{2\sigma^2} \sum_{i=0}^{N-1} (x_i - \hat{m})^2 \right] = 0 \quad \implies \quad \sum_{i=0}^{N-1} (x_i - \hat{m}) = 0 \\
\implies \quad \hat{m} = \frac{1}{N} \sum_{i=0}^{N-1} x_i. \quad (2.10)
\]

To derive the decision rule, (2.10) is substituted into the conditional probability density function under \( H_1 \). The exponent of this density is manipulated to obtain

\[
-\frac{1}{2\sigma^2} \sum_{i=0}^{N-1} \left( x_i - \frac{1}{N} \sum_{j=0}^{N-1} x_j \right)^2 = -\frac{1}{2\sigma^2} \left[ \sum_{i=0}^{N-1} x_i^2 - \frac{1}{N} \left( \sum_{i=0}^{N-1} x_i \right)^2 \right]. \quad (2.11)
\]
Note that the first term in (2.11) is exactly the same as the exponent of the conditional probability density function under \( H_0 \) (the denominator) in the likelihood ratio, the GLRT becomes

\[
\Lambda_1(x) = \exp \left\{ \frac{1}{2N\sigma^2} \left( \sum_{i=0}^{N-1} x_i \right)^2 \right\}.
\]

### 2.6 The Wiener Filter

The classical Wiener-filter problem is to find an estimate \( \hat{d} \) of a desired scalar signal \( d \) from an observed \( N \)-vector \( x \), as shown in Figure 2.2. The desired scalar signal \( d \) is assumed to be zero mean. The error signal \( \epsilon \) is given from this figure by

\[
\epsilon = d - \hat{d} = d - w^\dagger x,
\]

namely, the difference between the desired signal \( d \) and its linear estimate,

\[
\hat{d} = w^\dagger x.
\]
The Wiener filter is filter which is optimal in the sense of the minimum mean-square error (MMSE) performance measure. The problem is then to find the $N$-vector $\mathbf{w}$ which minimized the MSE. Thus the MSE is expressed as the follows:

$$
\min_{\mathbf{w}} \mathbb{E}\{\|e\|^2\} = \min_{\mathbf{w}} \mathbb{E}\{(d - \mathbf{w}^\dagger \mathbf{x})(d - \mathbf{w}^\dagger \mathbf{x})^\dagger\} = \min_{\mathbf{w}} \left\{ \sigma_d^2 - \mathbf{w}^\dagger \mathbf{r}_{xd} - \mathbf{r}_{xd}^\dagger \mathbf{w} + \mathbf{w}^\dagger \mathbf{R}_x \mathbf{w} \right\}, \quad (2.12)
$$

where the auto-correlation matrix $\mathbf{R}_x$ is given by,

$$
\mathbf{R}_x = \mathbb{E}\{\mathbf{x}\mathbf{x}^\dagger\},
$$

the cross-correlation vector $\mathbf{r}_{xd}$ is

$$
\mathbf{r}_{xd} = \mathbb{E}\{\mathbf{x}d^\star\},
$$

where $\mathbb{E}\{\cdot\}$ denotes the expected-value operator, and the variance of the desired signal $d$ is given by,

$$
\sigma_d^2 = \mathbb{E}\{\|d\|^2\}.
$$

The minimization of the expression in (2.12),

$$
J(\mathbf{w}) = \sigma_d^2 - \mathbf{w}^\dagger \mathbf{r}_{xd} - \mathbf{r}_{xd}^\dagger \mathbf{w} + \mathbf{w}^\dagger \mathbf{R}_x \mathbf{w},
$$

accomplished by taking its gradient and equating it to zero:

$$
\nabla_{\mathbf{w}} \{J(\mathbf{w})\} \triangleq \frac{\partial J(\mathbf{w})}{\partial \mathbf{w}} = 2\mathbf{R}_x \mathbf{w} - 2\mathbf{r}_{xd} = 0,
$$

19
where the complex gradient operator, denoted by $\nabla_w \{ \cdot \}$, represents the gradient with respect to the parameter $w$. The optimal Wiener weight vector $w_{\text{opt}}$ of the filter is provided by

$$w_{\text{opt}} = R_x^{-1}r_{xd},$$

and the MSE, $\xi$ is given by

$$\xi = J(w) = \sigma_d^2 - r_{xd}^\dagger R_x^{-1}r_{xd} + (w - R_x^{-1}r_{xd})^\dagger R_x (w - R_x^{-1}r_{xd})$$

$$= J(w_{\text{opt}}) + (w - w_{\text{opt}})^\dagger R_x (w - w_{\text{opt}}) \geq J(w_{\text{opt}}),$$

(2.13)

where the MMSE, $\xi_{\text{min}} = J(w_{\text{opt}}) = \sigma_d^2 - r_{xd}^\dagger R_x^{-1}r_{xd}$, is attained when $w = w_{\text{opt}}$. Note that the finding of the Wiener weight vector requires a matrix inversion which is impractical in some situations.
Chapter 3

An Asynchronous CDMA Receiver with Adaptive Beamforming and Nulling

3.1 Introduction

Spread-spectrum communication systems have been used successfully in military applications for several decades. Direct-sequence (DS) code-division multiple-access (CDMA), a specific form of spread-spectrum transmission, has become an important component of current mobile communication systems due to its attractive properties for wireless communications. In DS-CDMA communications, all users are allowed to transmit information at any time and to occupy the same broad-band frequency spectrum. This kind of channel allocation alleviates the problems commonly associated with other forms of channel allocation such as unused resources. However, the cross-correlations between different users’ spreading codes are generally not orthogonal to one another. As a consequence the receiver of the desired signal usually is embedded in the other users’ signals that are often received at higher power levels. This phenomenon of varying power levels of
multiple-access interference (MAI) is referred to as the near-far problem*. Figure 3.1 illustrates the near-far problem.

In CDMA, demodulation requires the suppression of two forms of noise: 1) ambient channel noise, which is an additive white Gaussian noise (AWGN), and 2) MAI, intersymbol interference (ISI) or multipath induced by the reflections in the wireless channel. The conventional receiver, which uses a simple correlation of the received signal with the desired-user spreading-code sequence, often does not take into account the existence of MAI and other non-white interference. The MAI caused by any one user is generally small, but as the number of interferers and their total power increases, the effects of MAI can become substantial. Hence, performance can degrade dramatically in the presence of strong MAI due to other users near the base station, i.e., near-far interference. The primary challenge of building mobile communication systems is to cure the problems caused by MAI.

Two types of adaptive near-far resistant detectors that uses multiple antennas for an asynchronous DS-CDMA system is presented here, one is developed in this chapter, the other more general approach is developed in Chapter 4. The only requirement for both types is knowledge of the desired spreading code sequence. Also there is no need of a special training sequence or period.

In this chapter synchronization or the acquisition of asynchronous DS-CDMA spreading signals is treated as a binary-hypothesis test, followed by a tracking the epochs of synchronization. Under the binary hypotheses, an adaptive generalized maximum likelihood ratio test (GLRT) is developed to acquire the multipath code timings of a single desired user in both a fading and a near-far interference environment. After the timing structure of the desired spreading code sequences

*Due to the differences in the distance between each user and the base station, the propagation path loss differences between the users and their base stations can be many tens of dB. The base station receives each user's signal with a different power. In CDMA the stronger power levels raise the noise floor at a receiver for the weaker signals, thereby decreasing the probability that the weaker signals can be received. This effect is called the near-far problem.
is identified a constrained maximum likelihood estimator (MLE) of the direction-of-arrival (DOA) is applied to the data that contains the spreading code pattern in order to refine the estimate of the arrival angle of the desired user. The estimated DOA is used by a Capon-type beamformer to further suppress the MAI and then information-bearing symbol is obtained by a conventional detector. Finally, the proposed detector is shown to have potential against multipath fading and to be insensitive to the problems of near-far interference by computer simulations of an asynchronous binary phase-shift keying (BPSK) DS-CDMA system.

3.2 Asynchronous DS-CDMA System Model

Consider an asynchronous DS-CDMA mobile radio network with $K$ users that employs $K$ spreading waveforms $s_1(t), s_2(t), \ldots, s_K(t)$ and their transmitted sequences of the BPSK symbols. The complex transmitted baseband signal of the $l$-th user is expressible in the form,
\[ r_l(t) = \sum_{m=-\infty}^{\infty} A_l d_l[m] s_l(t - mT_b), \]

for \( l = 1, 2, \ldots, K \). Here, \( s_l(t) \) is the spreading waveform of the \( l \)-th user, given by

\[ s_l(t) = \sum_{k=0}^{N-1} c_{l,k} p(t - kT_c), \quad 0 \leq t \leq T_b, \]

where \( p(t) \) is the chip waveform, assumed to be an approximately rectangular pulse of duration \( T_c \), given by

\[ p(t) = \begin{cases} \frac{1}{T_c} & , \quad t \in [0, T_c], \\ 0 & , \quad \text{otherwise}. \end{cases} \]

The parameters and other quantities are defined as follows:

- \( A_l \) The amplitude of user \( l \).
- \( d_l[m] \) The \( m \)-th information symbol of user \( l \) and \( d_l[m] \in \{ \pm 1, 0 \} \), where symbol zero denotes the fact that the \( l \)-th user is off (has either not started or has finished transmission).
- \( T_b \) The information symbol interval.
- \( \{c_{l,k}\}_{k=0}^{N-1} \) The spreading code sequence (signature vector) of \( \pm 1 \) of user \( l \).
- \( T_c \) The chip interval.
- \( N \) The processing (spreading) gain \( (\triangleq T_b/T_c) \).

On the assumption that each mobile user is equipped with a single antenna the baseband multipath channel between each user’s transmitter and the base station receiver is modeled as a single-input multiple-output (SIMO) channel with the following vector impulse response function [53]:

\[ h_l(t) = \sum_{j=1}^{K_l} b_{l,j} a_{l,j} \delta(t - \tau_{l,j}), \]
where $\delta(\cdot)$ denotes the continuous-time unit-impulse (or Dirac delta) function and $K_l$ is the number of paths of the $l$-th user’s channel. $a_{ij}$, $\tau_{ij}$, and $b_{ij} = [b_{ij}^1, b_{ij}^2, \ldots, b_{ij}^J]^T$ are, respectively, the complex gain, the propagation delay, and a direction vector that corresponds to the $j$-th path of the $l$-th user’s signal. Here $^{\text{T}}$ denotes the matrix transpose operator. A model which demonstrates the multipath nature of a wireless channel is presented in Figure 3.2. Assume that there are $K$ system users with $K_l$ propagation paths for the $l$-th user. All such user’s signals impinge on the receiving antenna array with $J$ sensors in the AWGN channel. Then the total received signal at the receiver can be written in the form,

$$r(t) = \sum_{l=1}^{K} h_l(t) * r_l(t) + n(t),$$

$$= \sum_{l=1}^{K} A_l \sum_{j=1}^{K_l} a_{lj} b_{lj} \sum_{m=-\infty}^{\infty} d_l[m] s_l(t - mT_b - \tau_{lj}) + n(t).$$

(3.1)

where “$*$” indicates the convolution operator. $n(t)$ is a $J$-vector complex Gaussian noise process that is assumed to be white in time and space, i.e., $n(t) = [n_1(t), n_2(t), \ldots, n_J(t)]^T$. 

Figure 3.2: A multipath channel model.
Figure 3.3: Construction of a typical DS signal. Illustrations (a) and (b) are parts of two information sequences, (c) and (d) are two 31-bit spreading waveforms, (e) and (f) are the two modulated waveforms to be transmitted, one for the desired user 1 and the other for the interfering user (user 2). Illustration (g) is the combined waveform of (e) and (f). Note that user 2 is assumed to be delayed by $\tau_2 - \tau_1 = 4T_c$ seconds with respect to user 1.
To understand how the asynchronous DS-CDMA signal is determined, an illustrative example for \( K = 2 \) and \( N = 31 \), is presented next. In the DS-CDMA system, the information-bearing sequence for each user is spread over a broader bandwidth by means of a spreading waveform that is unique to that user. Figure 3.3(a) shows two bits of an information sequence for user 1. The spreading waveform of user 1 is shown in Figure 3.3(c). It is constructed by modulating the chip waveform \( p(t) \) by means of a spreading code. In this example, a Gold code of length 31 is employed for the spreading code of user 1, given by \( s_1 = [-1, -1, -1, 1, 1, -1, 1, 1, -1, 1, 1, -1, 1, 1, 1, 1, 1, 1, -1, 1, -1, -1, -1, -1, -1, 1, -1, -1, -1, 1] \) (see Appendix A). Figure 3.3(e) shows the modulated baseband DS signal that corresponds to the multiplication (modulation) of the information sequence and the spreading waveform in Figures 3.3(a) and (c). The interfering information signal of user 2, that additively contributes to the received signal, is assumed to be offset in time from that of user 1 by \( 4T_c \) and is illustrated in Figure 3.3(b). Also Figures 3.3(d) and (f) show the spreading waveform and the modulated signal of user 2. The spreading code for user 2 is given by \( s_2 = [1, 1, 1, -1, 1, -1, -1, 1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, -1, -1, 1, -1, -1, -1, -1] \). The baseband transmitted signal for the time interval \([t, t + 2T_b]\), shown in Figure 3.3(g), is the sum of the modulated signals of both users. The sequences \( s_1 \) and \( s_2 \) in this example are derived by setting \( \nu \) in Eq.(A.1) equal to 0 and 1, respectively.

To facilitate the derivation of the receiver, the communication channel is assumed to approximate a time-invariant channel over some interval of time exceeding \( N \) chips, i.e., the channel is quasi-stationary. Here, the number of users \( K \), their powers and their delay profiles are assumed to be fixed (but unknown). Only the turning on-and-off of the information sequences, their \( d_i \)'s, and the AWGN are changing in time. In scenarios of this type, it turns out that the proposed receiver is able to adapt quickly to any such changing environment.
3.3 Receiver Structure

Consider a receiving array antenna with \( J \) receiver elements. For convenience, the proposed receiver uses a baseband-equivalent structure. Such a baseband complex signal process is physically achieved by the combination of quadrature demodulation and a phase-locked loop (PLL) ([33], Chapter 6). This converts the received radio frequency (RF) modulated signal to a baseband complex-valued signal. Then the received signal of each individual antenna sensor is passed through a filter that is matched to the square-wave chip waveform (or other type of waveform that is used to be the chip waveform). The output of the \( k \)-th chip matched filter is

\[
x_k(t) = \int_{t-T_c}^{t} r_k(t') dt' = \int_{0}^{T_c} r_k(t-t') dt',
\]

for \( k = 1, 2, \ldots, J \). Subsequently, the outputs of these chip MFs are sampled every \( T_s \) seconds, where \( S(= T_c/T_s) \) is assumed to be an integer and \( S \geq 1 \). This means that the sampling rate is greater than or equal to the chip rate. Then these discrete-time outputs are used as the inputs of \( J \) adaptive \( N \)-element tapped delay lines (TDLs) each with a \( T_c \) tap spacing to form \( J \) such \( N \)-element data vectors. Assume that all of the chip MFs are synchronized, and that the output signals of the chip MFs are sampled at the times \( iT_s \). The samples are taken overall symbol interval. The TDLs for the \( J \)-element antenna array are expressed as a \( J \times N \) data matrix given by

\[
X[i] = [x_1[i], x_2[i], \ldots, x_N[i]]
\]
\[
\begin{bmatrix}
  x_1(iT_s) & x_1(iT_s - T_c) & \cdots & x_1(iT_s - (N-1)T_c) \\
  x_2(iT_s) & x_2(iT_s - T_c) & \cdots & x_2(iT_s - (N-1)T_c) \\
  \vdots & \vdots & \ddots & \vdots \\
  x_J(iT_s) & x_J(iT_s - T_c) & \cdots & x_J(iT_s - (N-1)T_c)
\end{bmatrix}.
\]  

(3.4)

Hence, the \( k \)-th column vector \( \mathbf{x}_k[i] \) for \( k = 1, 2, \ldots, N \) in (3.4) is the vector,

\[
\mathbf{x}_k[i] = [x_1(iT_s - (k-1)T_c), x_2(iT_s - (k-1)T_c), \ldots, x_J(iT_s - (k-1)T_c)]^T,
\]

(3.5)

for \( k = 1, 2, \ldots, N \). In other words, the \( J \)-vector \( \mathbf{x}_k[i] \) for \( k = 1, 2, \ldots, N \) constitutes a data set or “window” of \( N \) vectors. The data matrix in (3.4) is passed through the time-synchronization and DOA acquisition systems in order to identify the time phase at which synchronization occurs and also to acquire the DOA of the desired-user signature vector. The \( J \)-vector of the relative complex gains of the \( J \) receivers of the \( J \)-element array is then determined later by the estimated DOA in Eq.(3.62) of Section 3.3.3 based on minimum output variance criterion and is expressed as a column vector of the form,

\[
\mathbf{w} = [w_1, w_2, \ldots, w_J]^T.
\]

(3.6)

The output vector of a Capon beamformer, \( \mathbf{z}[\hat{i}] \), is defined as the inner product of the weight vector \( \mathbf{w} \) and the column vectors of \( \mathbf{X}[\hat{i}] \) as follows:

\[
\mathbf{z}[\hat{i}] = \mathbf{w}^H \mathbf{X}[\hat{i}],
\]

(3.7)

where \( \hat{i} \) is the time phase at which synchronization occurs and \( ^H \) denotes the conjugate transpose (Hermitian) of a matrix. Then this output \( N \)-vector \( \mathbf{z}[\hat{i}] \) is exploited by the demodulation system, that uses a conventional detector, to obtain the information bit. Note that the time synchronization system can be
modeled conceptually as a filter bank constructed of \(NS\) parallel receivers in order to identify the time phase at which to demodulate the information symbols of the desired user.

### 3.3.1 Synchronization System

As it is shown in [6], the synchronization problem in communications is a signal detection problem analogous to radar detection. The detection of a single desired user's spreading code sequence that is embedded in MAIs can be modeled as a binary-hypothesis testing problem, where \(H_0\) corresponds to target-signal absence and \(H_1\) corresponds to target-signal presence. Thus at each time phase of the data matrix \(X[i]\) the time-synchronization system must distinguish between two hypotheses of the desired user, say user 1. In order to distinguish the two hypotheses, a GLRT, that is described by a probability density function (pdf), \(P\), on the sample space is developed next. To determine the unknown time phase of the desired spreading code pattern, the detector is designed by sliding a “search window” of length \(N\), sample-by-sample within the entire symbol of the received data. Thus the two hypotheses, that adaptive detector must distinguish at each sampling time, are given by

\[
\begin{align*}
H_0 & : \quad X[i] = V[i], \\
H_1 & : \quad X[i] = b s_1 + V[i].
\end{align*}
\]  

(3.8)

For simplicity of notation, the array response vector \(b \triangleq [b_1, b_2, \ldots, b_J]^T\) in (3.8) is used hereafter to combine the parameters of the amplitude, the complex gain, and the direction vector of the desired user. Let \(s_1 = [c_{1,0}, c_{1,1}, \ldots, c_{1,N-1}]\) denote the desired-user signature vector of length \(N\). \(V[i] = [v_1[i], v_2[i], \ldots, v_N[i]]\) is the data matrix when \(b = 0\), or hypothesis \(H_0\) is true. In other words, \(V[i]\)
represents interference-plus-noise-only matrix. $v_k[i] = [v_{k,1}[i], v_{k,2}[i], \ldots, v_{k,J}[i]]^\top$ represents the $k$-th column vector of $V[i]$. The $k$-th $J$-vector noise process $v[i]$, which impinges on the $J$-sensor receiver array, is assumed to be internal receiver noise plus external noise due to directional MAI and the spatially white noise. Such noise is assumed to be independent from sample-to-sample, that is the same assumption made in a Rake receiver, and can be approximated Gaussian noise with zero-mean [3] and its associated covariance matrix is defined as follows:

$$
R_k \triangleq E \{ x_k[i]x_k[i]^\top | H_0 \} = E \{ v_k[i]v_k[i]^\top \},
$$  

(3.9)

where $E \{ \cdot \}$ denotes the expected-value operator. Also, the stationarity time constant of the noise process is presumed to exceed window size $N$. Hence, under this assumption, $R_k$ in Eq.(3.9) is assumed to be approximately the constant covariance matrix for $k = 1, 2, \ldots, N$. This hypothesis test is equivalent to the type of test commonly used in radar to detect the presence of a coded subsequence such as in $s_1$, given above.

The random vector $x_k[i]$ is still an approximate complex Gaussian process under both hypotheses. Hence, the probability density function of $J$-variate vector $x_k[i]$ under hypothesis $H_0$ is given by

$$
P(x_k[i]|H_0) = \frac{1}{\pi^J|R|} e^{-\{x_k[i]R^{-1}x_k[i]\}},
$$  

(3.10)

for $k = 1, 2, \ldots, N$, where the mean value of $x_k[i]$ conditioned on $H_0$ is derived by

$$
E \{ x_k[i]|H_0 \} = E \{ v_k[i]|H_0 \} = 0.
$$
\(|R|\) indicates the determinant of \(R\). By Gaussianity and the independence of vectors \(x[k][i]\) for different \(k\), the joint probability density function of \(X[i]\) in (3.8) under hypothesis \(H_0\) can be derived by

\[
P(X[i]|H_0) = P(x_1[i], x_2[i], \ldots, x_N[i]|H_0) = \prod_{k=1}^{N} P(x_k[i]|H_0) = \frac{1}{\pi^{N/2}|R|^{N/2}} e^{-\left\{\sum_{k=1}^{N} x_k[i]^T R^{-1} x_k[i]\right\}}.
\] (3.11)

The exponent term in Eq.(3.11) is expressible in terms of the trace function which is defined as

\[
Tr(A) = \sum_{i=1}^{N} a_{ii},
\]

where \(A = [a_{ij}]\) represents an arbitrary \(N \times N\) matrix and \(Tr(\cdot)\) denotes the matrix trace operator. Since this exponent is a scalar and \(Tr(AB) = Tr(BA)\), where \(A\) and \(B\) are matrices, the exponent can be re-expressed as

\[
\sum_{k=1}^{N} x_k[i]^T R^{-1} x_k[i] = Tr\left(\sum_{k=1}^{N} x_k[i]^T R^{-1} x_k[i]\right)
= Tr\left(R^{-1} \sum_{k=1}^{N} x_k[i] x_k^*[i]\right)
= N \cdot Tr\left(R^{-1} \hat{R}_0\right).
\] (3.12)

\(\hat{R}_0\) in Eq.(3.12) denotes the sample covariance matrix of \(R\) over the \(N\) most recent samples under hypothesis \(H_0\) and is computed by

\[
\hat{R}_0 = \frac{1}{N} \sum_{k=1}^{N} x_k[i][x_k[i]]^* = \frac{1}{N} X[i][X[i]^*].
\] (3.13)
\( \hat{R}_0 \) in Eq.(3.13) is also known as the MLE of the unknown covariance matrix \( R \) under hypothesis \( H_0 \), see [28]. From Eqs.(3.12) and (3.13), Eq.(3.11) becomes

\[
P(X[\hat{\tau}]|H_0) = \frac{1}{\pi^{N/2} |R|^{N/2}} e^{-N \text{Tr}(R^{-1} \hat{R}_0)}.
\]  

(3.14)

Similarly, the conditional mean of \( x_k[\hat{\tau}] \) conditioned on the given hypothesis \( H_1 \) can be computed as follows:

\[
E \{ x_k[\hat{\tau}] | H_1 \} = E \{ bs_1(k) | H_1 \} + E \{ v_k[\hat{\tau}] | H_1 \} = bs_1(k),
\]

(3.15)

Then the joint probability density function of \( x_k[\hat{\tau}] \) under hypothesis \( H_1 \) is given by

\[
P(x_k[\hat{\tau}] | H_1) = \frac{1}{\pi^{N/2} |R|^{N/2}} e^{-\left\{ (x_k[\hat{\tau}] - bs_1(k))^T R^{-1} (x_k[\hat{\tau}] - bs_1(k)) \right\} }.
\]

(3.16)

Thus, the joint probability density function of \( X[\hat{\tau}] \) under hypothesis \( H_1 \) can be found in a similar way to be

\[
P(X[\hat{\tau}] | H_1) = P(x_1[\hat{\tau}], x_2[\hat{\tau}], \ldots, x_N[\hat{\tau}] | H_1) = \prod_{k=1}^{N} P(x_k[\hat{\tau}] | H_1)
\]

\[
= \frac{1}{\pi^{N/2} |R|^{N/2}} e^{-\sum_{k=1}^{N} \left\{ (x_k[\hat{\tau}] - bs_1(k))^T R^{-1} (x_k[\hat{\tau}] - bs_1(k)) \right\} }.
\]

(3.17)

Hence, Eq.(3.17) can be rewritten as

\[
P(X[\hat{\tau}] | H_1) = \frac{1}{\pi^{N/2} |R|^{N/2}} e^{-N \text{Tr}(R^{-1} \hat{R}_1)},
\]

(3.18)
where the sample averaging estimate of the covariance matrix $R$ under hypothesis $H_1$ is given by

$$
\hat{R}_1 = \frac{1}{N} \sum_{k=1}^{N} x_{k,b}[i] x_{k,b}^\dagger[i] \\
= \frac{1}{N} (X[i] - bs_1)(X[i] - bs_1)^\dagger,
$$

and the column vector of $x_{k,b}[i]$ in Eq.(3.19) is equal to

$$x_{k,b}[i] = x_k[i] - bs_1(k).$$

Here $\hat{R}_1$ denotes the MLE of the covariance matrix $R$ under hypothesis $H_1$. Note that the inverses of $\hat{R}_0$ and $\hat{R}_1$ exist with probability one only if $J < N$.

Let $X[i]$ be a point in sample space $X$, and $P(X[i], \Theta)$ be the probability density function of $X[i]$, where $\Theta$ is a parameter vector, defined next, in the set $\Omega$ of all possible vectors of $\Theta$. The parameter vector $\Theta$ consists of the $J$-vector $b$ and the $J(J + 1)/2$ elements of the unknown positive definite covariance matrix $R$. It is more convenient to express $\Theta$ as the pair $[b, R]$ so that the parameter set is the set

$$\Omega = \{\Theta\} = \{[b, R]|R > 0\},$$

where $R > 0$ denotes the fact that $R$ is Hermitian symmetric and positive definite. Suppose $\varpi_0$ denotes a subset in the parameter space $\Omega$, specified by hypothesis $H_0$, then the set $\Omega - \varpi_0$ is also a subset of $\Omega$, specified by hypothesis $H_1$. In terms of the subsets $\varpi_0$ and $\Omega - \varpi_0$ of $\Omega$, the alternative hypotheses $H_0$ and $H_1$ are defined as follows [51]:

$$H_0 \equiv [0, R] \in \varpi_0 \ and \ H_1 \equiv [b, R] \in \Omega - \varpi_0,$$
where
\[
\varpi_0 = \{ [0, R] | R > 0 \}, \\
\Omega - \varpi_0 = \{ [b, R] | b \neq 0, \ R > 0 \}.
\]

The generalized likelihood ratio test (GLRT) of the desired spreading code sequence \( \Lambda(X[i]) \), originally due to Neyman and Pearson [51], has the form ([6], Appendix A), ([39], Page 1762):
\[
\Lambda(X[i]) = \frac{\max_{X[i] \in \varpi_0} P(X[i]|H_0; R, b)}{\max_{X[i] \in \Omega - \varpi_0} P(X[i]|H_0; R)} \overset{H_i}{\underset{H_0}{\gtrless}} \gamma, \quad (3.22)
\]
where \( \gamma \geq 0 \) is the threshold of the test. If \( \hat{\Theta}_i \) are the maximum likelihood estimators (MLEs) of the parameter vector \( \Theta_i \) for a given \( X[i] \), conditioned on hypothesis \( H_i \) \( (i = 0, 1) \), then \( \Lambda(X[i]) \) in Eq.(3.22) is equivalent to
\[
\Lambda(X[i]) = \frac{P(X[i]; \hat{\Theta}_1)}{P(X[i]; \hat{\Theta}_0)} \overset{H_i}{\underset{H_0}{\gtrless}} \gamma. \quad (3.23)
\]
By Eqs.(3.21), (3.13), and (3.19) the MLEs of \( \Theta_i \) \( (i = 0, 1) \), holding \( b \) fixed, are
\[
\hat{\Theta}_0 = [0, \hat{R}_0] \quad \text{and} \quad \hat{\Theta}_1 = [b, \hat{R}_1]
\]
with respect to hypotheses \( H_0 \) and \( H_1 \), respectively. Thus a substitution of MLEs of \( R \) with respect to \( H_0 \) and \( H_1 \) into Eqs.(3.14) and (3.18), then Eq.(3.23) simplifies to
\[
\Lambda(X[i]) = \frac{|\hat{R}_0|^N}{\min_b |\hat{R}_1|^N} \overset{H_i}{\underset{H_0}{\gtrless}} \gamma. \quad (3.24)
\]
Taking the $N$-th root, this test is equivalent to

$$
\lambda(X[z]) = \frac{|\hat{R}_0|}{\min_b |\hat{R}_1|} \begin{cases} H_1 \\ \geq H_0 \end{cases} \gamma_1,
$$

(3.25)

where $\gamma_1 = \gamma^{1/N}$ is a new decision constant. By plugging Eqs. (3.13) and (3.19) into Eq. (3.25), the likelihood ratio in Eq. (3.25) is evidently equivalent to the form,

$$
\lambda(X[z]) = \frac{|X[z]X^\dagger[z]|}{\min_b [(X[z] - bs_1)(X[z] - bs_1)^\dagger]}
$$

(3.26)

$$
= \frac{|X[z]X^\dagger[z]|}{\min_b |F_b|}.
$$

(3.27)

Referring to Eqs. (3.26) and (3.27), expression $F_b$ is defined by

$$
F_b \triangleq (X[z] - bs_1)(X[z] - bs_1)^\dagger.
$$

(3.28)

To find $\min_b |F_b|$ in Eq. (3.27), one first expands $F_b$ in Eq. (3.28) and makes a change of variables for $F_b$ in Eq. (3.28) as follows:

$$
F_b = X[z]X^\dagger[z] - bs_1 X^\dagger[z] - X[z]s_1^\dagger b^\dagger + bb^\dagger(s_1 s_1^\dagger),
$$

(3.29)

where $s_1 s_1^\dagger$ is a positive scalar. Next the spreading code sequence is normalized by letting

$$
\hat{s}_1 = (s_1 s_1^\dagger)^{-1/2} s_1,
$$

(3.30)

which yields $\hat{s}_1 \hat{s}_1^\dagger = 1$, and

$$
\hat{b} \hat{s}_1 = bs_1,
$$

36
where \( \hat{b} \) is the modified complex gain vector of the \( J \)-element antenna array, given by

\[
\hat{b} = (s_i s_i^\dagger)^{1/2} b.
\]

The change of variables in \( F_b \) of \( b \) to \( \hat{b} \) and \( s_i \) to \( \hat{s}_i \) and also a completion of squares yields the expression,

\[
F_{\hat{b}} = X[i]X[i]^\dagger - \hat{b}\hat{s}_i X[i] - X[i]\hat{s}_i^\dagger \hat{b}^\dagger + \hat{b}\hat{b}^\dagger \quad (3.31)
\]

\[
= (\hat{b} - X[i]\hat{s}_i^\dagger)(\hat{b} - X[i]\hat{s}_i^\dagger)^\dagger + G, \quad (3.32)
\]

where

\[
G = X[i]X[i]^\dagger - (X[i]\hat{s}_i^\dagger)(X[i]\hat{s}_i^\dagger)^\dagger. \quad (3.33)
\]

Note here that matrix \( F_{\hat{b}} \) in Eq.(3.31) is \( F_b \) in Eq.(3.29) in terms of the modified gain vector \( \hat{b} \) and the "unit" signal vector \( \hat{s}_i \).

Under the signal-plus-noise hypothesis \( H_1 \) it further reduces the difficulty of minimizing \( F_b \) or its equivalent \( F_{\hat{b}} \) to transform the \( (J \times N) \)-data matrix \( X[i] \) to a \( (J \times N) \)-matrix \( Y[i] \) which has signal in only one of its columns, say the first column. Since signal \( \hat{s}_1 \) has unit magnitude, this is accomplished by the \( N \times N \) unitary transformation \( U \), defined by

\[
U = \begin{bmatrix} \hat{s}_1 \\ Q \end{bmatrix} \quad (3.34)
\]

as follows:

\[
\hat{s}_1 U^\dagger = \hat{s}_1 [\hat{s}_1^\dagger, Q^\dagger] = [\hat{s}_1 \hat{s}_1^\dagger, \hat{s}_1 Q^\dagger] = [1, 0, \ldots, 0],
\]
where $Q$ is a $(N-1) \times N$ matrix, composed of some set of orthonormal row vectors. Because $U$ is unitary, it must satisfy also

$$U^\dagger U = UU^\dagger = \begin{bmatrix} \hat{s}_1 \\ Q \end{bmatrix} \begin{bmatrix} \hat{s}_1 \\ Q \end{bmatrix}^\dagger = \begin{bmatrix} 1 & 0 \\ 0 & I_{N-1} \end{bmatrix} = I_N$$

so that the orthogonality relation,

$$\hat{s}_1 Q^\dagger = 0$$

holds, i.e., each element of the orthonormal set of row vectors, composing $Q$, is orthogonal to the normalized signal vector $\hat{s}_1$.

To see explicitly how unitary transformation $U$ in Eq.(3.34) acts upon signal vector $\hat{s}_1$ under $H_1$, denote

$$X[z] = \hat{b}\hat{s}_1 + V[z],$$

where $V[z]$ is data matrix when $b = 0$, or hypothesis $H_0$ is true. Then a multiplication of $X[z]$ on the right by $U^\dagger$ yields,

$$Y[z] = X[z]U^\dagger = [\hat{b}\hat{s}_1 + V[z]]\hat{s}_1^\dagger, Q^\dagger$$

$$= [\hat{b}\hat{s}_1 + V[z]]\hat{s}_1^\dagger, V[z]Q^\dagger$$

$$= [\hat{b} + V[z]\hat{s}_1^\dagger, V[z]Q^\dagger].$$

Evidently the action of $U$ on $X[z]$ is to send signal $\hat{b}\hat{s}_1$ to $\hat{b}[1,0,...,0]$ plus a noise-only term $V[z]\hat{s}_1^\dagger$ into the first column of $Y[z]$. The remaining $(N-1)$ columns of $Y[z]$, namely $V[z]Q^\dagger$, constitutes a signal-free $J \times (N-1)$ matrix from which an estimate of the covariance matrix $R$ in Eq.(3.9) can be found.
By the definition of $\mathbf{Y}[z]$ and $\mathbf{U}$ that

$$
\mathbf{Y}[z] = \mathbf{X}[z] \mathbf{U}^\dagger \mathbf{U} \mathbf{X}^\dagger [z] = \mathbf{X}[z] \mathbf{X}^\dagger [z]
$$

and also that

$$
\mathbf{Y}[z] = \mathbf{X}[z][\mathbf{s}_1^z, \mathbf{Q}^\dagger][\mathbf{s}_1^z, \mathbf{Q}^\dagger]^\dagger \mathbf{X}^\dagger [z]
$$

$$
= [(\mathbf{X}[z] \mathbf{s}_1^z, \mathbf{X}[z] \mathbf{Q}^\dagger)[\mathbf{X}[z] \mathbf{s}_1^z, \mathbf{X}[z] \mathbf{Q}^\dagger]^\dagger
$$

$$
= (\mathbf{X}[z] \mathbf{s}_1^z)(\mathbf{X}[z] \mathbf{s}_1^z)^\dagger + (\mathbf{X}[z] \mathbf{Q}^\dagger)(\mathbf{X}[z] \mathbf{Q}^\dagger)^\dagger. 
$$

Finally under $H_1$ by Eq. (3.37) one has

$$
\mathbf{Y}[z] = [(\mathbf{X}[z] \mathbf{s}_1^z, \mathbf{X}[z] \mathbf{Q}^\dagger)[(\mathbf{X}[z] \mathbf{s}_1^z, \mathbf{X}[z] \mathbf{Q}^\dagger)]^\dagger
$$

$$
= (\mathbf{X}[z] \mathbf{s}_1^z)(\mathbf{X}[z] \mathbf{s}_1^z)^\dagger + (\mathbf{X}[z] \mathbf{Q}^\dagger)(\mathbf{X}[z] \mathbf{Q}^\dagger)^\dagger. 
$$

The above identities (3.39) to (3.41) and (3.33) evidently establish the following relations:

$$
(\mathbf{X}[z] \mathbf{Q}^\dagger)(\mathbf{X}[z] \mathbf{Q}^\dagger)^\dagger = (\mathbf{V}[z] \mathbf{Q}^\dagger)(\mathbf{V}[z] \mathbf{Q}^\dagger)^\dagger
$$

$$
= \mathbf{X}[z] \mathbf{X}^\dagger [z] - (\mathbf{X}[z] \mathbf{s}_1^z)(\mathbf{X}[z] \mathbf{s}_1^z)^\dagger = \mathbf{G}. 
$$

Taking the expected value of the left side of Eq.(3.42) yields by a straightforward calculation which uses Eqs.(3.3), (3.9), (3.35), and the independence of the columns of $\mathbf{X}[z]$

$$
E \left\{ (\mathbf{X}[z] \mathbf{Q}^\dagger)(\mathbf{X}[z] \mathbf{Q}^\dagger)^\dagger \right\} = E \left\{ (\mathbf{V}[z] \mathbf{Q}^\dagger)(\mathbf{V}[z] \mathbf{Q}^\dagger)^\dagger \right\} = (N - 1)\mathbf{R},
$$
where $\mathbf{R}$ is the $J \times J$ covariance matrix of the discrete vector process $\mathbf{x}_k[i]$. This result shows that

$$
\hat{\mathbf{R}}_Q = \frac{1}{N-1} (\mathbf{X}[i] \mathbf{Q}^\dagger)(\mathbf{X}[i] \mathbf{Q}^\dagger)^\dagger
$$

(3.44)

is an unbiased estimate of $\mathbf{R}$ with $(N-1)$ degrees of freedom under both hypotheses $H_0$ and $H_1$, respectively. The $J(J+1)/2$ distinct elements of $\hat{\mathbf{R}}_Q$ are jointly Wishart distributed over the $J(J+1)/2$-dimensional space of positive definite matrices (see Appendix of [38]). Hence, $\hat{\mathbf{R}}_Q$ is positive definite with probability one. By Eqs.(3.42) and (3.44),

$$
\mathbf{G} = (\mathbf{X}[i] \mathbf{Q}^\dagger)(\mathbf{X}[i] \mathbf{Q}^\dagger)^\dagger = (N-1)\hat{\mathbf{R}}_Q.
$$

Since $\mathbf{G}^{-1}$ exists with probability one, the determinant of Eq.(3.32) can be expressed by

$$
|\mathbf{F}_b| = |\mathbf{G}| |\mathbf{I} + (\hat{\mathbf{b}} - \mathbf{X}[i] \hat{\mathbf{s}}_i^\dagger)(\hat{\mathbf{b}} - \mathbf{X}[i] \hat{\mathbf{s}}_i^\dagger)^\dagger \mathbf{G}^{-1}|
$$

$$
= |\mathbf{G}| |\mathbf{I} + \mathbf{a} \mathbf{a}^\dagger \mathbf{G}^{-1}|,
$$

(3.45)

where

$$
\mathbf{a} = \hat{\mathbf{b}} - \mathbf{X}[i] \hat{\mathbf{s}}_i^\dagger.
$$

(3.46)

Now multiply matrix $\mathbf{D} = \mathbf{I} + \mathbf{a} \mathbf{a}^\dagger \mathbf{G}^{-1}$ on the right by $\mathbf{a}$ as follows:

$$
\mathbf{D} \mathbf{a} = \mathbf{a} + \mathbf{a} (\mathbf{a}^\dagger \mathbf{G}^{-1} \mathbf{a}) = (1 + \mathbf{a}^\dagger \mathbf{G}^{-1} \mathbf{a}) \mathbf{a}.
$$

40
This shows that \( \mathbf{a} \) is an eigenvector of \( \mathbf{D} \) and its associated eigenvalue is \( 1 + \mathbf{a}^\dagger \mathbf{G}^{-1} \mathbf{a} \). Now let \( \{ \mathbf{v}_i \} \) for \( i = 1, 2, \ldots, J - 1 \) be some orthogonal set of vectors, all perpendicular to the vector \( \mathbf{G}^{-1} \mathbf{a} \). Then

\[
\mathbf{D} \mathbf{v}_i = (I + \mathbf{a} \mathbf{a}^\dagger \mathbf{G}^{-1}) \mathbf{v}_i = \mathbf{v}_i + \mathbf{a} [(\mathbf{G}^{-1} \mathbf{a})^\dagger \mathbf{v}_i] = \mathbf{v}_i
\]

so that \( \mathbf{v}_i \) is an eigenvector of \( \mathbf{D} \) with an associated eigenvalue equal to one for \( i = 1, 2, \ldots, J - 1 \). Since the determinant of an Hermitian symmetric matrix equals the product of its eigenvalues, the determinant in Eq.(3.45) is given by

\[
|\mathbf{F}_i| = |\mathbf{G}||1 + \mathbf{a}^\dagger \mathbf{G}^{-1} \mathbf{a}|
= |\mathbf{G}||1 + (\hat{\mathbf{b}} - \mathbf{X}[\varphi] \hat{\mathbf{s}}_i)\mathbf{G}^{-1}(\hat{\mathbf{b}} - \mathbf{X}[\varphi] \hat{\mathbf{s}}_i)|. \tag{3.47}
\]

If no default form is assumed for the antenna gain vector, the maximum solution of Eq.(3.27) can be achieved when the second term in Eq.(3.47) vanishes, i.e.,

\[
\hat{\mathbf{b}} = \mathbf{X}[\varphi] \hat{\mathbf{s}}_i, \tag{3.48}
\]

so that the likelihood ratio in Eq.(3.27) is given by

\[
\chi(\mathbf{X}[\varphi]) = \frac{|\mathbf{X}[\varphi] \mathbf{X}^\dagger[\varphi]|}{|\mathbf{G}|}
= \frac{|\mathbf{X}[\varphi] \mathbf{X}^\dagger[\varphi]|}{|\mathbf{X}[\varphi] \mathbf{X}^\dagger[\varphi] - (\mathbf{X}[\varphi] \hat{\mathbf{s}}_i)(\mathbf{X}[\varphi] \hat{\mathbf{s}}_i)^\dagger|} \tag{3.49}
= \frac{|\mathbf{X}[\varphi] \mathbf{X}^\dagger[\varphi]|}{|\mathbf{X}[\varphi] \mathbf{X}^\dagger[\varphi]| |I - (\mathbf{X}[\varphi] \hat{\mathbf{s}}_i)(\mathbf{X}[\varphi] \hat{\mathbf{s}}_i)^\dagger(\mathbf{X}[\varphi] \mathbf{X}^\dagger[\varphi])^{-1}|} \tag{3.50}
= \frac{1}{1 - (\mathbf{X}[\varphi] \hat{\mathbf{s}}_i)^\dagger(\mathbf{X}[\varphi] \mathbf{X}^\dagger[\varphi])^{-1}(\mathbf{X}[\varphi] \hat{\mathbf{s}}_i)}. \tag{3.51}
\]
where

\[ X[i]X^\dagger[i] = G + (X[i]s_1^\dagger)(X[i]s_1^\dagger)^\dagger \]

\[ = \left( I + (X[i]s_1^\dagger)(X[i]s_1^\dagger)^\dagger G^{-1} \right) G \triangleq AG. \tag{3.52} \]

\( X[i]X^\dagger[i] \) is evidently invertible since the inverse matrices of both \( A \) and \( G \) in Eq.(3.52) exist with probability one. Hence, Eq.(3.49) can achieve considerable simplification by factoring out \( X[i]X^\dagger[i] \). Eq.(3.51) is then derived from Eq.(3.50) which follows a well-known determinant identity proved in ([27], Appendix A, i.e., \( |I + XX^\dagger| = 1 + X^\dagger X \)). Thus, using Eq.(3.51), the test in Eq.(3.27) is equivalent to the new test statistic given by

\[ y(X[i]) = (X[i]s_1^\dagger)^\dagger (X[i]X^\dagger[i]^{-1}(X[i]s_1^\dagger) \]

\[ = \frac{(X[i]s_1^\dagger)^\dagger (X[i]X^\dagger[i]^{-1}(X[i]s_1^\dagger))}{s_1s_1^\dagger} \]

\[ \begin{array}{c}
H_1 \\
H_0
\end{array} \overset{\gamma_2}{\colonslash} \gamma_2, \tag{3.53} \]

where \( y(X[i]) \) is related to \( \lambda(X[i]) \) by \( \lambda(X[i]) = \frac{1}{1 - \gamma_1} \), with threshold \( \gamma_2 = 1 - \frac{1}{\gamma_1} \). Note that the maximum solution of test in Eq.(3.27) is attained without adding any assumption on a known array geometry, i.e., no special format like in Eq.(3.56) will be assumed to the array response vector while a uniformly spaced linear array is employed, and this is the reason that the estimated array response vector \( \hat{b} \) in Eq.(3.48) is termed as “the non-constrained MLE of the DOA”.

The test statistic in Eq.(3.53) is used to test at each time phase within time period \( NT_c \) for the existence of the signal of the desired user. If

\[ Y[i] = \{ y(X[i]), y(X[i-1]), \ldots, y(X[i - NS + 1]) \} \tag{3.54} \]
is the set of such tests in decreasing time sequence of the form, given in Eq.(3.53) over the just-past spreading code period total NS clock time, then the integer clock time

\[ \hat{i} \in \{i, i - 1, \ldots, i - NS + 1\}, \]

such that

\[ y(X[\hat{i}]) = \max_{k \in \{0, 1, \ldots, NS-1\}} y(X[i - k]) \quad (3.55) \]

is attained, is the discrete time that synchronization is most likely to occur within the interval \((i - NS + 1, i)\). The “greatest-of”, sequence-in-time test in Eq.(3.55), detects sequentially the maximum of all likelihood tests in the set \(Y[i]\) in Eq.(3.54).

### 3.3.2 DOA Acquisition System

#### 3.3.2.1 A Constrained MLE of DOA

Due to its comparative simplicity, the non-constrained MLE of DOA in Eq.(3.48) is used for the fast acquisition of the desired user-spreading code sequence. However, the information of the DOA obtained by the non-constrained MLE of DOA may not be sufficiently accurate.

For the constrained the MLE of the DOA it is assumed for simplicity that the direction vector \(b\) is that of a linear array which has the form:

\[ b = [1, e^{j\phi}, \ldots, e^{j(J-1)\phi}]^\top, \quad (3.56) \]
where

\[ \phi = \frac{2\pi d \sin \theta}{\lambda}. \]  \hspace{1cm} (3.57)

Here, \( \lambda \) is the signal-carrier wavelength, \( d \) is the spacing between antenna elements, and \( \theta \) is the DOA of the desired user.

The DOA of a specified spreading code sequence can be determined by finding the minimum value of the denominator of Eq.(3.27), which is given in Eq.(3.32). Since \( G \) in Eq.(3.47) does not depend on the variable \( \phi \), the minimization of Eq.(3.32) is equivalent to finding \( \theta \) such that

\[ \hat{\theta} = \min_{\hat{\theta}} [(\hat{b} - X[\hat{\theta}]\hat{s})^\dagger G^{-1} (\hat{b} - X[\hat{\theta}]\hat{s})], \]  \hspace{1cm} (3.58)

where \( X[\hat{\theta}] \), where \( \hat{\theta} \) is the time phase at which synchronization occurs, is the observed data matrix obtained upon synchronization. An exhaustive search within 180° is the most direct manner to obtain the solution of Eq.(3.58). In a manner similar to many high-resolution signal-parameter estimation schemes, such as MUSIC [46], Minimum-Norm (MN) [19], etc., the exhaustive search is in a 1-dimensional space. The computational complexity may be high, but the processing remains linear. Although the constrained MLE of DOA is developed for a uniformly spaced linear array in this proposal, the algorithm can be extended to accommodate any other antenna array with a different structure.

### 3.3.2.2 A Constrained MLE of DOA with the ESPRIT Algorithm

There exists a range of different approaches of high-resolution signal parameters estimation schemes that can be used for DOAs acquisition. Most schemes can efficiently convert the exhaustive search in high-dimensional parameters spaces into
only 1-dimensional space and still remain the high resolution on the estimated parameters, such as MUSIC and MN. Although MUSIC and MN can appropriately alleviate the searching burden, performing an exhaustive search in 1-dimensional space is still unavoidable. A variety of modifications to MUSIC and MN have been proposed to increase their resolution performance and decrease the computational complexity but they do not always yield sufficient accuracy. ESPRIT [40] is a computational efficient and robust parametric array processing method for DOAs acquisition with only knowledge of antenna array geometry and eigen-decomposition operation. Therefore, to alleviate the computational load and avoid the exhaustive search process, ESPRIT can be employed. ESPRIT derives its advantages by requiring that the sensor array have a structure that can be decomposed into two equal-sized identical subarrays with the corresponding elements of the two subarrays displaced from each other by a fixed translational distance $d$. ESPRIT formulations under a uniform linear array (ULA) assumption have been mentioned in [40]. While the translational distance is assumed to be $d$, the acquiring processes of DOAs can be summarized as follow:

$$\Psi_k = \text{eigenvalues of } \Psi, \ k = 1, 2, \ldots, Q,$$  \hspace{1cm} (3.59)

so that

$$\psi_k = \arcsin\left[\frac{\text{arg}(\Psi_k)}{2\pi d/\lambda}\right].$$  \hspace{1cm} (3.60)

where $\Psi$ is as the definition in [40]. Here $Q$ denotes the number of the signals contained in the observation data matrix. From Eqs. (3.59) and (3.60), ESPRIT eliminates the search procedure inherent in most DOA estimation methods and produces the DOA estimates directly in terms of the eigenvalues.
Nevertheless, in [56] Guanghan Xu et al. also successfully developed a beamspace transformation version of ESPRIT which is utilized not only to achieve computational savings over the element-space ESPRIT but to overcome the potential problem of a ULA losing its displacement invariance structure after the beamspace transformation. Usually beamspace ESPRIT is used with signal subspace finding technique called fast signal decomposition (FSD) [55] to assure the best performance in compariance with its alternatives. The possibility of an order computational saving in compariance with beamspace Root-MUSIC [59] has been achieved in some experiment results when a larger antenna array is employed. However, like most high-resolution estimation schemes, ESPRIT can acquire all of the users’ DOAs, but to identify the DOA of the desired spreading code sequence, the following search process needs to be performed, namely,

$$\hat{\theta} = \min_{\psi_k, 1 \leq k \leq Q} \left[ (\hat{b} - X[\psi_k]\hat{s}_k^\dagger)\tilde{G}^{-1}(\hat{b} - X[\psi_k]\hat{s}_k^\dagger) \right]$$

over the data in which the desired spreading code sequence is detected, where $\psi_k$ is defined in Eq.(3.60).

### 3.3.3 Minimum Variance Beamforming and Demodulation Systems

The sensitivity of an antenna array to jammers can be suppressed efficiently by processing the outputs of the individual array elements. To accomplish this special purpose, the adaptive array processing technique (i.e., beamforming technique) is inspired by solving a mean-squared constrained optimization problem. Let $X[\hat{\theta}]$ be the observed data matrix obtained upon synchronization and $\hat{b}$ denote the estimated DOA of the desired user which is acquired by the constrained
MLE of the DOA. This adaptive algorithm based on optimization problem solution consists of applying a weight vector \( \mathbf{w} \) to the data matrix \( \mathbf{X}[\hat{\mathbf{v}}] \) with the intent of minimizing the mean-square value of the weighted observations \( E \left\{ \| \mathbf{w}^\dagger \mathbf{X}[\hat{\mathbf{v}}] \|_2^2 \right\} \) subject to the constraint which is \( \mathbf{w}^\dagger \hat{\mathbf{b}} = 1 \), where the notation of \( \| \cdot \| \) denotes the norm of a vector. By applying the weight vector \( \mathbf{w} \) to the receiving array output, any signal from the propagation direction specified by \( \hat{\mathbf{b}} \) is emphasized, whereas signals and noise propagating from other directions are suppressed. The constraint \( \mathbf{w}^\dagger \hat{\mathbf{b}} = 1 \) ensures that the signal on the look direction passes to the beamformer with unity gain. The weight vector of this Minimum Variance Beamformer is the solution to the problem

\[
\min_{\mathbf{w}} E \left\{ \| \mathbf{w}^\dagger \mathbf{X}[\hat{\mathbf{v}}] \|_2^2 \right\} = \min_{\mathbf{w}} \mathbf{w}^\dagger \mathbf{M} \mathbf{w}, \quad \text{subject to } \mathbf{w}^\dagger \hat{\mathbf{b}} = 1. \tag{3.61}
\]

where the autocorrelation matrix of the input data \( \mathbf{X}[\hat{\mathbf{v}}] \) is \( \mathbf{M} = E \{ \mathbf{X}[\hat{\mathbf{v}}] \mathbf{X}[\hat{\mathbf{v}}]^\dagger \} \). Using Lagrange multipliers, the optimal weight vector that satisfies Eq.(3.61) can be shown to be [13]

\[
\hat{\mathbf{w}} = \frac{\hat{\mathbf{M}}^{-1} \hat{\mathbf{b}}}{\hat{\mathbf{b}}^\dagger \hat{\mathbf{M}}^{-1} \hat{\mathbf{b}}}.	ag{3.62}
\]

where \( \hat{\mathbf{M}} \) indicates the sample-average estimate of \( \mathbf{M} \).

To see explicitly how the weight vector \( \mathbf{w} \) acts upon the data matrix \( \mathbf{X}[\hat{\mathbf{v}}] \) under hypothesis \( H_1 \), a multiplication of \( \mathbf{X}[\hat{\mathbf{v}}] \) on the left by \( \mathbf{w}^\dagger \) yields

\[
\mathbf{w}^\dagger \mathbf{X}[\hat{\mathbf{v}}] = \mathbf{w}^\dagger (\hat{\mathbf{b}} \hat{\mathbf{s}}_1 + \mathbf{V}[\hat{\mathbf{v}}]) = \mathbf{w}^\dagger \hat{\mathbf{b}} \hat{\mathbf{s}}_1 + \mathbf{w}^\dagger \mathbf{V}[\hat{\mathbf{v}}] = \hat{\mathbf{s}}_1 + \mathbf{w}^\dagger \mathbf{V}[\hat{\mathbf{v}}]. \tag{3.63}
\]

Referred to Eqs.(3.61) and (3.63), the signal from the desired direction specified by \( \hat{\mathbf{b}} \) is sustained, whereas interferers propagating from other directions plus Gaussian noise are suppressed by minimizing the energy of \( E \left\{ \| \mathbf{w}^\dagger \mathbf{V}[\hat{\mathbf{v}}] \|_2^2 \right\} \). Note
that the optimum weight vector not only depends on the spatial correlation matrix of the observation but also on the estimated DOA of the desired signal, which is \( \hat{b} \). Thus, as various directions are scanned, the weight vector changes to adapt to the signal and noise components in the observation data. This beamformer that minimizes the output variance of the beamformer, subject to the constraint, can be also termed the Capon beamformer.

Once beamforming is completed, information-symbol demodulation is exploited by the use of a conventional detector. Then an estimate of the information symbol \( d_1 \) is obtained from the test statistic as follows:

\[
\hat{d}_1 = \text{sgn} \left( \text{Re} \left\{ \hat{w}^H X[i] \hat{s}_1^+ \right\} \right) = \text{sgn} \left( \text{Re} \left\{ z[i] \hat{s}_1^+ \right\} \right),
\]
where $\text{sgn}$ is the sign operator and $\text{Re}\{\cdot\}$ denotes the real part. The output vector of the minimum variance beamformer is given by

$$z[\hat{\tau}] = \hat{w}^T X[\hat{\tau}] .$$

The time acquisition process to find the estimate $\hat{\tau}$, defined in Eq. (3.55), is illustrated by the conceptual block diagram, shown in the Figure 3.4. The observation data matrices are obtained by sliding a window of length $N$ through the original data.

### 3.4 Numerical Results

In this section, three simulation examples are conducted to demonstrate the performance of the proposed detector for code-timing acquisition and information-symbol demodulation of DS-CDMA signals. First considered are two examples of an asynchronous BPSK DS-CDMA system with the number of users $K = 6$. Users 1, 3, and 5 are assumed to generate the multipath signals due to multipath reflections. The spreading code sequence of each user in the active mode is a Gold sequence of length $N = 31$ (see Appendix A). The detector employs a uniformly spaced linear array with twelve elements of half-wavelength spacing. A single user of interest, say user 2, is acquired by the proposed detector in the presence of MAI with different near-far ratios. The level of MAI is designated in terms of the ratio of the power of any interfering user to the 1st path of the desired user. In DS-CDMA Scenario I, all interfering users are assumed to have the same power ratio with respect to the desired user while it is selected randomly in
Table 3.1: Simulated channel parameters for DS-CDMA Scenarios I and II.

<table>
<thead>
<tr>
<th>Users</th>
<th>Delays (in ( T_c ))</th>
<th>DOAs (in degrees)</th>
<th>NFRs (in dB) (Scenario I)</th>
<th>NFRs (in dB) (Scenario II)</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
<td>1</td>
<td>3</td>
<td>5</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>15</td>
<td>0</td>
<td>-3</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
<td>-15</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>30</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>-45</td>
<td>0</td>
<td>-3</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>-5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
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<td></td>
<td>13</td>
<td>-30</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>21</td>
<td>60</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

the DS-CDMA Scenario II. This power advantage is denoted by a quantity called the *near-far ratio* (NFR) given by

\[
NFR = \frac{||\alpha_{i,j}||^2}{||\alpha_{2,1}||^2},
\]

where \( \alpha_{i,j} = A_l a_{i,j} d_l \) and the subscript ‘\( i,j \)’ denotes the \( j \)-th path of the \( l \)-th user. ‘\( 2,1 \)’ indicates the 1st path of the desired user, say user 2. The multipath propagation delays, the DOAs and the NFRs of all users in first two simulated examples are tabulated in Table 3.1. The relative delay listed in Table 3.1 is relative to the beginning of the 1st path of user 3. It is assumed for convenience that the propagation delays of the different users are multiples of \( T_c \) in these experiments. This assumption is not necessary if the sampling rate is higher than \( 1/T_c \). All experimental curves are derived by performing 1000 independent trials.
3.4.1 DS-CDMA Scenario I

In Figure 3.5, the probability of correct synchronization is presented as a function of the generalized signal-to-noise ratio (GSNR) (i.e., \( \text{GSNR} \triangleq \frac{\hat{s}_2 s_2^\dagger b_{2,1}^\dagger R_n^{-1} b_{2,1}}{} \)) under perfect power control scenario, i.e., all users have the same signal power (NFR = 0dB). Synchronization results in Figure 3.5 show that the performance of the proposed detector is substantially improved by a larger antenna array. When the number of antenna elements is slightly over 5, the synchronization performance approaches that of the case of 12 antenna elements. In Figure 3.5 the conventional detector, that uses the sample cross-correlation of the received signal with the user-spreading code sequence of interest, fails to identify the desired code timing as the curves show.

Figure 3.7(Upper) demonstrates the probability of correct demodulation in terms of the GSNR under the stringent power control in DS-CDMA Scenario
I. The cases that use 10 and 12 antenna sensors are considered under the assumptions of the synchronization being known or unknown \textit{a priori}. For the synchronization-absence case, the proposed detector is employed to acquire the synchronization information. The results show that no significant improvement can be achieved when the antenna sensors are increased from 10 to 12. The performance of demodulation is better when the synchronization is known \textit{a priori} than that of no synchronization information, which is acquired by the proposed detector, when GSNR is lower than 10dB, but has no significant improvement when GSNR is higher than 10dB.

3.4.2 DS-CDMA Scenario II

In order to consider the near-far situation, power ratios with respect to the desired user are assumed random, i.e., no stringent power control is employed, in DS-CDMA Scenario II. The other parameters are set identically with DS-CDMA Scenario I. Observe in Figure 3.6 that the performance of acquiring synchronization degrades under this near-far situation except for the larger antenna array case. In the absence of power control, the number of antenna elements must be increased to achieve the same performance as the power control case.

The parameters which are chosen to derive Figure 3.7(Lower) are remained the same as in Figure 3.7(Upper) except the poor power control environment is assumed. Compare two figures in Figure 3.7, it is evident that the proposed detector performs better in the perfect power control case than the poor power control case. Moreover, the proposed detector is much better than the conventional detector no matter what kind of environment is assumed when the GSNR increases.
3.4.3 DS-CDMA Scenario III

In Scenario III, an asynchronous BPSK DS-CDMA system with the number of users $K = 8$ is considered. The spreading code sequence of each user in the active cell is a Gold sequence of length $N = 31$ while the spreading codes for intercell interferers are random codes. Users 1, 2, and 3 are assumed to generate the multipath signals. A single user of interest with 3 multipath duplicates, say user 1, is acquired by the proposed detector in the presence of strong MAI and near-far environment. The multipath delays, the DOAs and the NFRs of all users in this tested scenario is tabulated in Table 3.2. The other parameters are set identically to DS-CDMA Scenario I. In this scenario power ratios with respect to user 1 are chosen randomly and set much worse than the values used in DS-CDMA Scenario II. The capability of the proposed detector to detect the multipath code timing of the desired user is also evaluated.

The probability of correct synchronization for an even-number of antenna sensors is plotted as a function of the GSNR under the conditions of stringent and poor power control scenarios. All three versions generated by user 1 must be identified correctly to be counted as a “correct” detection. Observe in Figure 3.8, the performance of synchronization is achieved almost the same for both tested cases of stringent and poor power control when a 12-element antenna array is employed. Both cases have perfect detection when GSNR is around 20dB. The performance of synchronization degrades slightly as the number of antenna elements is decreased to 8 for both cases. Perfect detection is still available for the stringent power control case, but not for the condition of poor power control. Moreover, the drastic degradation on the performance of synchronization is observed as the antenna number goes down to 6, especially for the poor power control case. In Figure 3.9, the proposed algorithm compares and identifies the multipath code timing of the desired spreading sequence accurately, while it is
difficult for the sample cross-correlator to make the correct decision. Therefore, multiple-access interferers are suppressed successfully in the cases that a large antenna array is employed. This remarkable result is observed especially under the poor power control scenario.

3.5 Conclusions

An adaptive algorithm for an asynchronous DS-CDMA system that exploits spatial-temporal diversity is proposed for detecting the multipath timing structure and information-symbol demodulation of a single desired user under multipath fading and a near-far environment. It should be noted that the requirement for the proposed detector is only prior knowledge of the desired-user spreading-code sequence. No side information of system users is necessary while focusing on a given user. Also, the proposed algorithm can be extended to multiuser detection by forming all of the single-user detectors in parallel if the other users’ spreading code sequences are available. In other words, the proposed algorithm can be used to identify the multipath timings for either multiuser or single-user case. The performance of timing-acquisition and symbol-demodulation is shown to be magnificently improved when a larger antenna array is employed and to have potential against channel fading and near-far problems.
Figure 3.5: Synchronization results for DS-CDMA Scenario I, (Upper): odd-number of antenna elements, (Lower): even-number of antenna elements.
Figure 3.6: Synchronization results for DS-CDMA Scenario II, (Upper): odd-number of antenna elements, (Lower): even-number of antenna elements.
Figure 3.7: Demodulation results for (Upper): DS-CDMA Scenario I and (Lower): DS-CDMA Scenario II.
Figure 3.8: Synchronization results using even-number of antenna elements for DS-CDMA Scenario III under the condition of (Upper): stringent power control, (Lower): poor power control.
Figure 3.9: Synchronization results of the sample cross-correlation algorithm (Upper) and the proposed algorithm (Lower). (K=8, J=12, SNR=6dB, and MAI≈14dB in a Near-Far Interference Environment, i.e., DS-CDMA Scenario III)
Chapter 4

Space-Time Adaptive Multistage Wiener Filtering for Asynchronous CDMA

4.1 Introduction

In the present chapter an adaptive near-far-resistant array receiver for asynchronous space-time DS-CDMA signals is developed. The primary difference to the receiver developed in Chapter 3 is that the independence assumption in the temporal domain presumed for the observed data matrix in (3.4) is released. Thus an array receiver that deals with the space-time joint observed data is presented in this chapter. The only requirement needed here for the proposed receiver timing is knowledge of the spreading code of the desired user. Moreover, there is no need for a pilot signal, a side channel, a long training period, or signal-free observations. Furthermore, a considerably lower complexity version of the proposed receiver that utilizes the recently developed reduced-rank multistage Wiener filter (MWF) of Goldstein and Reed [11] is presented. This latter technique obviates the necessity of either a covariance matrix inversion or an eigen-decomposition. As a consequence the computational complexity of the system is reduced substantially from $\mathcal{O}((JNS)^3)$ to $\mathcal{O}(M(JNS))$ for each computing cycle of clock time [41], where $M$ is the number of stages of the reduced-rank
multistage Wiener filter, where $1 \leq M \leq JNS - 1$. In fact $M$ often can be chosen such that $M \ll JNS$.

The computational complexity achieved by the proposed array receiver is comparable to the complexity $O(JNS)$ of the MMSE CDMA receiver that uses the adaptive least mean squares (LMS) coefficients update algorithm [58]. But the proposed receiver does not have the drawback of convergence instability and sluggishness of an LMS-based algorithm, which is due principally to its dependence on the eigenvalue spread. Moreover, the achieved computational efficiency is much better than that of the adaptive recursive least-squares (RLS) taps-update algorithm used in the linear MMSE CDMA receiver (with $O((JNS)^2)$ operations) [58]. Also this multistage adaptive filtering scheme achieves a rapid adaptive convergence under limited observation-data support. These important features contribute significantly to the reduction of the computational cost and the amount of data sample support needed to accurately estimate a covariance matrix.

This chapter is organized as follows: In Section 4.2 the asynchronous DS-CDMA system is introduced. Binary phase-shift keying (BPSK) modulation is developed and used in Section 4.2 and the remainder of this paper. However, the proposed receiver can be applied to many other modulation schemes that utilize other symbol alphabets and pseudo-noise spreading-sequence generators. Also, the proposed receiver is not limited by the type of spreading code or the code length of the desired user or of other users. In Sections 4.3.1 and 4.3.2 the code-timing acquisition and demodulation systems of the receiver are derived in detail. For demodulation an adaptive MMSE detector is implemented. Also the adaptive MMSE detector is proved to play the same role as the maximum a posteriori probability (MAP) detector and maximum-likelihood (ML) detector if the source is equi-probable. A low complexity version of the receiver that is based on the multistage representation of the adaptive Wiener filter is presented.
in Section 4.3.3. In Section 4.3.4 the estimate of the steering vector that is used in the acquisition and demodulation systems is obtained by the aid of a training sequence followed by a decision-directed adaptation. The adaptive multistage realization of the reduced-rank receiver is developed in Section 4.4. A more efficient IIR approach, i.e., an iterative algorithm, is used to simplify the implementation of the adaptive reduced-rank receiver in Section 4.5. Finally, simulation results and conclusions are provided in Sections 4.6 and 4.7, respectively.

4.2 Asynchronous DS-CDMA System Model

Consider an asynchronous DS-CDMA system that uses BPSK modulation with \( K \) simultaneous users on an additive white Gaussian noise (AWGN) baseband channel. All such user's signals are combined linearly on the receiving antenna array which has \( J \) sensor elements. The received signal can be written as a received \( J \)-vector \( \mathbf{r}(t) \) as follows:

\[
\mathbf{r}(t) = \sum_{l=1}^{K} A_l a_l \mathbf{b}_l \sum_{m=-\infty}^{\infty} d_l[m] s_l(t - mT_b - \tau_l) + \mathbf{n}(t).
\] (4.1)

Here in (4.1) the parameters of this \( J \)-vector \( \mathbf{r}(t) \) are defined as follows:

- \( A_l \) The amplitude of user \( l \).
- \( a_l \) The complex gain of the channel that corresponds to the signal of user \( l \).
- \( \mathbf{b}_l \) The array-response (or antenna) \( J \)-vector of user \( l \).
- \( d_l[m] \) The \( m \)-th information symbol of user \( l \) and \( d_l[m] \in \{ \pm 1, 0 \} \), where symbol zero denotes the fact that the \( l \)-th user is off (has either not started or has finished transmission).
- \( T_b \) The information symbol interval.
- \( \tau_l \) The propagation delay that corresponds to user \( l \).
- \( \mathbf{n}(t) \) Additive white Gaussian noise (AWGN) vector.
Here, $s_l(t)$ is the spreading code waveform of the $l$-th user, given by

$$s_l(t) = \sum_{k=0}^{N-1} c_{l,k} p(t - kT_c), \quad 0 \leq t \leq T_b,$$

for $l = 1, 2, \ldots, K$, where $T_c$ is the time width of a chip and $p(t)$ is the chip waveform, assumed for simplicity to be a rectangular signal of duration $T_c$. The spreading code waveforms are assumed to have unit amplitude and, except for sign, occur periodically with the time period $T_b$. In one symbol period there are $N = T_b/T_c$ chips, modulated with the spreading code $(c_{l,0}, c_{l,1}, \ldots, c_{l,N-1})$. Here $N$ is called the spreading (or processing) gain. Also the array response vector $b_l$ is the $J$-component vector $[b_{l,1}, b_{l,2}, \ldots, b_{l,J}]^\top$ for the receiver array of $J$ elements (sensors). The symbol "$^\top$" denotes the matrix transpose operator, and $n(t)$ denotes a $J$-dimensional Gaussian noise and interference process, i.e., $n(t) = [n_{1}(t), n_{2}(t), \ldots, n_{J}(t)]^\top$, for the $J$-element antenna.

### 4.3 Receiver Structure

Consider a receiving-array antenna with $J$ receiving elements. For convenience the proposed receiver is described by means of a baseband-equivalent structure. Such a baseband complex signal process is achieved physically by phase-locked loops (PLLs) and $J$ quadrature demodulators, e.g., see [33], Chapter 6. This converts the received radio frequency (RF) modulated signal vector to a baseband complex-valued signal vector. The received signal of each individual antenna element, e.g., dipole, waveguide slot, etc., is passed through a filter that is matched to the square-wave chip waveform (or other type of waveform that is used for the
chip waveform). If \( p(t) \) is a rectangular pulse and \( r_k(t) \) is the \( k \)-th component of \( r(t) \) in (4.1), the output of the \( k \)-th antenna element is

\[
x_k(t) = \int_{-\infty}^{t} p(t - t') r_k(t') dt' = \int_{0}^{T_c} r_k(t - u) du,
\]

(4.2)

for \( k = 1, 2, \ldots, J \). Subsequently, the output of this chip MF is sampled every \( T_s \) seconds, where \( S \) (\( \triangleq T_c/T_s \)) is the number of samples in each chip interval \( T_c \), which is assumed here for purposes of simulation to be an integer and \( S \geq 1 \). This means that the sampling rate is greater than or equal to the chip rate.

Subsequently, the output of this chip MF is sampled every \( T_s \) seconds, where the sampling rate \( S \) (\( \triangleq T_c/T_s \)) is assumed for simplicity to be an integer and \( S \geq 1 \). This means that the sampling rate is greater than or equal to the chip rate.

When \( S = 1 \), the chip MF output is sampled at the chip rate. The effect of chip asynchronism may lead to an average loss, on the assumption that the delay is uniformly distributed in \([0, T_b]\), of \( 10\log_{10}(\frac{3}{2}) = 1.76 \) dB to the signal-to-noise ratio (SNR) with a worst-case loss of approximately 3 dB for a particular user when the timing misalignment for that user is exactly \( \frac{1}{2} T_c \) [2]. Fortunately, the loss in SNR can be alleviated by letting \( S > 1 \). However, the amount of data support and the hardware requirements grow proportionally as \( S \) increases and so does the computational complexity. As a consequence the choice of \( S \) is generally made by a tradeoff between algorithmic performance and computational complexity.

These discrete-time outputs are used as the inputs of \( J \) adaptive, \( NS \)-element, tapped delay lines (TDLs) with a tap spacing of \( T_s \) to form \( J \) such \( NS \)-element data vectors. If \( S \) is not an integer, the number of samples in a symbol period is \( \lfloor NT_c/T_s \rfloor \) (or \( \lfloor T_b/T_s \rfloor \)), where the notation \( \lfloor x \rfloor \) indicates the greatest integer less than \( x \). Assume that the output signals of the chip MFs are sampled at the
times $i T_s$. The samples are taken over all $NS$ time intervals. The TDLs for the $J$-element antenna array are expressed as a $J \times NS$ data matrix, given by

$$
Z[i] = 
\begin{bmatrix}
  x_1(i T_s) & x_1((i - 1) T_s) & \cdots & x_1((i - NS + 1) T_s) \\
  x_2(i T_s) & x_2((i - 1) T_s) & \cdots & x_2((i - NS + 1) T_s) \\
  \vdots & \vdots & \ddots & \vdots \\
  x_J(i T_s) & x_J((i - 1) T_s) & \cdots & x_J((i - NS + 1) T_s)
\end{bmatrix}.
$$

(4.3)

To avoid cumbersome two-dimensional filtering operations and notation, $Z[i] \in \mathbb{C}^{J \times NS}$ is “vectorized” by sequencing all matrix rows to form a $JNS$-vector as follows:

$$
x[i] = Vec \{Z[i]\} = [z_1[i], z_2[i], \ldots, z_{JNS}[i]]^T.
$$

(4.4)

The vector $x[i]$ in (4.4) denotes the joint space-time data of the $C^{JNS \times 1}$ complex vector domain, and the $z_n[i]$ for $n = 1, 2, \ldots, JNS$ are the data components of the vector $x[i]$.

Similarly the adaptive weight vector of a filter for vector $x[i]$ is expressed as the column vector,

$$
w[i] = [w_1[i], w_2[i], \ldots, w_{JNS}[i]]^T.
$$

(4.5)

The components of the weight vector $w[i]$ as an optimum Wiener filter are determined explicitly later in Eq.(4.39) of Section 4.3.1.

The output of the tapped delay-line adaptive filter for $x[i]$ is the inner product of the vectors in Eqs.(4.4) and (4.5) as follows:

$$
y[i] = w^\dagger[i] x[i] = \sum_{n=1}^{JNS} w_n^a[i] z_n[i],
$$

(4.6)
where "\(^*\)" denotes the conjugate (Hermitian) transpose of a matrix and "\(^*\)" denotes the conjugate of a complex number. This output is passed through the time-synchronization acquisition system to obtain the required information about synchronization. This time acquisition system can be modeled conceptually as a filter bank constructed of \(N\) filters in sequence with each filter being of the type, shown in (4.6), in order to identify the "optimum" time phase at which to demodulate the information symbol of the desired user.

### 4.3.1 Test Statistic for Code-Timing Acquisition

As it is shown in [6], the synchronization problem in communications can be treated as a signal detection problem analogous to radar detection. The detection of a single desired spreading code sequence, say of user 1, hereafter called the target signal that is embedded in the MAI plus noise, can also be modeled as a binary-hypothesis testing problem, where \(H_0\) corresponds to the MAI plus receiver noise process with the target-signal absent, and \(H_1\) corresponds to the presence of the target signal in the same noise process. Thus at each time phase of the \(\text{JNS}\)-vector \(x[i]\) the time-synchronization system must distinguish between the two hypotheses, \(H_0\) for noise only and \(H_1\) for noise plus target signal, i.e., the spreading code sequence for user 1. The target-signal vector under hypothesis \(H_1\) is given by the \(\text{JNS}\)-vector \(A_1 a_1 d_1 (b_1 \otimes s_1)\), where \(A_1\) is the amplitude of user 1, \(a_1\) denotes the complex gain introduced by the receiver following the antenna array, \(d_1\) is the information bit of user 1, \(b_1 = [b_{11}, b_{21}, \ldots, b_{J1}]^\top\) represents the direction vector of user 1, and \(s_1 = [c_{1,0}, c_{1,1}, \ldots, c_{1,\text{NS}-1}]^\top\) is the sampled signature spreading-code vector of user 1. Here \(\otimes\) denotes the Kronecker product of vectors [12], defined by

\[
b_1 \otimes s_1 = [b_{11}, b_{21}, \ldots, b_{J1}]^\top \otimes [c_{1,0}, c_{1,1}, \ldots, c_{1,\text{NS}-1}]^\top,
\]
\[
= [b_{11}c_{1,0}, \ldots, b_{11}c_{1,NS-1}, b_{21}c_{1,0}, \ldots, b_{J1}c_{1,NS-1}]^T.
\]

(4.7)

For a linear array and identical element patterns, \( \mathbf{b}_1 \) has the form:

\[
\mathbf{b}_1 = [1, e^{j\phi_1}, \ldots, e^{(J-1)j\phi_1}]^T,
\]

(4.8)

where

\[
\phi_1 = \frac{2\pi d \sin \theta_1}{\lambda}.
\]

(4.9)

Here, \( \lambda \) is the signal-carrier wavelength, \( d \) is the spacing between antenna elements, and \( \theta_1 \) is the angular antenna-boresight bearing of user 1 in radians.

To determine the unknown time phase of the desired spreading code pattern a detector is designed by sliding a “search window” of length \( NS \), sample-by-sample, along the entire received data sequence of \( NS \) samples. Thus the two hypotheses, that the adaptive detector must distinguish at each sampling time, are given by

\[
H_0 : \quad \mathbf{x}[i] = \mathbf{v}[i],
\]

\[
H_1 : \quad \mathbf{x}[i] = d_1 \mathbf{u} + \mathbf{v}[i],
\]

(4.10)

where

\[
\mathbf{u} = g_1 (\mathbf{b}_1 \otimes \mathbf{s}_1)
\]

(4.11)

is what is called the steering vector and the complex scalar \( g_1 \) in (4.11) is \( g_1 = A_1 a_1 \). Also in (4.10) the vector \( \mathbf{v}[i] = [v_1[i], v_2[i], \ldots, v_{NS}[i]]^T \) represents the interference-plus-noise environment without the target signal \( d_1 \mathbf{u} \) for user 1. By the law of large numbers the interference-plus-noise process \( \mathbf{v}[i] \) is assumed to
approximate zero-mean, non-white, complex Gaussian noise (also see [38]), where the associated covariance matrix is defined as follows:

$$R_{\mathbf{v}}[i] \overset{\Delta}{=} E\{\mathbf{v}[i]\mathbf{v}^*[i]\},$$

where $E\{\cdot\}$ denotes the expected-value operator. This test is equivalent to the type of test commonly used in space-time adaptive (STAP) radar [5] to detect the presence of a target signal such as $d_1 \mathbf{u}$, given above.

It is difficult to separate the presence of desired signal $d_1 \mathbf{u}$ under hypothesis $H_1$ from the interference-plus-noise vector $\mathbf{v}[i]$ under hypothesis $H_0$. This is particularly true since the interference in the process $\mathbf{x}[i]$, which the sampled version of the filtered original received signal $\mathbf{r}(t)$ in Eq. (4.1) is composed of $K$ signals all of which are of the same nature as the designated target signal $s_1(t)$. To simplify this separation process, consider a general invertible linear transformation $\mathbf{T}$ that operates on $\mathbf{x}[i]$ in the test given in Eq. (4.10) as follows:

$$H_0 : \quad \mathbf{Tx}[i] = \mathbf{Tv}[i],$$

$$H_1 : \quad \mathbf{Tx}[i] = d_1 \mathbf{Tu} + \mathbf{Tv}[i]. \quad (4.12)$$

The random vector $\mathbf{Tx}[i]$, when conditioned on the first user’s information symbol $d_1$ ($d_1 = \pm 1$), is still an approximate complex Gaussian process under both hypotheses. Thus the conditional probability densities of $\mathbf{Tx}[i]$ are given by

$$P(\mathbf{Tx}[i]|H_1, d_1) = \frac{1}{\pi JNS |\mathbf{TR}_{\mathbf{v}}[i]\mathbf{T}^*|^2} e^{-(\mathbf{Tx}[i] - d_1 \mathbf{Tu})^*(\mathbf{TR}_{\mathbf{v}}[i]\mathbf{T}^*)^{-1}(\mathbf{Tx}[i] - d_1 \mathbf{Tu})}, \quad (4.13)$$

$$P(\mathbf{Tx}[i]|H_0) = \frac{1}{\pi JNS |\mathbf{TR}_{\mathbf{v}}[i]\mathbf{T}^*|^2} e^{-(\mathbf{Tx}[i])^*(\mathbf{TR}_{\mathbf{v}}[i]\mathbf{T}^*)^{-1}(\mathbf{Tx}[i])}, \quad (4.14)$$
where \( |\mathbf{R}| \) denotes the determinant of \( \mathbf{R} \). From Eq. (4.13) the conditional probability density of \( \mathbf{Tx}[\mathbf{z}] \) given \( H_1 \) is expressed in terms of the conditional probabilities \( P(\mathbf{Tx}[\mathbf{z}]|H_1,d_1) \) for \( d_1 = 1 \) or \(-1\) as follows:

\[
P(\mathbf{Tx}[\mathbf{z}]|H_1) = \sum_{d_1} P(d_1) \cdot P(\mathbf{Tx}[\mathbf{z}]|H_1,d_1),
\]

(4.15)

where it is assumed that \( P(d_1 = 1) = P(d_1 = -1) = 1/2 \). Thus, the Bayes-optimum likelihood-ratio test (LRT) [50] by Eq.(4.15) evidently takes the form,

\[
\Lambda = \frac{P(\mathbf{Tx}[\mathbf{z}]|H_1)}{P(\mathbf{Tx}[\mathbf{z}]|H_0)}
= \frac{1}{2} \left( \frac{P(\mathbf{Tx}[\mathbf{z}]|H_1,d_1 = 1) + P(\mathbf{Tx}[\mathbf{z}]|H_1,d_1 = -1)}{P(\mathbf{Tx}[\mathbf{z}]|H_0)} \right).
\]

(4.16)

This is equivalent to

\[
\Lambda = Z \cdot \frac{e^{(\mathbf{U}^\dagger \mathbf{U})} + e^{-(\mathbf{U}^\dagger \mathbf{U})}}{2},
= Z \cdot \frac{e^{2\text{Re}(\mathbf{U})} + e^{-2\text{Re}(\mathbf{U})}}{2},
= Z \cdot \cosh(2\text{Re}(\mathbf{U})),
= Z \cdot \cosh \left( 2\text{Re} \left\{ (\mathbf{T_u}^\dagger (\mathbf{T_R}_\mathbf{x}[\mathbf{z}]^\dagger \mathbf{T})^{-1} \mathbf{T x}[\mathbf{z}] \} \right) \right)^{H_1 \gtrless H_0} \gamma,
\]

(4.17)

where

\[
Z = e^{-(\mathbf{T_u}^\dagger (\mathbf{T_R}_\mathbf{x}[\mathbf{z}]^\dagger \mathbf{T})^{-1} (\mathbf{T_u})},
U = (\mathbf{T_u}^\dagger (\mathbf{T_R}_\mathbf{x}[\mathbf{z}]^\dagger \mathbf{T})^{-1} \mathbf{T x}[\mathbf{z}].
\]

Here \( \text{Re} \{ \cdot \} \) denotes the real part and \( \gamma \) is the detection threshold. Evidently this test no longer depends on the values of \( d_1 \). The hyperbolic cosine function \( \cosh (\cdot) \) is a monotonically increasing function in the magnitude of its argument. Also for
the constant $Z$ one has $Z > 0$ so that $Z$ can be absorbed into the threshold of the test. Thus the test in Eq. (4.17) is equivalent to the test,

$$\| \text{Re} \left\{ (Tu)^\dagger (TR_v[i]T^\dagger)^{-1}Tx[i] \right\} \| \ \overset{H_1}{\gtrsim} \ \overset{H_0}{\lesssim} \ \gamma_1,$$

(4.18)

where $\gamma_1$ is another threshold constant. Since $T$ is an invertible operator, the test statistic in Eq. (4.18) is equivalent to the test,

$$\| \text{Re} \left\{ u^\dagger R_{v[i]}^{-1}x[i] \right\} \| \ \overset{H_1}{\gtrsim} \ \overset{H_0}{\lesssim} \ \gamma_1,$$

(4.19)

which is independent of the invertible transformation $T$. This is the likelihood ratio test that would be obtained from the original data without using transformation $T$ [50]. Thus, the likelihood ratio test is not changed by the introduction of such an invertible transformation $T$. In a CDMA-type system the quantities $u$ and $R_v[i]$ are usually not known \textit{a priori}. Thus to perform the test in (4.19), it is necessary to find estimates $\hat{u}[i]$ and $\hat{R}_v[i]$ to substitute for $u$ and $R_v[i]$, respectively.

In a CDMA system, the computationally cumbersome direct estimation of the matrix $R_v[i]$ is avoided next by introducing the appropriate nonsingular transformation $T_1$. Consider the $JNS \times JNS$ invertible linear transformation matrix $T_1[i]$, given by

$$T_1[i] = \begin{bmatrix} \hat{u}_1[i] \\ B_1[i] \end{bmatrix},$$

(4.20)

where $u_1[i] = u/\sqrt{u^\dagger u}$ is the unit vector in the direction of $u$, defined in (4.11) and estimated in (4.69), and the $(JNS-1) \times JNS$ matrix $B_1[i]$ is known as a blocking matrix whose rows are composed of any orthonormal basis set of the orthogonal
complement of the space (or span) of the signal \( u \). Thus, \( B_1[i]u[i] = B_1[i]u = 0 \). Hence the transformed output vector can be expressed in the form,

\[
T_1[i]x[i] = \begin{bmatrix} u[i]x[i] \\ B_1[i]x[i] \end{bmatrix} = \begin{bmatrix} \delta_1[i] \\ x_1[i] \end{bmatrix},
\]

where \( \delta_1[i] = u[i]x[i] \) and \( x_1[i] = B_1[i]x[i] \). Here, the data vector \( x[i] \) is split by the transformation \( T_1[i] \) into two channels, namely, the scalar channel \( \delta_1[i] \) and the \((JNS-1)\)-vector channel \( x_1[i] \). The data vector \( x_1[i] \) contains no information about signal \( u[i] \) since it is obtained by a projection of the data onto the orthogonal complement of the signal subspace. Actually, the \( \delta_1[i] \) process is obtained by cross-correlating the data vector \( x[i] \) with the signal \( u[i] \) so that the residual interference in \( \delta_1[i] \) is suppressed by the low autocorrelation and cross-correlation properties of the spreading Gold or other pseudo-noise codes with the other users' codes of the same class. The \( \delta_1[i] \) channel is the same process which would be obtained from the conventional cross-correlation detector. The “auxiliary” channel \( x_1[i] \), which contains information only about the interference-plus-noise process, is used to cancel MAI with a Weiner filter which estimates the non-white residual noise process in the \( \delta_1[i] \) channel. The correlation matrix of the transformed vector \( T_1[i]x[i] \) is expressed next as the partitioned matrix,

\[
T_1[i]R_x[i]T_1^*[i] = \begin{bmatrix} u[i]R_x[i]u[i] & u[i]R_x[i]B_1[i] \\ B_1[i]R_x[i]u[i] & B_1[i]R_x[i]B_1[i] \end{bmatrix}
= \begin{bmatrix} \sigma^2_\delta[i] & r_{x_1\delta_1}[i] \\ r_{x_1\delta_1}[i] & R_{x_1}[i] \end{bmatrix},
\]

where

\[
\sigma^2_\delta[i] = u[i]R_x[i]u[i] = E\{\delta_1[i]\delta_1^*[i]\},
\]

71
\[ \mathbf{r}_{x_i\delta_i}[\ell] = \mathbf{B}_1[\ell] \mathbf{R}_x[\ell] \mathbf{u}_1[\ell] = E \{ x_i[\ell] \delta_i^*[\ell] \}, \quad (4.24) \]

\[ \mathbf{R}_{x_i}[\ell] = \mathbf{B}_1[\ell] \mathbf{R}_x[\ell] \mathbf{B}_1^*[\ell] = E \{ x_i[\ell] x_i^*[\ell] \}. \quad (4.25) \]

Next the signal-free correlation matrix \( \mathbf{R}_v[\ell] \), needed in Eq.(4.18), evidently is expressed in terms of \( \mathbf{R}_x[\ell] \) under hypothesis \( H_1 \) by the relation,

\[ \mathbf{R}_v[\ell] = \mathbf{R}_x[\ell] - \mathbf{u} \mathbf{u}^\dagger \]

\[ = \mathbf{R}_x[\ell] - (g_1(b_1 \otimes s_1))(g_1(b_1 \otimes s_1))^\dagger, \quad (4.26) \]

where \( \mathbf{u} \mathbf{u}^\dagger \) in (4.26) is the \( JNS \times JNS \) outer product matrix of vector \( \mathbf{u} \) in (4.11) with itself. If one defines the positive scalar (norm), \( \Delta_1[\ell] = \sqrt{\mathbf{u}^\dagger \mathbf{u}} \), one obtains, using Eq.(4.26), the relations,

\[ \mathbf{T}_1[\ell] \mathbf{R}_v[\ell] \mathbf{T}_1^*[\ell] = \begin{bmatrix} \mathbf{u}_1^\dagger[\ell] \mathbf{R}_v[\ell] \mathbf{u}_1[\ell] & \mathbf{u}_1^\dagger[\ell] \mathbf{R}_v[\ell] \mathbf{B}_1^*[\ell] \\ \mathbf{B}_1[\ell] \mathbf{R}_v[\ell] \mathbf{u}_1[\ell] & \mathbf{B}_1[\ell] \mathbf{R}_v[\ell] \mathbf{B}_1^*[\ell] \end{bmatrix} \]

\[ = \begin{bmatrix} \sigma_1^2[\ell] - \Delta_1^2[\ell] & \mathbf{r}_{x_i\delta_i}[\ell] \\ \mathbf{r}_{x_i\delta_i}[\ell] & \mathbf{R}_{x_i}[\ell] \end{bmatrix}. \quad (4.27) \]

By the aid of the matrix inversion lemma [45], the inverse of \( \mathbf{T}_1[\ell] \mathbf{R}_v[\ell] \mathbf{T}_1^*[\ell] \) is computed to be

\[ \left( \mathbf{T}_1[\ell] \mathbf{R}_v[\ell] \mathbf{T}_1^*[\ell] \right)^{-1} = \kappa_1^{-1}[\ell] \begin{bmatrix} 1 & -\mathbf{r}_{x_i\delta_i}[\ell] \mathbf{R}_{x_i}^{-1}[\ell] \\ -\mathbf{R}_{x_i}^{-1}[\ell] \mathbf{r}_{x_i\delta_i}[\ell] & \mathbf{R}_{x_i}^{-1}[\ell] \left( \kappa_1[\ell] \mathbf{I} + \mathbf{r}_{x_i\delta_i}[\ell] \mathbf{r}_{x_i\delta_i}^\dagger[\ell] \mathbf{R}_{x_i}^{-1}[\ell] \right) \end{bmatrix}, \quad (4.28) \]

where

\[ \kappa_1[\ell] = \zeta_1[\ell] - \Delta_1^2[\ell], \quad (4.29) \]
Figure 4.1: Values of parameter \( (\kappa^{-1}_1[i] \Delta_1[i]) \) as a function of the time phase \( i \) for NFR = 3dB under various SNR values.

with \( \xi_1[i] \) defined by

\[
\xi_1[i] = \sigma_{\delta_1}^2[i] - r_{X_1 \delta_1}^+[i] R_{X_1}^{-1}[i] r_{X_1 \delta_1}^-[i].
\] (4.30)

When hypothesis \( H_0 \) is true, \( R_\nu[i] \) is equivalent to \( R_x[i] \) due to the absence of the target-signal \( g_1 d_1(\mathbf{b}_1 \otimes \mathbf{s}_1) \) in (4.26). Now the correlation matrix \( T_1[i] \mathbf{R}_\nu[i] T_1^T[i] \) of the transformed vector \( T_1[i] \mathbf{v}[i] \) has the same form as \( T_1[i] \mathbf{R}_x[i] T_1^T[i] \) in (4.22). The matrix \( (T_1[i] \mathbf{R}_\nu[i] T_1^T[i])^{-1} \) is then obtained using (4.28) with \( \kappa_1[i] = \xi_1[i] \).

Hence the test statistic in Eq.(4.19) for both hypotheses is obtained now as

\[
\| \text{Re} \left\{ u^\dagger R_\nu^{-1}[i] x[i] \right\} \| = \| \text{Re} \left\{ u^\dagger T_1[i] \left( T_1[i] \mathbf{R}_\nu[i] T_1^T[i] \right)^{-1} T_1[i] x[i] \right\} \| \quad (4.31)
\]

\[
= \| \text{Re} \left\{ \kappa_1^{-1}[i] \Delta_1[i] \left( u_1[i] - w_{\text{GSC}}^T[i] \mathbf{B}_1[i] \right) x[i] \right\} \| \quad (4.32)
\]

\[
= \| \text{Re} \left\{ \omega_1[i] \left( u_1[i] - w_{\text{GSC}}^T[i] \mathbf{B}_1[i] \right) x[i] \right\} \|, \quad (4.33)
\]

where

\[
w_{\text{GSC}}^T[i] = r_{X_1 \delta_1}^+[i] R_{X_1}^{-1}[i] \quad (4.34)
\]
with

\begin{align}
\omega_{1}[i] &= \kappa_{1}^{-1}[i] \Delta_{1}[i], \quad (4.35) \\
\kappa_{1}[i] &= \xi_{1}[i] - \Delta_{2}^2[i] \quad \text{under } H_1 \quad (4.36) \\
&= \xi_{1}[i] \quad \text{under } H_0. \quad (4.37)
\end{align}

where \( \xi_{1}[i] \) is defined in (4.30). The quantity \( \omega_{1}[i] = (\kappa_{1}^{-1}[i] \Delta_{1}[i]) \) in Eq. (4.35) is proved to be positive because the scalar \( \kappa_{1}^{-1}[i] \) is one of the diagonal elements of the positive-definite matrix \( (T_{1}[i]R_{y}[i]T_{1}^{T}[i])^{-1} \) (see Appendix C). This fact is also demonstrated experimentally in Figure 4.1 for the simulated Scenario \( (K = 6, \; N = 31) \) with the assumption of \( 3(JN) \) data samples and a 2-element receiving antenna under various SNR values. This implies that the term \( \omega_{1}[i] \) is a positive scalar over the symbol period, and as a consequence it can be ignored when the information-bearing symbol is explored. Thus the test statistic in Eq. (4.31) is the log-likelihood ratio test, given as follows:

\[
\| \text{Re} \{ y[i] \} \| = \| \text{Re} \{ \omega_{1}[i] \left( \left( u_{1}^{T}[i] - w_{GSC}[i]B_{1}[i] \right) x[i] \right) \} \| \quad \overset{H_{1}}{\underset{H_{0}}{\gtrless}} \gamma_{1}, \; (4.38)
\]

where

\[
y[i] = \omega_{1}[i] \left( \left( u_{1}^{T}[i] - w_{GSC}[i]B_{1}[i] \right) x[i] \right). \; (4.39)
\]

The block-diagram structure of the test statistic is shown in Figure 4.2. Evidently this test statistic has the form of the classical generalized sidelobe canceler (GSC) [1] that was used originally to suppress or cancel interferers or jamming of radars and communication systems.
The test statistic in Eq. (4.38) is used to test at each time phase within time period $NT_c$ for the existence of the signal of the desired user. If

$$Y[i] = \{||\Re \{y[i]\}||, ||\Re \{y[i - 1]\}||, \ldots, ||\Re \{y[i - NS + 1]\}||\}$$  \hspace{1cm} (4.40)

is the set of such tests in decreasing time sequence of the form, given in Eq. (4.38) over the just-past spreading code period, a total of $NS$ clock times. Then the integer clock time,

$$i \in \{i, i - 1, \ldots, i - NS + 1\}$$  \hspace{1cm} (4.41)

such that

$$||\Re \{y[i]\}|| = \max_{k \in \{0, 1, \ldots, NS-1\}} ||\Re \{y[i - k]\}||$$  \hspace{1cm} (4.42)

is attained, is the discrete time that synchronization is most likely to occur within the interval $(i - NS + 1, i)$ for $i = 0, 1, \ldots, NS - 1$. The “greatest-of”, sequence-in-time test in Eq. (4.42) detects sequentially the maximum of all likelihood tests in the set $Y[i]$ in Eq. (4.40).

![Diagram](image)

Figure 4.2: The structure of the test statistic.
4.3.2 Demodulator

4.3.2.1 Minimum Mean-Squared Error (MMSE) Detector

Due to its simple implementation and good performance, the minimum mean-
squared error (MMSE) detector often is employed [54] to explore the desired
information symbol. Let \( \hat{t} \) be the synchronization time phase or clock time found
and defined in Eqs. (4.41) and (4.42). Also let \( x[\hat{t}] \) be the observation vector
obtained at time phase \( \hat{t} \) upon coarse synchronization. The linear MMSE detector
has the form

\[
\hat{d}_1 = \text{sgn} \left( \text{Re} \left( w^\dagger[\hat{t}] x[\hat{t}] \right) \right), \tag{4.43}
\]

where the weight vector \( w[\hat{t}] \in \mathbb{C}^{J_N S \times 1} \) is chosen to minimize the mean-square
error (MSE)

\[
\text{MSE} \triangleq E \left\{ (d_1 - w^\dagger[\hat{t}] x[\hat{t}])^2 \right\}. \tag{4.44}
\]

The solution of the weight vector \( w[\hat{t}] \) to minimize MSE in (4.44) is given by the
vector

\[
w_{\text{MMSE}}[\hat{t}] = E \left\{ x[\hat{t}] x^\dagger[\hat{t}] \right\}^{-1} E \left\{ d_1^* x[\hat{t}] \right\}
= R_x^{-1}[\hat{t}] (g_1 (b_1 \otimes s_1))
= R_x^{-1}[\hat{t}] u. \tag{4.45}
\]

Then the estimate of the information symbol \( d_1 \) in (4.43) has the form,

\[
\hat{d}_1 = \text{sgn} \left( \text{Re} \left( u^\dagger R_x^{-1}[\hat{t}] x[\hat{t}] \right) \right), \tag{4.46}
\]
where $\text{sgn}$ denotes the $\text{sign}$ operator. However, a small training signal with a separated training period is needed in advance to obtain an estimate of the required steering vector, i.e., $\mathbf{u}$ in (4.46). To find this estimate, one utilizes a multiple of code-symbol length of the desired spreading code sequence as preamble to find an estimate of vector $\mathbf{u}$. With this selection the code-timing acquisition detector and the demodulation detector can be achieved with the same test statistic in (4.32), which will be shown next. As a consequence it is unnecessary to develop a new test statistic for demodulation. Also this selection is equivalent to the finding of an estimate $\hat{\mathbf{u}}[\hat{i}]$ of $\mathbf{u}$ in (4.11) when $d_1$ is set to 1. The decision statistic in (4.46) can be modified by using the methodology in Section 4.3.1 to the test as follows:

$$
\text{Re} \left\{ \mathbf{u}^\dagger \mathbf{R}_x^{-1} \tilde{x}[\hat{i}] \right\} = \text{Re} \left\{ \mathbf{u}^\dagger \mathbf{T}_1[\hat{i}] \left( \mathbf{T}_1[\hat{i}] \mathbf{R}_x[\hat{i}] \mathbf{T}_1[\hat{i}] \right)^{-1} \mathbf{T}_1[\hat{i}] \mathbf{x}[\hat{i}] \right\},
$$

$$
= \text{Re} \left\{ \xi_1^{-1}[\hat{i}] \Delta_1[\hat{i}] \left( \left( \mathbf{u}^\dagger[\hat{i}] - \mathbf{w}_{\text{GSC}}^\dagger[\hat{i}] \mathbf{B}_1[\hat{i}] \right) \mathbf{x}[\hat{i}] \right) \right\}. \quad (4.47)
$$

Thus, the estimate of the information symbol $d_1$, using (4.47), can be obtained by ignoring the positive scalar $(\xi_1^{-1}[\hat{i}] \Delta_1[\hat{i}])$ in (4.47), which is the same as $\omega_1[\hat{i}]$ under $H_0$, and replacing the weight vector $\mathbf{u}^\dagger \mathbf{R}_x^{-1} \tilde{x}[*]$ in (4.46) by the new weight vector $(\mathbf{u}^\dagger[\hat{i}] - \mathbf{w}_{\text{GSC}}^\dagger[\hat{i}] \mathbf{B}_1[\hat{i}])$ in (4.47), to obtain

$$
d_1 = \text{sgn} \left( \text{Re} \left\{ \left( \mathbf{u}^\dagger[\hat{i}] - \mathbf{w}_{\text{GSC}}^\dagger[\hat{i}] \mathbf{B}_1[\hat{i}] \right) \mathbf{x}[\hat{i}] \right\} \right). \quad (4.48)
$$

### 4.3.2.2 Maximum-Likelihood (ML) Detector

If the source is equi-probable, the ML criterion is equivalent to the maximum $a \ posteriori$ probability (MAP) criterion, which ensures the minimum bit-error probability. Let $\mathbf{x}[\hat{i}]$ be the observation vector at time phase $\hat{i}$ obtained upon
coarse synchronization. Since the BPSK modulation is considered, the received vector \( \mathbf{x}[\hat{\theta}] \) is observed under two equally probable hypotheses, namely,

\[
H_0 : \quad \mathbf{T} \mathbf{x}[\hat{\theta}] = -\mathbf{T} \mathbf{u} + \mathbf{T} \mathbf{v}[\hat{\theta}],
\]

\[
H_1 : \quad \mathbf{T} \mathbf{x}[\hat{\theta}] = \mathbf{T} \mathbf{u} + \mathbf{T} \mathbf{v}[\hat{\theta}].
\]

(4.49)

The probability density of \( \mathbf{x}[\hat{\theta}] \) under each hypothesis follows easily:

\[
P \left( \mathbf{T} \mathbf{x}[\hat{\theta}] \bigg| H_1 \right) = \frac{e^{-(\mathbf{T} \mathbf{x}[\hat{\theta}] - \mathbf{T} \mathbf{u})^\dagger (\mathbf{T} \mathbf{R}_v[\hat{\theta}] \mathbf{T}^\dagger)^{-1} (\mathbf{T} \mathbf{x}[\hat{\theta}] - \mathbf{T} \mathbf{u})}}{\pi^{JNS} \det \left( \mathbf{T} \mathbf{R}_v[\hat{\theta}] \mathbf{T}^\dagger \right)},
\]

(4.50)

\[
P \left( \mathbf{T} \mathbf{x}[\hat{\theta}] \bigg| H_0 \right) = \frac{e^{-(\mathbf{T} \mathbf{x}[\hat{\theta}] + \mathbf{T} \mathbf{u})^\dagger (\mathbf{T} \mathbf{R}_v[\hat{\theta}] \mathbf{T}^\dagger)^{-1} (\mathbf{T} \mathbf{x}[\hat{\theta}] + \mathbf{T} \mathbf{u})}}{\pi^{JNS} \det \left( \mathbf{T} \mathbf{R}_v[\hat{\theta}] \mathbf{T}^\dagger \right)}.
\]

(4.51)

Thus, the likelihood ratio is given by

\[
\Lambda = \frac{P \left( \mathbf{T} \mathbf{x}[\hat{\theta}] \bigg| H_1 \right)}{P \left( \mathbf{T} \mathbf{x}[\hat{\theta}] \bigg| H_0 \right)} = \frac{e^{-(\mathbf{T} \mathbf{x}[\hat{\theta}] - \mathbf{T} \mathbf{u})^\dagger (\mathbf{T} \mathbf{R}_v[\hat{\theta}] \mathbf{T}^\dagger)^{-1} (\mathbf{T} \mathbf{x}[\hat{\theta}] - \mathbf{T} \mathbf{u})}}{e^{-(\mathbf{T} \mathbf{x}[\hat{\theta}] + \mathbf{T} \mathbf{u})^\dagger (\mathbf{T} \mathbf{R}_v[\hat{\theta}] \mathbf{T}^\dagger)^{-1} (\mathbf{T} \mathbf{x}[\hat{\theta}] + \mathbf{T} \mathbf{u})}}.
\]

(4.52)

Canceling common terms and taking the logarithm yields the log-likelihood ratio given by

\[
I(\Lambda) \triangleq \ln(\Lambda) = 2 \Re \left\{ \mathbf{(T u)^\dagger (T R_v[\hat{\theta}] T^\dagger)^{-1} T x[\hat{\theta}] } \right\}.
\]

(4.53)

Hence, the likelihood ratio test is

\[
\Re \left\{ \mathbf{(T u)^\dagger (T R_v[\hat{\theta}] T^\dagger)^{-1} T x[\hat{\theta}] } \right\}_{H_0}^{H_1} \geq 0.
\]

(4.54)

Then the estimate of the information symbol \( d_1 \) in (4.54) has the form,

\[
\hat{d}_1 = \text{sgn} \left( \Re \left\{ \mathbf{u^\dagger R_v^{-1} x[\hat{\theta}] } \right\} \right),
\]

(4.55)

78
The decision statistic in (4.55) can be modified by the techniques used in Section 4.3.1 to the test function given as follows:

$$\text{Re} \left\{ u^\dagger R^{-1}_v \hat{y} x[\hat{y}] \right\} = \text{Re} \left\{ u^\dagger T_{\hat{y}}^\dagger \left( T_{\hat{y}} \hat{y} R_v \hat{y} T_{\hat{y}}^\dagger \right)^{-1} T_{\hat{y}} \hat{y} x[\hat{y}] \right\},$$

$$= \text{Re} \left\{ \kappa^{-1}_{\hat{y}} \Delta_{\hat{y}} \left( \left( u^\dagger \hat{y} - w^\dagger_{\text{GSC}} \hat{y} B_{\hat{y}} \right) x[\hat{y}] \right) \right\}. \quad (4.56)$$

The estimate of the information symbol $d_1$, using (4.56), can be obtained by ignoring the positive scalar $\omega_{\hat{y}} (\kappa^{-1}_{\hat{y}} \Delta_{\hat{y}})$ in (4.56) and replacing the weight vector $u^\dagger R_v \hat{y}$ in (4.55) by the new weight vector $(u^\dagger \hat{y} - w^\dagger_{\text{GSC}} \hat{y} B_{\hat{y}})$ in (4.56), to obtain

$$d_1 = \text{sgn} \left( \text{Re} \left\{ \left( u^\dagger \hat{y} - w^\dagger_{\text{GSC}} \hat{y} B_{\hat{y}} \right) x[\hat{y}] \right\} \right). \quad (4.57)$$

The term $(u^\dagger \hat{y} - w^\dagger_{\text{GSC}} \hat{y} B_{\hat{y}}) x[\hat{y}]$ is needed in common with both the acquisition and the demodulation operations. This term can be computed and stored during the adaptive acquisition and synchronization process. It does not need to be recomputed for demodulation.

It should be noted that the MAP test in (4.54) is based on the inner product between the vector $x[\hat{y}]$ and the linear filter $w_{\text{SNR}}[\hat{y}]$, $w_{\text{SNR}}[\hat{y}] = R_v^{-1}[\hat{y}] u$ that maximizes the output signal-to-interference plus noise ratio (SINR$_e[\hat{y}]$). The observation implies that the decision rule based on the maximum output SINR filter $w_{\text{SNR}}[\hat{y}]$ guarantees the minimization of the decision-error probability. Furthermore, since the Wiener filter is equivalent to $w_{\text{SNR}}[\hat{y}]$ except a gain factor [13, 7], one can conclude that the MMSE performance measure is equivalent to the maximum output SINR and minimum decision-error probability measures.

Also the proposed receiver is illustrated by the conceptual block diagram, shown in the Figure 4.3. The observation data vectors are obtained by sweeping a moving window of length $N$ through the original data matrix. The timing
acquisition process to find the estimate $\hat{\iota}$, defined in Eq.(4.42), is presented in the Figure 4.3. Once synchronization is established, the demodulation of the information symbol is accomplished by performing the \textit{sign} operation on the term $\text{Re}\{y[\hat{\iota}]\}$.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{block_diagram.png}
\caption{Conceptual block diagram of the proposed receiver.}
\end{figure}

\subsection{4.3.3 Lower-Complexity Multistage Representation of The Test}
To achieve the desired multistage decomposition of the likelihood ratio test in Eq.(4.38), successive orthogonal decompositions are applied in the following to the observed data. To accomplish the second stage of this decomposition, the same technique is applied to the terms $x_1[\iota] = B_1[\iota]x[\iota]$ and $w_{\text{GSC}}[\iota]$, shown in Figure 4.2. First define $u_2[\iota]$ as the unit vector,

$$u_2[\iota] \triangleq \frac{r_{X_1,\hat{\delta}_1}[\iota]}{\sqrt{r_{X_1,\hat{\delta}_1}[\iota]r_{x_1,\hat{\delta}_1}[\iota]}}.$$ (4.58)
Then in a manner similar to the first stage define a new nonsingular transformation \( T_2[v] \) as follows:

\[
T_2[v] = \begin{bmatrix}
  u_2^T[v] \\
  B_2[v]
\end{bmatrix},
\]

(4.59)

where \( B_2[v]u_2[v] = 0 \). Thus, the new transformed vector under the transformation \( T_2[v] \) is represented by

\[
T_2[v]x_1[v] = \begin{bmatrix}
  u_2^T[v]x_1[v] \\
  B_2[v]x_1[v]
\end{bmatrix} = \begin{bmatrix}
  \delta_2[v] \\
  x_2[v]
\end{bmatrix}.
\]

(4.60)

The likelihood ratio in Eq. (4.32) is re-written as follows:

\[
y[v] = \omega_1[v] \left( (u_1^T[v] - w_{GSC}[v]B_1[v])x[v] \right)
  = \omega_1[v] \left( (u_1^T[v] - \omega_2[v](r_{\theta 2\delta_2}[v]R_{\theta 2\delta_2}^{-1}[v]B_2[v])B_1[v])x[v] \right)
  = \omega_1[v] \left( \delta_1[v] - \omega_2[v](r_{\theta 2\delta_2}[v]R_{\theta 2\delta_2}^{-1}[v]x_2[v]) \right),
\]

(4.61)

in two stages, where \( \omega_2[v] \triangleq \frac{\Delta_2[v]}{\xi_2[v]} \), \( \Delta_2[v] = \sqrt{r_{\theta 1\delta_1}[v]r_{\theta 1\delta_1}[v]} \) and \( \xi_2[v] = \sigma_{\delta_2[v]}^2 - r_{\theta 2\delta_2}[v]R_{\theta 2\delta_2}^{-1}[v]r_{\theta 2\delta_2}[v] \).

A continuation of this decomposition process, i.e., extending Eq. (4.61), yields finally the \( JNS \)-stage likelihood ratio test in terms of a sequence of only scalar quantities in a form given as follows:

\[
y[v] = \omega_1[v](\delta_1[v] - \omega_2[v](\ldots(\delta_{JNS-1}[v] - \omega_{JNS}[v]\delta_{JNS}[v])\ldots)).
\]

(4.62)

More complete details of the multistage decomposition can be found in [11]. Rank reduction is realized as a truncation of the full-scale structure of the formula
in Eq. (4.62) at the $M$-th stage, where $M < JNS$. Figure 4.4 illustrates the likelihood ratio test for $M = 4$.

![Block diagram of the multistage decomposition of the likelihood ratio test for $M = 4$.](image)

Figure 4.4: Block diagram of the multistage decomposition of the likelihood ratio test for $M = 4$.

Let $S_M[i]$ denote the $JNS \times M$ matrix with column vectors forming an orthonormal basis for a $M$-dimensional subspace of the reduced-rank MWF. Clearly, the $M$ basis vectors for the $M$-stage reduced-rank MWF are given by

$$S_M[i] = \begin{bmatrix} u_1[i] & B_1^T[i]u_2[i] & \cdots & B_1^T[i]u_M[i] \end{bmatrix}.$$  

(4.63)

With the $S_M[i]$ given in (4.63), the reduced-rank Wiener solutions $w_M[i] \in \mathbb{C}^{M \times 1}$ for maximizing output SINR and minimizing MSE are obtained as

$$w_{\text{SINR}}[i] = (S_M^T[i]R_v[i]S_M[i])^{-1}S_M^T[i]u,$$  

(4.64)

$$w_{\text{MMSE}}[i] = (S_M^T[i]R_x[i]S_M[i])^{-1}S_M[i]u.$$  

(4.65)
The resulting (SINR)\(_M\) and MMSE\(_M\), associated with the rank-\(M\) Wiener filters \(w_M[\hat{z}]\) in Eqs.(4.64) and (4.65), are given by

\[
(SINR)\(_M\)[\hat{z}] = u^*[\hat{z}]S_M[\hat{z}](S_M^*[\hat{z}]R_v[\hat{z}]S_M[\hat{z}])^{-1}(S_M^*[\hat{z}]u[\hat{z}]'),
\]

\[
MMSE\(_M\)[\hat{z}] = \sigma_{d1}^2 - u^*[\hat{z}]S_M[\hat{z}](S_M^*[\hat{z}]R_x[\hat{z}]S_M[\hat{z}])^{-1}S_M^*[\hat{z}]u[\hat{z}],
\]

where \(\sigma_{d1}^2\) is equal to 1 when the modulation format is BPSK.

### 4.3.4 Parameter Estimation

To see explicitly how to find the estimate \(\hat{u}\)[\hat{z}] of the vector \(u\), first correlate the received data matrix \(Z[\hat{z}]\) in (4.3) under hypothesis \(H_1\) with the spreading-code vector \(s_1\) of the desired user. The \(J \times NS\) data matrix \(Z[\hat{z}]\), the sampled version of the filtered received \(J\)-vector \(r(t)\), under hypothesis \(H_1\) is shown next to be the sum of the target signal matrix \(g_1d_1b_1s_1^\dagger\) and the interference-plus-noise matrix \(V[\hat{z}]\) as follows:

\[
Z[\hat{z}] = g_1d_1b_1s_1^\dagger + V[\hat{z}], \tag{4.66}
\]

where the \((J \times NS)\)-matrix \(V[\hat{z}]\) in (4.66) is the matrix form of the vector \(v[\hat{z}]\) in (4.10). Remember that \(V[\hat{z}]\) and \(v[\hat{z}]\) are related by

\[
v[\hat{z}] = Vec\{V[\hat{z}]\}.
\]

The expected value of the correlation between \(Z[\hat{z}]\) in (4.66) and the desired spreading-code vector \(s_1\) is computed as follows:

\[
E\left\{Z[\hat{z}] \cdot \frac{s_1}{s_1^\dagger s_1}\right\} = E\left\{(g_1d_1b_1s_1^\dagger + V[\hat{z}]) \cdot \frac{s_1}{s_1^\dagger s_1}\right\}
\]
\[
= E \left\{ g_1 d_1 b_1 s_1^T \cdot \frac{s_1}{s_1^T s_1} \right\} + E \left\{ V[i] \cdot \frac{s_1}{s_1^T s_1} \right\} \\
= g_1 d_1 b_1 .
\] (4.67)

Note that the Kronecker-product vector in (4.68) of the above vector \((Z[i] \cdot \frac{s_1}{s_1^T s_1})\) and the desired spreading code vector \(s_1\), denoted by \(\hat{u}_d[i]\), is shown next by Eq.(4.11) to be an unbiased estimate of the target signal vector \(d_1 u\). This is seen from the following expected-value identity:

\[
E \left\{ \hat{u}_d[i] \right\} = E \left\{ (Z[i] \cdot \frac{s_1}{s_1^T s_1}) \otimes s_1 \right\} \\
= g_1 d_1 (b_1 \otimes s_1) \\
= d_1 u .
\] (4.68)

This identity in (4.68) implies that the quantity, \(\hat{u}_d[i]\) under the expected value in (4.68), is an unbiased estimate of \(d_1 u\) defined in (4.11). That is,

\[
\hat{u}_d[i] = (Z[i] \cdot \frac{s_1}{s_1^T s_1}) \otimes s_1
\] (4.69)

is the desired estimate of \(d_1 u\). Even though the difference of a sign may exist between \(\hat{u}_d[i]\) in (4.69) and the vector \(u\) in (4.11) when \(d_1 = -1\), they can be used interchangeably for the magnitude test in (4.19) for the time-synchronization acquisition. Hence, the proposed acquisition system that uses the vector \(\hat{u}_d[i]\) in (4.69) can be used to to combine with most multiuser DS-CDMA algorithms that require precise knowledge of synchronization.

Note that the estimate of the steering vector \(u\) in (4.11), which is used for the acquisition and the demodulation system, can be obtained by removing the information symbol \(d_1\) from \(\hat{u}_d[i]\) in (4.69). It can be achieved by the aid of a training sequence followed by a decision-directed adaptation. The estimated information symbol \(\hat{d}_1\) can be utilized as feedback information in order to provide

84
a more precise estimate about the steering vector $\mathbf{u}$ in (4.11) which is employed to explore the successive acquisition and information symbols. In other words, the array receiver continues to adapt the vector $\mathbf{u}$ in a decision-directed (DD) adaptive manner, in which symbol decisions that are made by the receiver are fed back for further adaptation of vector $\mathbf{u}$. In fact, a more efficient recursive formula for updating the estimate of vector $\mathbf{u}$ can be used for coarse synchronization and demodulation within the $k$-th symbol interval, denoted by $\hat{\mathbf{u}}^{(k)}[i]$. Such an iterative algorithm for $\hat{\mathbf{u}}^{(k)}[i]$ is given by

$$\hat{\mathbf{u}}^{(k)}[i] = (1 - \frac{1}{k}) \hat{\mathbf{u}}^{(k-1)}[i] + \frac{1}{k} \hat{d}_1^{(k-1)} \hat{u}_d[i], \quad (4.70)$$

where $\hat{\mathbf{u}}^{(k-1)}[i]$ is the estimate of the vector $\mathbf{u}$ in (4.11) of the $(k-1)$-th symbol interval, and the term $\hat{d}_1^{(k-1)} \hat{u}_d[i]$ is updated by the observed data within the $(k-1)$-th symbol interval. Here $\hat{d}_1^{(k-1)}$ is the estimated $(k-1)$-th information bit and $\hat{d}_1^{(0)} = 1$ denotes the information bit that is used for preamble. The term $\hat{u}_d[i]$ in (4.70) is the vector which has to be calculated by (4.69) every symbol interval. Figure 4.5 illustrates the proposed MMSE receiver.

In order to test the adaptively learning capability of the proposed iterative procedure for the vector $\mathbf{u}$, we consider a system with $K = 6$ users, $N = 31$,
Figure 4.6: Convergence dynamics of the steering vector of the proposed receiver implementation with system parameters $J = 2$, $K = 6$, $M = 2$, $N = 31$, SNR = 10dB, and $NFR = 10^{J/10}$, where $\Gamma_i \sim N(4, 16)$.

NFR = 3dB, and SNR = 12dB. In Figure 4.6 the following normalized correlation coefficient

$$\rho(k) = \frac{||u^\dagger \cdot \hat{u}^{(k)}||}{||u||||\hat{u}^{(k)}||},$$

(4.71)
is shown versus the number of samples used in the recursive adaptation. The estimate of the steering vector is significantly improved by the decision-directed (DD) adaptive mode as the number of iterations increases.

### 4.4 Adaptive Multistage Realization of The Test

In this paper the performance of the proposed detector is investigated by using a training-based algorithm for the multistage decomposition introduced in [14, 21]. By this algorithm one does not need to directly calculate an estimate of the
covariance matrix. Also the coefficients of the proposed detector can be estimated using only the received data vectors, and the dimensions of the blocking matrices \( \hat{\mathbf{B}}_j[k] \) can be kept constant (i.e., in a \( JNS \times JNS \) matrix form) for every stage in this algorithm. To make this possible a blocking matrix of the form [21],

\[
\hat{\mathbf{B}}_j[k] = \mathbf{I} - \hat{\mathbf{u}}_j[k] \hat{\mathbf{u}}_j[k]^\dagger,
\]

is used, where \( \hat{\mathbf{u}}_j[k] \) is the estimate of unit vector \( \mathbf{u}_j[k] \) in Table 4.1. In this manner, the lengths of the registers needed to store the blocking matrices and vectors can be kept the same at every stage, a fact that is very desirable for either a hardware or software realization. The rank-\( M \) algorithm shown in [39, 21] is summarized in Table 4.1. Note that this new structure no longer requires the calculation of a blocking matrix and the computational complexity is reduced significantly [41].

### 4.5 An Iterative Version of the Multistage Filter

Instead of estimating the required parameters in the training-based mode, the required parameters also can be estimated iteratively. By this procedure the coefficients of the filter are updated every time that new data are received. Thus, the detection system more readily can keep track adaptively of the changes in the interference environment. In order to follow the statistical variations of the observed data, when the receiver operates in a non-stationary environment, a weighting factor is often used to ensure that data in the distant past are forgotten. A special form of weighting that is commonly used is the exponential weighting
factor. As it is shown in [13], an estimate of the cross-correlation vector \( \hat{r}_{x_j \delta_j}[i] \) can be expressed as the infinite series,

\[
\hat{r}_{x_j \delta_j}[i] = \lim_{k \to \infty} \sum_{m=1}^{k} (1 - \alpha)^{k-m} x_j^{(m)}[i] \delta_j^{(m)}[i]^*,
\]

where \( \alpha \) is a positive constant less than 1. The inverse of \( \alpha \) is, roughly speaking, a measure of the memory of the algorithm. The special case, when \( \alpha \) approaches zero, corresponds to infinite memory. The summation on the right hand side of Eq.(4.73) can be written as the difference or iterative equation,

\[
\hat{r}_{x_j \delta_j}^{(k)}[i] = (1 - \alpha) \left[ \sum_{m=1}^{k-1} (1 - \alpha)^{k-1-m} x_j^{(m)}[i] \delta_j^{(m)}[i]^* \right] + x_j^{(k)}[i] \delta_j^{(k)}[i]^*,
\]

\[
= (1 - \alpha) \hat{r}_{x_j \delta_j}^{(k-1)}[i] + x_j^{(k)}[i] \delta_j^{(k)}[i]^*,
\]

where \( \hat{r}_{x_j \delta_j}^{(k-1)}[i] \) is the previous value of \( \hat{r}_{x_j \delta_j}^{(k)}[i] \), and the term \( x_j^{(k)}[i] \delta_j^{(k)}[i]^* \) is obtained from the new data. Hence, for each iteration, only two quantities (\( \hat{r}_{x_j \delta_j}^{(k-1)}[i] \) and \( x_j^{(k)}[i] \delta_j^{(k)}[i]^* \)) are necessary for updating \( \hat{r}_{x_j \delta_j}^{(k)}[i] \). The length of the observation-data that contributes to the estimate, \( \hat{r}_{x_j \delta_j}^{(k)}[i] \), is determined by the weighting factor \( \alpha \triangleq \frac{1}{L} \), where \( L \) is the time constant corresponding to the length of the training-based algorithm in Table 4.1. The exponential weighting factor \( \alpha \) is substituted in the algorithm in Table 4.1. This results in the iterative algorithm shown in Table 4.2. The algorithms in Tables 4.1 and 4.2 do not require matrix multiplications.

The error signal \( e_j[i] \) at the \( j \)-th stage is given by

\[
e_j^{(k)}[i] = \delta_j^{(k)}[i] - \omega_{j+1}^{(k)}[i] e_{j+1}^{(k)}[i],
\]

\[88\]
so that in a manner similar to the estimate of the cross-correlation vector \( \hat{r}_{x_jz}[\nu] \) in (4.74), the variance \( \hat{\xi}_j[\nu] \) of the scalar error signal \( e_j[\nu] \) is computed iteratively by

\[
\hat{\xi}_j^{(k)}[\nu] \triangleq (\hat{\sigma}_{e_j}^{(k)}[\nu])^2 = (1 - \alpha)\hat{\xi}_j^{(k-1)}[\nu] + |e_j^{(k)}[\nu]|^2, \tag{4.76}
\]

with the definition, \( \hat{\xi}_j^{(0)}[\nu] = (\hat{\sigma}_{e_j}^{(0)}[\nu])^2 \). Here \( \hat{\xi}_j^{(k-1)}[\nu] \) is the previous value of \( \hat{\xi}_j^{(k)}[\nu] \) and the values of the scalar Wiener filters are found from the Wiener-Hopf equation to be

\[
\hat{\omega}_j^{(k)}[\nu] = \frac{\hat{\lambda}_j^{(k)}[\nu]}{\hat{\xi}_j^{(k)}[\nu]}, \tag{4.77}
\]

where \( j = 1, 2, \ldots, M \).

### 4.6 Numerical Results

In this section simulations were conducted to demonstrate the performance of the proposed asynchronous CDMA receiver.

#### 4.6.1 DS-CDMA Scenario I and II

An asynchronous BPSK DS-CDMA system with the number of users \( K = 6 \) is considered in these simulations. The spreading sequence of each user is a Gold sequence of length \( N = 31 \). The desired user, say user 1, is the user one wants to be acquired in the presence of MAI. The receiver to be simulated employs a uniformly spaced linear array antenna with multiple elements of half-wavelength spacing. Also the performance of the asynchronous CDMA receiver that employs an antenna with a single element is derived for purposes of comparison. The
Table 4.1: Training-Based Multistage Decomposition Algorithm.

Let $\mathbf{X}_0[i] \triangleq [x^{(1)}[i], x^{(2)}[i], \ldots, x^{(L)}[i]]$ denote $L$ independent samples.

**Forward Recursion**

Initial: Let $\hat{u}_i[i] = \frac{\hat{u}^{(k)}[i]}{\sqrt{\hat{n}^{(k)}[i]}}$ and $x_0[i] = x[i]$, where $\hat{u}^{(k)}[i]$ is for the $k$-th symbol interval.

For $j = 1$ to $(M - 1)$

Calculate $\delta_j[i]$ and $x_j[i]$.

\[
\delta_j[i] = \hat{u}^{(k)}[i] x_{j-1}[i] \\
x_j[i] = x_{j-1}[i] - \hat{u}^{(k)}[i] \delta_j[i] \\
d_j[i] \triangleq \left[ \delta_j^{(1)}[i], \delta_j^{(2)}[i], \ldots, \delta_j^{(L)}[i] \right] = \hat{u}^{(k)}[i] x_{j-1}[i] \\
X_j[i] \triangleq \left[ x_j^{(1)}[i], x_j^{(2)}[i], \ldots, x_j^{(L)}[i] \right] = X_{j-1}[i] - \hat{u}^{(k)}[i] d_j[i]
\]

Calculate the $(j + 1)$-th stage basis vector,

\[
r_{x_j \delta_j}[i] = \frac{1}{L} \sum_{m=1}^{L} x_j^{(m)}[i] \delta_j^{(m)}[i] = \frac{1}{L} X_j[i] d_j[i] \\
\Delta_{j+1}[i] = \sqrt{n_{x_j \delta_j}[i]} r_{x_j \delta_j}[i] \\
\hat{u}_{j+1}[i] = \frac{r_{x_j \delta_j}[i]}{\Delta_{j+1}[i]}
\]

End

Compute $\delta_M[i]$ and set it equal to $\epsilon_M[i]$

\[
d_M[i] \triangleq \left[ \delta_M^{(1)}[i], \delta_M^{(2)}[i], \ldots, \delta_M^{(L)}[i] \right] = \hat{u}_M[i] = \hat{u}^{(k)}[i] X_{M-1}[i]
\]

**Backward Recursion**

\[
\hat{\sigma}_M^2[i] = \frac{1}{L} \sum_{m=1}^{L} \left| \delta_M^{(m)}[i] \right|^2 = \epsilon_M[i], \ \omega_M[i] = \hat{\epsilon}_M[i] \Delta_M[i]
\]

For $j = (M - 1)$ to $1$

Estimate the variance of $\delta_j[i]$ by

\[
\hat{\sigma}_j^2[i] = \frac{1}{L} \sum_{m=1}^{L} \left| \delta_j^{(m)}[i] \right|^2
\]

Estimate the variance of $\epsilon_j[i]$ by

\[
\hat{\epsilon}_j[i] \triangleq \hat{\sigma}_j^2[i] = \hat{\sigma}_j^2[i] - \hat{\epsilon}_j+1[i] \Delta_j+1[i]
\]

If $j \geq 2$,

Calculate the $j$-th scalar Wiener filter $\hat{\omega}_j[i]$ by

\[
\hat{\omega}_j[i] = \frac{\Delta_j[i]}{\hat{\epsilon}_j[i]}
\]

Calculate the $(j - 1)$-th error signal $\epsilon_{j-1}[i]$ by

\[
\epsilon_{j-1}[i] = \delta_{j-1}[i] - \hat{\omega}_j[i] \epsilon_j[i]
\]

If $j = 1$, $\hat{\omega}_j[i] = \frac{\Delta_1[i]}{\hat{\epsilon}_1[i] - \Delta_1[i]}
\]

End

The outputs for the acquisition and demodulation systems are $y[i] = \hat{\omega}_1[i] \epsilon_1[i]$ and $\epsilon_1[i]$, respectively.
Table 4.2: Iterative Multistage Decomposition Algorithm.
Let $k$ be the $k$-th clock time, where $k = 1$ is the first time the data is observed.

Let $\alpha = \frac{1}{\tau}$ be the memory or time constant of the algorithm.

**Initialization:**
\[
\begin{align*}
\hat{r}_{x, j}(^0)[i] &= 0, & \hat{e}_{j}(^0)[i] &= 0 & \forall j
\end{align*}
\]
For $k = 1$ to $n$
\[
\begin{align*}
\hat{r}_{x, j}(^k)[i] &= \hat{u}^{(k)}[i], & \hat{u}^{(k)}[i] &= \frac{\hat{u}^{(k)}[i]}{\sqrt{\hat{u}^{(k)}[i]^*\hat{u}^{(k)}[i]}}, & \hat{\Delta}_1[1] &= \sqrt{\hat{u}^{(k)}[i]^*\hat{u}^{(k)}[i]}, & \hat{x}_0^{(k)}[i] & \text{is the received vector.}
\end{align*}
\]

**Forward Recursion**
For $j = 1$ to $(M - 1)$
\[
\begin{align*}
\hat{s}_j^{(k)}[i] &= \hat{u}_j^{(k)}[i] \hat{x}_{j-1}^{(k)}[i] \\
\hat{x}_j^{(k)}[i] &= \hat{x}_{j-1}^{(k)}[i] - \hat{u}_j^{(k)}[i] \hat{\delta}_j^{(k)}[i] \\
\text{Calculate the $(j+1)$-th stage basis vector,} & \hat{r}_{x, j+1}^{(k)}[i] = (1 - \alpha) \hat{r}_{x, j+1}^{(k-1)}[i] + \hat{x}_j^{(k)}[i] \hat{\delta}_j^{(k)}[i]^* \\
\hat{\Delta}_{j+1}[i] &= \sqrt{\hat{r}_{x, j+1}^{(k)}[i]^* \hat{r}_{x, j+1}^{(k)}[i]} \\
\hat{u}_{j+1}^{(k)}[i] &= \frac{\hat{r}_{x, j+1}^{(k)}[i]}{\hat{\Delta}_{j+1}[i]}
\end{align*}
\]

End

\[
\begin{align*}
\hat{e}_M^{(k)}[i] &= \hat{s}_M^{(k)}[i]^* \hat{u}_M^{(k)}[i] \hat{x}_{M-1}^{(k)}[i]
\end{align*}
\]

**Backward Recursion**
For $j = M$ to $1$
\[
\begin{align*}
\hat{\delta}_j^{(k)}[i] &= \hat{\delta}_j^{(k)}[i] + \hat{e}_j^{(k-1)}[i] \\
\text{Estimate the variance of} \hat{\delta}_j^{(k)}[i] \text{by} & \hat{\xi}_j^{(k)}[i] = (\hat{\delta}_j^{(k)}[i])^2 = (1 - \alpha) \hat{\xi}_j^{(k-1)}[i] + \hat{e}_j^{(k)}[i]^2 \\
\text{If} \ j \geq 2, & \hat{\omega}_j^{(k)}[i] = \frac{\hat{\Delta}_j^{(k)}[i]}{\hat{\xi}_j^{(k)}[i]} \\
\text{Calculate the} \ j \text{-th scalar Wiener filter} \hat{\omega}_j^{(k)}[i] \text{by} & \hat{\xi}_{j-1}^{(k)}[i] = \hat{\delta}_{j-1}^{(k)}[i] - \hat{\omega}_j^{(k)}[i] \hat{e}_j^{(k)}[i] \\
\text{Calculate the} \ (j - 1) \text{-th error signal} & \hat{\delta}_{j-1}^{(k)}[i] = \hat{\delta}_{j-1}^{(k)}[i] - \hat{\omega}_j^{(k)}[i] \hat{e}_j^{(k)}[i] \\
\text{If} \ j = 1, & \hat{\omega}_j^{(k)}[i] = \frac{(\hat{\delta}_j^{(k)}[i])^2}{\hat{\xi}_j^{(k)}[i]} - (\hat{\delta}_j^{(k)}[i])^2
\end{align*}
\]

End

The outputs for the acquisition and demodulation systems at the $k$-th time phase are $y_1^{(k)}[i] = \hat{e}_1^{(k)}[i] \hat{\omega}_1^{(k)}[i]$ and $e_1^{(k)}[i]$, respectively.

End
Table 4.3: Simulated channel parameters for DS-CDMA Scenarios I and II.

<table>
<thead>
<tr>
<th>Users</th>
<th>Delays (in $T_c$)</th>
<th>DOAs (in degrees)</th>
<th>NFRs (in dB)</th>
<th>NFRs (in dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Scenario I</td>
<td>Scenario II</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>-5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>60</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>-15</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>-45</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>30</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>-75</td>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>

power ratios of the five interfering users with the desired user constitute the levels of MAI. In this simulation all interfering users are assumed to have the same power ratio relative to the desired user. Such a power ratio is denoted by a quantity called the *near-far ratio* (NFR), defined by

$$NFR = \frac{\| A_1a_l \|^2}{\| A_1^T a_1 \|^2} = \frac{\| g_l \|^2}{\| g_1 \|^2},$$  \hspace{1cm} (4.78)

where the subscript ‘$l$’ denotes user $l$, $l = 1$ indicates the user of interest. The relative propagation delays, the direction of arrivals (DOAs), and the NFRs of all users that are used in the simulations are tabulated in Table 4.3. The relative delays of the different users, listed in Table 4.3, are with respect to user 1. For convenience and simplicity it is assumed in the simulations that the relative delays of the different users are multiples of $T_c$. This assumption is not necessary if the sampling rate is higher than $1/T_c$. All experimental curves are obtained using 1000 independent trials.

First, the acquisition and demodulation performance of the proposed DS-CDMA receiver as a function of the signal-to-white noise ratio ($E_b/N_0$) is obtained for a full rank covariance matrix. The results are shown in Figure 4.7 for a $J$-element array antenna ($J = 1, 2, 4,$ and $6$), and NFR = 0dB and 3dB, under
the assumption that the channel parameters of all users, such as amplitudes, the complex gains of channel, propagation delays, DOAs, etc., are known at the receiver. Hence, the precise covariance matrix is assumed to be available at the receiver. For these cases the dimension of the full-rank receiver is equal to $J \times N$. The simulations in Figure 4.7 provide an upper-bound on the acquisition and demodulation performance of the proposed asynchronous DS-CDMA receiver.

The performance of the adaptive implementation of the proposed receiver for the training-based algorithm (FIR), given in Table 4.1, is shown in Figures 4.8, 4.9, 4.10, 4.11, 4.12, and 4.13. Figure 4.8 shows that the acquisition and BER performance versus the number $M$ of stages of the multistage Wiener filter for asynchronous users. The proposed receiver provides superior performance as an increasing function of the size of the $J$-element antenna array. Full-rank performance is achieved at a remarkably low ranks and is independent of the number of signals.

In Figures 4.9 the probability of correct acquisition and demodulation of a 2-element array receiver is presented as a function of the number of training data samples $L$ for the stringent power control scenario of NFR = 0dB under the assumption of SNR = 14dB. In Figure 4.10 the probability of correct demodulation of a 2-element array receiver is presented as a function of the number of training data samples $L$ for the near-far scenario of NFR = 3dB in the upper figure under the assumption of SNR = 14dB. Also in the lower figure this comparison is made for a number of cases for the NFR values, NFR = 0dB, 3dB, 6dB, 9dB, and 12dB. It is demonstrated in the upper figure that the filters of lower rank have a rapid adaptive convergence while a larger number of data samples is required for the case of a full rank filter. That is, the adaptive truncated multistage filters converge significantly faster than the adaptive full-rank filter. Moreover, no significant degradation is observed in the lower figure, when the case of rank-4 is
compared for a wide range of NFR values, 0dB to 12dB. This shows that the proposed receiver still performs well under conditions of poor power control. Hence, a stringent power control mechanism is not required for the proposed receiver.

Figure 4.11 (Upper and Lower) demonstrates the probability of correct acquisition and bit-error-rate performance of a rank 2 filter* of a 2-element antenna receiver for various numbers of data samples as a function of $E_b/N_0$ under the near-far scenario of NFR = 3dB. These results demonstrate that when a larger number of data samples is available, the better is the acquisition and demodulation performance. It is obvious that a more precise estimate of the covariance matrix is obtained by a larger number of training samples.

In Figure 4.12 (Upper and Lower) the probability of correct acquisition and bit-error-rate performance of a rank 2 filter for various number of antenna elements is presented in terms of $E_b/N_0$ under data size $L = 6JN$ and NFR = 3dB. A better performance is obtained when an antenna of more elements is employed. This is made possible because the multiple-access interference (MAI) is suppressed successfully by adaptively placing nulls in the directions of the stronger interferers. Moreover, a 2-element array receiver achieves a performance that is comparable with receivers that have even a larger antenna array. Such receivers also yield a substantial improvement in acquisition and demodulation when only a single element antenna ($J = 1$) is utilized.

In Figure 4.13 (Upper and Lower) the acquisition-error-rate and bit-error-rate performance of a rank 2 filter for various number of antenna elements is presented in terms of the number of users $K$ for data size $L = 6JN$ and SNR = 14dB. Each interfering user is assumed to have the same signal power as the desired user, i.e., NFR = 0dB. A better performance is accomplished when a larger antenna

*The number of adaptive receiver-filter coefficients $w_k$ in (4.5) is reduced by projecting the received vector $x_k$ onto a lower dimensional subspace. Here a rank 2 filter means $x_k$ is projected onto the subspace spanned by $u_k$ and $B_k u_k$ as precisely the case of $M = 2$ shown in Figure 4.4.
is employed. Also these results demonstrate that a significant capacity increase
can be achieved under a fixed performance requirement when a larger antenna is
used.

Finally, the performance of the iterative or IIR algorithm of the proposed
receiver in Table 4.2 is shown in Figures 4.14 and 4.15. When compared to the
results of the training-based algorithm in Figures 4.11 and 4.12, the iterative algo-
rithm has substantially lower complexity both computationally and in hardware,
but it does not perform quite as well as the training-based algorithm. To obtain a
better performance, the iterative algorithm requires the same number of samples
as the full-rank training-based case but demonstrates a lower complexity. Possibly
a more sophisticated iterative (or IIR) adaptive-weight filter algorithm could
be used to obtain a better performance. However, simulation results show that
in all cases the reduced-rank adaptive filter bank outperforms the conventional
matched-filter receiver. Notably also when compared with the computational effi-
ciency to the LMS and RLS coefficients update algorithms that are used in MMSE
CDMA receivers, this IIR algorithm has a complexity of $O(M(JNS))$ similar to
the $O(JNS)$ of the LMS-based algorithm and outperforms the $O((JNS)^2)$ of the
RLS-based algorithm when the condition of $M \ll JNS$ is met.

4.6.2 DS-CDMA Scenario III

An asynchronous BPSK DS-CDMA system with the number of users $K = 6$ is
considered. The power ratios between each of the five interfering users and the
desired user are randomly chosen from the log-normal distribution with a mean
6dB and a standard deviation of 6dB in this tested scenario. That is, this power
ratio is denoted by a quantity called the *near-far ratio* (NFR), defined by

$$NFR = \frac{||g_i||^2}{||g_1||^2} = 10^{\Gamma_i/10},$$
where \( \Gamma_i \sim N(4, 16) \). Here the subscript \( 'I' \) denotes user \( I \). The other parameters are set identically with DS-CDMA Scenario I and II.

Figure 4.16 shows that the acquisition and BER performance versus stages \( M \) of the MWF for asynchronous users. The proposed receiver provides superior performance as an increasing function of the size of the \( J \)-element antenna array. The full-rank performance is achieved at remarkably low ranks and is nearly independent on the number of signals. In Figure 4.17 the acquisition and BER performance of a rank-4 filter is presented as a function of SNR parameterized by \( L \) for \( J = 2 \). Results demonstrate that when a larger number of data samples is available, the better is the acquisition and demodulation performance. In Figure 4.18 the acquisition-error-rate and BER performance of a rank-4 filter for various number of antenna elements is presented in terms of SNR for \( L = 6JN \). A better performance is achieved when a larger antenna is employed. This is made possible because MAI can be suppressed successfully by placing nulls in the directions of the stronger interferers. Moreover, a 2-element antenna receiver achieves a substantial improvement in demodulation in comparison with a single-element antenna.

### 4.7 Conclusions

An adaptive near-far resistant array-antenna receiver for asynchronous DS-CDMA signals is developed without the need for prior synchronization. Only knowledge of the spreading code sequence of the desired user is needed. This new array receiver eliminates the need for the strict power control, that is needed in the conventional receiver, due to its near-far resistant property.

A considerably lower complexity version of the proposed receiver is developed that utilizes the concept of the multistage reduced-rank Wiener filter. This results in a substantial reduction of the computational cost and a rapid adaptive
convergence for the filter coefficients. These facts demonstrate an enhancement in bandwidth efficiency of the communication system and a potential performance gain when the amount of sample support is limited. Remarkably the computational efficiency of the proposed system is significantly improved from $O((JNS)^3)$ to $O(M(JNS))$ (where $M \ll JNS$) for each clock phase. This complexity reduction approaches that of the LMS algorithm and is significantly superior in performance to that of the RLS algorithm. When compared to the performance to the full-rank MMSE receiver, the proposed multistage receiver accomplishes a similar performance level without the requirement of a prior synchronization. Also the proposed algorithm can be extended to multiuser detection by forming all of the single-user detectors in parallel if the other users' signature sequences are available at the receiver.

The acquisition detector can be used individually to acquire the code-timing structure of a single user with only knowledge of the desired signature vector. No training period of signal-free observations is required. These are the same requirements as the conventional CDMA detector that uses a standard matched filter. Moreover, the acquisition detector is anticipated to combine with most multiuser DS-CDMA algorithms that require precise knowledge of synchronization, assumed in most coherent or noncoherent modulation/demodulation schemes.

Simulation results show that the proposed DS-CDMA receiver achieves a superior performance under the environment of a lower filter rank and a smaller number of data samples. This makes it possible to design a lower-complexity receiver without a large loss in performance in comparison with the full-rank system. Furthermore the proposed receiver obtains a better performance than that of the conventional receiver in all simulations and accomplishes a substantial improvement in acquisition and demodulation when a larger antenna array is employed. Notably a 2-element antenna receiver has a performance that is comparable to a larger antenna ($J = 4$). It also achieves a significant improvement
in acquisition and demodulation in comparison with a single-element antenna receiver.

Compare the results of this chapter with those of Chapter 3, the proposed multistage array receiver outperforms the receiver in Chapter 3 regardless of the use of a much critical simulated Scenario and a significantly lower rank of filter in Chapter 4 due to the joint benefits of time diversity and receiver antenna diversity. This fact is verified by comparing the results in Figures 3.5, 3.7(Upper), and 4.12. Although the proposed array-receiver is developed on the assumption of uplink reception (from mobiles to a base station) and a $J$-element antenna array, all its achieved properties make the new receiver meet the requirements of a lower-complexity, small-size, and light-weight receiver that most mobile users (downlink) demand today.
Figure 4.7: (Upper): The bit-error-rate performance of full rank vs. SNR parameterized by $J$, for $K = 6$, $N = 31$, and NFR is 0dB, (Lower): The bit-error-rate performance of full rank vs. SNR parameterized by $J$, for $K = 6$, $N = 31$, and NFR is 3dB, when the channel parameters of all users are known.
Figure 4.8: The acquisition and BER performance vs. stages $M$ parameterized by $J$ for $K = 6, L = 6JN, N = 3l, SNR = 7dB,$ and NFR is $3dB$. 
Figure 4.9: Training-based algorithm in Table 4.1 with the parameters $J = 2$, $K = 6$, $N = 31$, SNR = 14dB, and NFR is 0dB, (Upper): The probability of correct acquisition vs. the number of training data samples $L$ parameterized by $M$. (Lower): The probability of correct demodulation vs. the number of training data samples $L$ parameterized by $M$. 

101
Figure 4.10: Training-based algorithm in Table 4.1 with the parameters $J = 2$, $K = 6$, $N = 31$, and SNR = 14dB, (Upper): The probability of correct demodulation vs. the number of training data samples $L$ parameterized by $M$ for NFR is 3dB. (Lower): The probability of correct demodulation vs. the number of training data samples $L$ parameterized by NFR for $M = 4$. 
Figure 4.11: Training-based algorithm in Table 4.1 with the parameters $J = 2$, $K = 6$, $M = 2$, $N = 31$, and NFR is 3dB. (Upper): The probability of correct acquisition vs. SNR parameterized by $L$. (Lower): The bit-error-rate performance vs. SNR parameterized by $L$. 

103
Figure 4.12: Training-based algorithm in Table 4.1 with the parameters $K = 6$, $L = 6JN$, $M = 2$, $N = 31$, and NFR is 3dB. (Upper): The probability of correct acquisition vs. SNR parameterized by $J$. (Lower): The bit-error-rate performance vs. SNR parameterized by $J$. 

104
Figure 4.13: Training-based algorithm in Table 4.1 with the parameters $L = 6(JN)$, $M = 2$, $N = 31$, SNR = 14dB, and NFR is 0dB, (Upper): The acquisition-error-rate performance vs. the number of users $K$ parameterized by $J$. (Lower): The bit-error-rate performance vs. the number of users $K$ parameterized by $J$. 

105
Figure 4.14: Iterative algorithm in Table 4.2 with the parameters $J = 2$, $K = 6$, $M = 2$, $N = 31$, and NFR is $3\text{dB}$. (Upper): The probability of correct acquisition vs. SNR parameterized by $\alpha$. (Lower): The bit-error-rate performance vs. SNR parameterized by $\alpha$. 
Figure 4.15: Iterative algorithm in Table 4.2 with the parameters $\alpha = 1/(6JN)$, $K = 6$, $M = 2$, $N = 31$, and NFR is 3dB. (Upper): The probability of correct acquisition vs. SNR parameterized by $J$. (Lower): The bit-error-rate performance vs. SNR parameterized by $J$. 
Figure 4.16: The acquisition and BER performance vs. stages $M$ parameterized by $J$ for $K = 6$, $L = 6JN$, $N = 31$, and SNR = 8dB.
Figure 4.17: Training-based algorithm in Table 4.1 with the parameters $J = 2$, $K = 6$, $M = 4$, and $N = 31$. (Upper): The acquisition-error-rate vs. SNR parameterized by $L$. (Lower): The bit-error-rate performance vs. SNR parameterized by $L$. 

109
Figure 4.18: Training-based algorithm in Table 4.1 with the parameters $K = 6$, $L = 6JN$, $M = 4$, and $N = 31$, (Upper): The acquisition-error-rate vs. SNR parameterized by $J$. (Lower): The bit-error-rate performance vs. SNR parameterized by $J$. 
Chapter 5

A Cochannel TDMA Array Receiver

5.1 Introduction

TDMA is a digital multiplexing technique that has been widely used in cellular/PCS communication systems such as the IS-136 (Interim Standard 136) [36] and GSM (Global System for Mobile Phones) [37]. Both IS-136 and GSM belong to a frequency re-use system which means that in a given coverage area there are several cells that use the same set of frequencies. These cells are called cochannel cells and the interference between the signals in these cells is called cochannel interference (CCI). The cochannel TDMA communication interferes in bursts with each other in a somewhat random manner. Also, the presence of reflecting objects and scatterers in the channel creates a constantly changing environment that scrambles the signal energy and as a consequence dissipates the signal in amplitude, phase, and time synchronism. These effects lead to multiple contaminated versions of the transmitted signals that arrive at the receiver end. Multipath propagation creates intersymbol interference (ISI) which adversely affects receiver performance. CCI and ISI are the two major problems in a TDMA-based communication system. In a greater transmission congestion these problems become even more serious.
A receiver antenna algorithm that uses an adaptive spatial-temporal processing technique is presented here to suppress CCI, time dispersion, and multipath fading that occur in a TDMA-based mobile radio. An adaptive GLRT is formulated to acquire frame synchronization and the DOAs of cochannel TDMA bursts. The directivity pattern of the beamformer is calculated by means of estimated DOAs in order to precisely perform beamforming, to lock onto the signal of interest (SOI) and to reject cochannel interferers (CCIs) in overlapped TDMA bursts. Finally, a linear equalizer is employed to remove the ISI introduced by the transmit filter. The potential performance of the proposed cochannel TDMA receiver is illustrated by computer simulations that use cochannel IS-136 signals.

5.2 TDMA System Properties

In a TDMA-based mobile system the downlink from a base station to the mobile units assumes a time division multiplexing (TDM) transmission, where each mobile unit processes only the slot to which it is assigned. The uplink from the mobile units to the base station always occurs in random bursts. Cochannel interference is much more severe in the uplink. Thus, the uplink is considered to be the most challenging.

The base station receives synchronized bursts from users within its cell and randomly timed bursts from users in distant cells. In the sequel, cochannel signals are treated the same as a signal of interest. Our goal is to separate and estimate all signals that impinge on the antenna array. For physical reasons, an antenna array may generally be available only at the base station.

In an uplink, the cochannel interference and additive white Gaussian noise (AWGN) effect the transmitted signal channel and impinge on an \( J \)-element receiving array at the receiver end. As in many studies, as well as the present study, the received data of each antenna sensor is first downconverted and then
sampled and A/D converted at the Nyquist rate. The output digitized data sequences of the A/D converters are then passed to the adaptive GLRT algorithm for frame synchronization and beamforming/equalization.

5.3 Cochannel TDMA System Model

Suppose that $K$ cochannel users, each with $K_i$ propagation paths, impinge on a $J$-element receiving array, then the signals, received at the base station, can be modeled in the following manner:

$$x(t) = \sum_{i=1}^{K} \sum_{j=1}^{K_i} a_{ij} b\left(\theta_{ij}\right) A_i(t - \tau_{ij}) s_i(t - \tau_{ij}) + n(t),$$

where

$$s_i(t) = \sum_{n=-\infty}^{\infty} d_i(n) g(t - nT),$$

$d_i(k)$ denotes the transmitted data stream of the $i$-th cochannel user, $g(\cdot)$ is the pulse shaping function (i.e., the transmit filter), and $T$ denotes the symbol period. A square-root raised-cosine (SRRC) filter with roll-off factor $\alpha = 0.35$ is used for the TDMA-based IS-136. $A_i(t)$ is the amplitude function of the $i$-th user. $a_{ij}$, $\theta_{ij}$, and $\tau_{ij}$ denote, respectively, an attenuation factor, the direction of arrival (DOA), and the propagation delay of the $j$-th path of user $i$. $b\left(\theta_{ij}\right)$ is an array response vector that depends on the $\theta_{ij}$, i.e., $b_{ij}$ is a function of $\theta_{ij}$. $n(t)$ represents a baseband white Gaussian noise vector.
5.4 Receiver Structure

At the receiver, the received data is first downconverted to baseband and then A/D converted at the Nyquist rate. These digitized data sequences are passed to an adaptive GLRT algorithm for frame synchronization and then through the beamforming/equalization process, shown in Figure 5.1.

5.4.1 Synchronization System

A typical TDMA frame contains a fixed length of synchronization symbols for each time slot that are known at the receiver and can be used for channel parameters estimation. Since the frame/slot structure is fixed in TDMA-based communication systems, knowledge of the starting times of the synchronization code sequences for bursts in a block of data can provide information about the cochannel scenario for the entire block. The near-zero cross-correlation synchronization
code sequences are designed for initial frame acquisition. Here the detection of the multiple active users' synchronization code sequences that are embedded in the cochannel interference is treated as a multiple-hypothesis test. In order to distinguish the multiple hypotheses, a GLRT, that is described by a probability density function (pdf), $P$, defined on the sample space $X$ with a parameter set $\Omega$, is derived next.

The received data is first downconverted to baseband and then A/D converted at the Nyquist rate at the receiver end. The digitized data sequences are then passed through the adaptive GLRT algorithm for frame synchronization. Consider a receiving antenna array with $J$ sensors and suppose that the received signal $x(t)$ is sampled at the Nyquist rate, then the $k$-th sample of the receiver array is represented as the column vector, given by

$$x(k) = [x_1(k), x_2(k), \ldots, x_J(k)]^T,$$ (5.1)

for $k = 1, 2, \ldots, N$, where "$^T$" denotes the matrix transpose and $N$ equals to the length of the filtered synchronization code sequences. Hence, a $J \times N$ observed data matrix $X_t$ formed at sampling time instant $t$ is constituted by $N$ successive $J$-vectors of the receiver array as follows:

$$X_t = [x(1), x(2), \ldots, x(N)].$$ (5.2)

Let

$$t_{fa}(k) = \sum_{n=0}^{N-1} t_s(n)g((k - n)T), \quad k = 1, 2, \ldots, N$$

denote the filtered synchronization code sequence, which is generated by convolving the code sequence $t_s$ of length $N$ with the transmit waveform filter $g(\cdot)$. 

115
respectively. Then sample the convolved sequence at the Nyquist rate. The subscripts \( f \) and \( i \) denote respectively the filtered and the \( i \)-th synchronization code sequence. The filtered synchronization code sequence is usually represented by a row vector of \( N \) components as follows:

\[
t_{f,i} \triangleq [t_{f,i}(1), t_{f,i}(2), \ldots, t_{f,i}(N)]^T, \quad i \in \{1, 2, \ldots, I\},
\]

where \( I \) denotes the number of the different code sequences used in the TDMA-based communication system. Usually \( I \) is 6 for IS-136 and 8 for GSM in the half-rate operating mode which means each active user is assigned only one slot in a frame.

To search for the unknown location of each code pattern, the adaptive detector is designed by sliding a “moving window”, sample-by-sample, through all of the received data. Thus at each sampling time, the detector of the filtered synchronization code sequence, \( t_{f,i} \), which observes a new data set of \( N \) samples, must distinguish between multiple hypotheses by the adaptive GLRT algorithm. A set of multiple hypotheses are defined as follows: the null hypothesis \( H_0 \) is composed of an interference-plus-noise only process, and the signal-interference-plus-noise hypotheses \( H_i \), for \( i = 1, 2, \ldots, I \) are the processes given by

\[
H_0 : \quad X_t = V_t,
\]

\[
H_i : \quad X_t = b_{t,i} t_{f,i} + V_t, \quad (5.3)
\]

where \( b_{t,i} = [b_1, b_2, \ldots, b_J]^T \) represents the receiving array response vector of the burst with the \( i \)-th synchronization code pattern embedded at the search time instant \( t \). \( V_t \) represents the cochannel interferers plus the white Gaussian noise in the search window.
The \( k \)-th sample of the noise process \( \mathbf{v}_t \), i.e., \( \mathbf{v}_t(k) = [v_{t,1}(k), \ldots, v_{t,J}(k)]^\top \), that impinges on the \( J \)-sensor receiver array is composed of vectors that are due to the combined effects of directional CCI and the spatially white Gaussian noise can be modeled as Gaussian zero-mean complex random variables \([4]\). It is approximately independent from spatial sample to sample. Also, assume that the stationarity time constant of the noise process exceeds window size \( N \). The conditional mean and the unknown covariance matrix of \( \mathbf{x}(k) \) for the given hypotheses \( H_{i^l}, i^l = 0,1, \ldots, I \), can be computed as follows:

\[
E[\mathbf{x}(k)|H_0] = E[\mathbf{v}_t(k)|H_0] = \mathbf{0},
\]

\[
E[\mathbf{x}(k)|H_i] = E[\mathbf{b}_{i,i^l},\mathbf{v}_t(k)|H_i] = \mathbf{b}_{i,i^l} + \mathbf{v}_t(k), \quad i = 1,2,\ldots, I,
\]

\[
\mathbf{R} \triangleq E[(\mathbf{x}(k) - E[\mathbf{x}(k)])(\mathbf{x}(k) - E[\mathbf{x}(k)])^\dagger|H_{i'}], \quad i' = 0,1,\ldots, I.
\]

The generalized likelihood ratio function of the \( i \)-th synchronization code sequence \( L_i(\mathbf{X}_t) \), which depends on the maximum likelihood estimate \( \hat{\Theta}_i \) of the parameter vector \( \Theta_i \), under the multiple hypotheses, takes the form ([6], Appendix A), ([39], Page 1762):

\[
L_i(\mathbf{X}_t) = \frac{\text{max}_{\mathbf{R}_0} \mathbf{b}_{i^l} \mathbf{P}(\mathbf{X}_t|H_i)}{\text{max}_{\mathbf{R}} \mathbf{P}(\mathbf{X}_t|H_0)} \quad (5.4)
\]

\[
= \frac{\mathbf{P}(\mathbf{X}_t; \hat{\Theta}_i(\mathbf{X}_t))}{\mathbf{P}(\mathbf{X}_t; \hat{\Theta}_0(\mathbf{X}_t))} \quad (5.5)
\]

\[
= \frac{|\hat{\mathbf{R}_i}|^N}{\text{min}_{b_{i^l}} |\hat{\mathbf{R}_i}|^N}, \quad (5.6)
\]

where

\[
\mathbf{P}(\mathbf{X}_t|H_i) = \frac{1}{\pi^{NJ} |\mathbf{R}|^{N^2} e^{-N^2 \text{Tr}(\mathbf{R}^{-1}\hat{\mathbf{R}_i})}},
\]

\[
\mathbf{P}(\mathbf{X}_t|H_0) = \frac{1}{\pi^{NJ} |\mathbf{R}|^{N^2} e^{-N^2 \text{Tr}(\mathbf{R}^{-1}\hat{\mathbf{R}_0})}},
\]

117
\[
\hat{R}_0 = \frac{1}{N} \sum_{k=1}^{N} x(k)x^\dagger(k) = \frac{1}{N} X_t X_t^\dagger,
\]
\[
\hat{R}_i = \frac{1}{N} \sum_{k=1}^{N} x_{b_{t,i}}(k)x_{b_{t,i}}^\dagger(k) = \frac{1}{N} (X_t - b_{t,i}t_{f,i})(X_t - b_{t,i}t_{f,i})^\dagger,
\]
\[
x_{b_{t,i}}(k) = x(k) - E[x(k)|H_i] = x(k) - b_{t,i}t_{f,i}(k).
\]

\(P(X_t|H_i)\) and \(P(X_t|H_0)\) represent the joint probability density functions of \(X_t\) in terms of the trace function, \(T_r\), and the estimates of \(R\) under multiple hypotheses. \(\hat{R}_i\) and \(\hat{R}_0\) are the well-known MLEs of the unknown covariance matrix \(R\) and the unknown array response vector \(b_{t,i}\) under hypotheses \(H_i\) and \(H_0\), respectively; see [28].

Take the \(N\)-th root, the likelihood ratio in Eq.(5.6) can be simplified to

\[
\ell_i(X_t) = \frac{|X_t^t X_t|}{\min_{b_{t,i}} |F_{b_{t,i}}|},
\]  
(5.7)

where

\[
|F_{b_{t,i}}| = |G_i||1 + (\hat{b}_{t,i} - X_t\hat{t}_{f,i})^\dagger R_{b_{t,i}}^{-1}(\hat{b}_{t,i} - X_t\hat{t}_{f,i})| = |G_i||1 + R_{b_{t,i}}|,
\]  
(5.8)

\[
G_i = X_t X_t^\dagger - (X_t^\dagger t_{f,i})(X_t^\dagger t_{f,i})^\dagger,
\]  
(5.9)

\[
R_{b_{t,i}} = (\hat{b}_{t,i} - X_t\hat{t}_{f,i})^\dagger G_i^{-1}(\hat{b}_{t,i} - X_t\hat{t}_{f,i}).
\]

Note that a general invertible linear transformation \(T\) can be introduced in advance to simplify the separation process under multiple hypotheses in Eq.(5.3). The generalized likelihood ratio function \(\ell_i\) in Eq.(5.7) is shown to be independent of the linear transformation \(T\), i.e., the likelihood ratio test takes the same form either with or without the introduction of an invertible linear transformation \(T\).

If no default form is assumed for the antenna pattern \(b_{t,i}\), the maximum solution of Eq.(5.7) is achieved when the second term in Eq.(5.8) vanishes, i.e.,
Figure 5.2: A non-constrained MLE of DOA.

\[ R_{b_{t,i}} = 0 \quad \Rightarrow \quad \hat{t}_{i,t} = X^t_i t_{i,t}^\dagger ; \quad (5.10) \]

so that

\[ \ell_i(X_t) = \frac{|X_t X_i^\dagger|}{|G_i|} = \frac{1}{1 - (X_t t_{i,t}^\dagger)^\dagger (X_t X_i^\dagger)^{-1} (X_t t_{i,t}^\dagger)} . \quad (5.11) \]

Evidently, the test in (5.7) plays the same role as the test statistic \( y_i \) given by

\[ y_i(X_t) = (X_t t_{i,t}^\dagger)^\dagger (X_t X_i^\dagger)^{-1} (X_t t_{i,t}^\dagger) = \frac{(X_t t_{i,t}^\dagger)^\dagger (X_t X_i^\dagger)^{-1} (X_t t_{i,t}^\dagger)}{(t_{i,t} t_{i,t}^\dagger)} . \quad (5.12) \]

The test statistic \( y_i \) is used to test at each time phase for the existence of the \( i \)-th synchronization code pattern. A filter bank which contains the test statistics \( y_i, \ i = 1, 2, \ldots, I \), for each synchronization code sequence is formed in parallel to detect the existence of all available synchronization code patterns within the cell.

The decision on which the synchronization code pattern is most likely to occur at a given time phase is made by finding the maximum value over the filter bank of tests in \( Y_i \triangleq \{y_i(X_t) | i = 1, 2, \ldots, I \} \) then comparing the maximum value with a given threshold, as depicted in Figure 5.2.
Due to its comparative simplicity, the non-constrained MLE of DOA is used for the fast acquisition of the synchronization code sequences. However, the information about the DOAs obtained by the non-constrained MLE of DOA may not be sufficiently accurate. To prove this, an example of simulated cochannel TDMA scenario with signals in IS-136 format is conducted. Figures 5.7(a-c) show the signal constellations using the direction vector derived from the estimate DOAs of the non-constrained MLE of DOA, i.e., \( \hat{\mathbf{b}}_{\text{b,i}} \) in Eq.(5.10). Since the results do not show any similarity to the standard DQPSK signal constellations of IS-136, it is assured that the directivity pattern of the beamformer computed by means of \( \hat{\mathbf{b}}_{\text{b,i}} \) does not perform well to effectively suppress the CCIs and lock-up onto the SOI. In other words, further refinement procedures on the DOA for each cochannel signal need to be performed. Therefore, a constrained MLE of DOA is developed to achieve the refinement of the DOA by operating on the data set in which the synchronization code pattern is acquired.

### 5.4.2 DOAs Acquisition System

Compressed signal constellations for cochannel TDMA signals primarily relies on precise estimation on the DOAs. There are two alternatively approaches for acquiring DOAs to be investigated.

#### 5.4.2.1 A Constrained MLE of DOAs

For the constrained MLE of DOA, it is assumed for simplicity and an example that a line array with equally spaced \( J \)-elements is employed, and that the array response vector \( \mathbf{b}_{\text{r,d}} \) has the form:

\[
[1, e^{j\phi}, \ldots, e^{j(J-1)\phi}]^T,
\]

(5.13)
where
\[
\phi = \frac{2\pi d \sin \theta}{\lambda}. \tag{5.14}
\]

Here, \(\lambda\) is the signal-carrier wavelength, \(d\) is the spacing between antenna elements, and \(\theta\) is the DOA of the cochannel burst.

The DOA of a specified synchronization code sequence whose location is marked by the non-constrained MLE of the DOA can be determined by finding the minimum value of the denominator of Eq.(5.7), which is given in Eq.(5.8). Since \(G_i\) does not depend on variable \(\phi\), the minimization of Eq.(5.8) is equivalent to finding \(\theta\) such that
\[
\hat{\theta}_i = \min_{\hat{\theta}} \left[ (\hat{B}_{l,i} - X_i t_{j,i})^\dagger G_i^{-1} (\hat{B}_{l,i} - X_i t_{j,i}) \right], \tag{5.15}
\]
\[
= \min_{\hat{\theta}} [R_{B_l}]. \tag{5.16}
\]

The exhaustive search within 180\(^\circ\) is in a 1-dimensional space.

### 5.4.2.2 A Constrained MLE of DOAs with the ESPRIT Algorithm

To alleviate the computational load and avoid the exhaustive search process, the ESPRIT algorithm can be used along with the constrained MLE of DOAs. The acquiring process of DOAs which has been mentioned in Chapter 3 is briefly summarized as follows:

\[
\Psi_k = eigenvalues \ of \ \Psi, \ k = 1, 2, \ldots, Q, \tag{5.17}
\]

so that
\[
\psi_k = \arcsin \left( \frac{\arg(\Psi_k)}{2\pi d/\lambda} \right). \tag{5.18}
\]
Figure 5.3: A constrained MLE of DOA with/without the ESPRIT algorithm.

However, ESPRIT can acquire all bursts’ DOAs, but to identify DOA of the \( i \)-th synchronization code sequence, the following search process has to be performed over the data in which the \( i \)-th synchronization code pattern is confirmed,

\[
\hat{\theta}_i = \min_{\psi_k} \psi_k \left[ \left( \hat{b}_{t,i} - X_t \hat{\nu}_{f,i}^\dagger \right)^\dagger G_i^{-1} \left( \hat{b}_{t,i} - X_t \hat{\nu}_{f,i}^\dagger \right) \right],
\]  

(5.19)

where \( \psi_k \) is defined in Eq.\( (5.18) \). The block diagram of the constrained MLE of the DOA with the ESPRIT algorithm which is utilized to refine the DOA of the desired synchronization code pattern is shown in Figure 5.3.

### 5.4.3 Beamforming and Equalization System

The sensitivity of an antenna array to interfering sources can be reduced by suitably processing the outputs of the individual array sensors. To accomplish this, the received data are partitioned into blocks to achieve beamforming. The length of each block is determined by the detected scenario and the number of antenna array sensors. In general, a \( N \)-sensor antenna array can be used to simultaneously
null up to \(N - 1\) interferers. The length of a beamforming block can be broadened
to accommodate cochannel signals up to the number of antenna array elements.
By making a full use of this antenna array property, the computational burden
to find the proper beamforming weights can be significantly reduced. The beam-
forming weights are computed by means of the estimated DOAs of the cochannel
signals in a specific block. In order to increase the output SINR, SOI is tuned to
the mainlobe, the desired look direction of the beampattern, and nulls are placed
on the directions of the CCIs in order to reject the interferers occurring outside
the mainlobe.

The observed data matrix \(X_t\), in which the \(i\)-th synchronization code sequence
with the impinging angle \(\theta_{t,i}\) is detected, can be modeled as follows:

\[
X_t = [b(\theta_{t,i})]b(\theta_2)\cdots b(\theta_Q)]S_t + N_t = B_tS_t + N_t, \tag{5.20}
\]

where the remaining DOAs of the cochannel bursts which appear in \(X_t\) are de-
noted by \(\theta_2, \ldots, \theta_Q\).

Define

\[
B_t = [b(\theta_{t,i})][b(\theta_2), \ldots, b(\theta_Q)] = [b(\theta_{t,i})]B_{t,i}, \quad \text{and}
\]

\[
S_t = \begin{bmatrix}
\frac{t_{f,i}}{v_2} \\
v_2 \\
\vdots \\
v_Q
\end{bmatrix} = \begin{bmatrix}
\frac{t_{f,i}}{S_{t,i}} \\
v_2 \\
\vdots \\
v_Q
\end{bmatrix}, \quad \text{where} \quad S_{t,i} = \begin{bmatrix}
v_2 \\
\vdots \\
v_Q
\end{bmatrix}. \tag{5.21}
\]
Note that \( S_t \) denotes the \( Q \) cochannel signals contained in \( X_t \). \( N_t \) stands for the noise matrix in the search window. From Eqs. (5.20) and (5.21), \( X_t \) can be re-expressed as

\[
X_t = b_{t,i} f_{j,i} + B_{t,i} S_{t,i} + N_t.
\]  

(5.22)

In the absence of noise, the subspace spanned by the direction vectors in \( B_t \) (i.e., \( b(\theta_1), b(\theta_2), \ldots, b(\theta_Q) \)) is exactly the same as that spanned by the columns of \( X_t \), i.e., \( \text{span}(B_t) = \text{span}(X_t) \). Therefore, \( X_t \) can be approximated by the rank-reduced matrix \( \hat{X}_t = B_t S_t \) with the space the same as the space spanned by the direction vectors \( B_t \). As a consequence, the optimal beamformer weights can be obtained by solving the following least-squares (LS) problem:

\[
\hat{w}_{b_{t,i}} = \min_{w_{b_{t,i}}} J(w_{b_{t,i}}) = \min_{w_{b_{t,i}}} \| t_{f,i} - \hat{X}_t \|_2^2.
\]

(5.23)

By plugging \( \hat{X}_t \) into Eq. (5.23), the cost function \( J \) which is defined as the sum of error squares can be rewritten as

\[
J(w_{b_{t,i}}) = \| (1 - w_{b_{t,i}}^\dagger b_{t,i}) t_{f,i} - w_{b_{t,i}}^\dagger B_{t,i} S_{t,i} \|^2.
\]

(5.24)

The solution to (5.24) can be approximated by

\[
J(w_{b_{t,i}}) = \| w_{b_{t,i}}^\dagger B_{t,i} S_{t,i} \|^2, \text{ subject to } w_{b_{t,i}}^\dagger b_{t,i} = 1.
\]

(5.25)

The cross-term in Eq. (5.24) is ignored. Eq. (5.25) is then minimized by projecting \( b_{t,i} \) onto the null space of \( \hat{B}_{t,i} \), which is spanned by the direction vectors of cochannel interferers. The beamformer weights \( \hat{w}_{b_{t,i}} \), which can be employed to
beamform the SOI with the $i$-th synchronization code pattern and to null the cochannel bursts, are calculated as follows:

$$
\mathbf{\hat{w}}_{bi,i} = \frac{(I - \mathbf{P}_{\hat{B}_{t;i}})\hat{b}_{t;i}}{\hat{b}_{t;i}^\dagger (I - \mathbf{P}_{\hat{B}_{t;i}})\hat{b}_{t;i}} = \frac{\mathbf{P}_{\hat{B}_{t;i}}^\perp \hat{b}_{t;i}}{\hat{b}_{t;i}^\dagger \mathbf{P}_{\hat{B}_{t;i}}^\perp \hat{b}_{t;i}},
$$

where

$$
\mathbf{P}_{\hat{B}_{t;i}} = \hat{B}_{t;i} (\hat{B}_{t;i}^\dagger \hat{B}_{t;i})^{-1} \hat{B}_{t;i}^\dagger \quad \text{and} \quad \mathbf{P}_{\hat{B}_{t;i}}^\perp = I - \mathbf{P}_{\hat{B}_{t;i}}.
$$

The matrix $\mathbf{P}_{\hat{B}_{t;i}}$ in (5.26) denotes the projection matrix of $\text{span}(\hat{B}_{t;i})$ and $\mathbf{P}_{\hat{B}_{t;i}}^\perp$ is its orthogonal complement. The weight vector can be interpreted as a spatial filter, that means it is designed to match the impinging angle $\theta_{t;i}$ of the desired user and to suppress other cochannel interferers simultaneously. By calculating the beamformer weights for each co-existing signal in turn, all cochannel signals that impinge on the antenna array can be estimated and separated. The block diagram of beamforming and equalization is shown in Figure 5.1.

For instance: Over the range $[t_a, t_b]$ of the scenario in Figure 5.4, there are 4 cochannel signals are included which is exactly the same number as the antenna sensors employed. The estimated DOAs of cochannel signals in $[t_a, t_b]$ are derived from the synchronization code patterns contained in $[t_A, t_B]$. The estimated DOA of burst 5, $\hat{b}_{t_5}$, as the SOI is projected onto the null space of $\hat{B}_{t_5}$, which is spanned by the estimated DOAs of the cochannel bursts $\hat{b}_{t_3}$, $\hat{b}_{t_4}$, and $\hat{b}_{t_6}$. The beamformer weights, $\mathbf{\hat{w}}_{b_{t_5}}$, calculated by Eq.(5.26) are employed to beamform the data of burst 5 and to null the cochannel interferers simultaneously over the range of $[t_a, t_b]$, i.e., the bursts 3, 4, and 6. By calculating the beamformer weights for each cochannel signal in turn, all signals can be estimated and separated.
Once the beamforming process is completed, a follow-on linear equalizer is adopted to remove the ISI introduced by the transmit filter for the CCI-free output of the beamformer. The reason is that the beamformer weights are designed to null CCI for cochannel TDMA but not to compensate for the ISI which is induced by the transmit filter. Therefore, the linear equalizer weight vector denoted by \( \mathbf{u} \triangleq [u_1, u_2, \ldots, u_N]^T \) is computed by the LS method operating on the original synchronization code sequence (i.e., the synchronization code sequence is not processed by the transmit filter) instead of the filtered synchronization code sequence. For the burst with the \( i \)-th synchronization code pattern,

\[
\mathbf{u}_i \mathbf{z}(k) \overset{LS}{=} t_i(k), \quad k \in [k_i^{\text{start}}, k_i^{\text{end}}],
\]

where

\[
\mathbf{u}_i = \left( \sum_{n=0}^{N-1} \mathbf{z}(k_i^{\text{start}} + n) \mathbf{z}^\top(k_i^{\text{start}} + n) \right)^{-1} \left( \sum_{n=0}^{N-1} \mathbf{z}(k_i^{\text{start}} + n) t_i(n) \right).
\]

\( \mathbf{z}(k) = [z(k), z(k+1), \ldots, z(k+N-1)]^T \) represents the output vector of the beamformer. The starting and end points of the \( i \)-th synchronization code pattern are denoted by \( k_i^{\text{start}} \) and \( k_i^{\text{end}} \), respectively. The output of the linear equalizer corresponds to the estimate of the transmitted signal patterns.

### 5.5 Numerical Results

Simulations are performed for the uplink of the IS-136 digital cellular standard. IS-136 TDMA system uses \( \pi/4 \)-shifted differential quadrature phase-shift keying (DQPSK) modulation so that there are two bits per symbol. A typical IS-136 TDMA frame contains \( N = 14 \) synchronization symbols for each time slot which are known at the receiver and therefore can be used for frame synchronization and beamformer/equalizer training. Two representative IS-136 scenarios in which the
cochannel signals are assumed operating in the full-rate mode, i.e., three users are assigned two slots each in a A-B-C-A-B-C pattern for a total six slots within each frame, are conducted in this chapter. The synchronization code sequences which are used for both tested scenarios are listed in Table 1. In the first scenario, the cochannel signals impinge on a four-element array with an inter-element spacing of \( \lambda/2 \). In the second scenario, the number of antenna elements and cochannel interferers are increased simultaneously and the attenuation factor for each burst is also considered. The relative delays listed in Figures 5.4 and 5.5 are relative to the beginning of the first burst in the search window. The synchronization code sequence used for each burst and the synchronized sequences corresponding to each burst are also indicated.

### 5.5.1 IS-136 Scenario I

Scenario I and its channel parameters are shown in Figure 5.4. In this scenario, only a single propagation path is assumed for each user. Three layers of cochannel
Table 1: Synchronization Code Sequences of the IS-54/136 Standard

<table>
<thead>
<tr>
<th>Sync. Seqs.</th>
<th>Phase Change (radians)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{-4x}{4}$ $\frac{3x}{4}$ $\frac{3x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$</td>
</tr>
<tr>
<td>4</td>
<td>$\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$ $\frac{-4}{4}$</td>
</tr>
<tr>
<td>5</td>
<td>$\frac{-4}{4}$ $\frac{3x}{4}$ $\frac{3x}{4}$ $\frac{3x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$</td>
</tr>
<tr>
<td>6</td>
<td>$\frac{-4x}{4}$ $\frac{3x}{4}$ $\frac{3x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$ $\frac{-4x}{4}$</td>
</tr>
</tbody>
</table>

Signals are overlapped in which the top two layers from distant cochannel cells are asynchronous with the local cell. The signal-to-noise ratio (SNR) is 20dB and signal-to-interference-plus-noise ratio (SINR) for each slot is -3dB because each cochannel signal is assumed to have an unit power. The frame synchronization results of the adaptive GLRT approach are presented in Figure 5.6(c). The detected synchronization code sequence number is labeled above the location of each peak. The sample cross-correlation method and the SQ algorithm for frame synchronization are also tested using the same parameters as Scenario I and their results are shown in Figure 5.6(a-b), respectively. The SQ and the proposed GLRT algorithm compare and identify the locations of the synchronization code sequences accurately, while it is hard for the sample cross-correlation method to recognize the peaks.

The signal constellations of two-frame durations using the estimated DOAs obtained from the non-constrained MLE of the DOA alone and the two MLEs of the DOA for the cochannel bursts 1, 5, and 7 in IS-136 Scenario I are shown in Figure 5.7(a-c) and (d-f), respectively. The signal constellation of burst 5 is the only one to be observed in Figure 5.7(a-c) and shows sufficient tight signal
constellation to make correct demodulation possible, but not clear for the others. Therefore, the non-constrained MLE of the DOA alone fails to acquire the DOAs precisely, and the accurate beamforming/equalization processing becomes unexpected. The acquisition results of the constrained MLE of the DOA for the cochannel bursts 1, 5, and 7 are presented in Figure 5.8(a-c). Applying the constrained MLE of the DOA alone on a few data sets in which contain synchronization code sequences, an improved impinging angle estimate of each user is derived by an identification of the maximization of the GLRT values. Then the beamformer weights are computed by means of the information of the estimated DOAs and used to enhance the effects of locking onto the SOI and rejecting the CClIs, thereby making the decoding process much more precise, as the results shown in Figure 5.7(d-f). It should be noted that the signal constellation for each cochannel user is much compressed than those of the guard-ramp (GR) algorithm presented in [20] and the SQ algorithm presented in [18] due to the accuracy of the estimated DOAs. The beampatterns derived from the two MLEs of the DOA are applied to beamforming the data of burst 5 during the time interval \([t_a, t_b]\) shown in Figure 5.4 and the results are presented in Figure 5.8(d). The beamformer weights result in the successes of locking onto the SOI and suppressing the CClIs that occur simultaneously. The constellation results of the cochannel bursts 1, 5, and 7 obtained by the constrained MLE of the DOA along with the ESPRIT are shown in Figure 5.7(g-i).

In this chapter, a use of the "look and null" beampattern can be controlled precisely by utilizing the information of the estimated DOAs. This assures that the performance of the proposed GLRT algorithm outperforms the GR and the SQ algorithms.
Table: Channel Parameters for Scenario II

<table>
<thead>
<tr>
<th>User</th>
<th>Relative Delay (in symbol)</th>
<th>Synchronized with users</th>
<th>Sync. Code Sequence</th>
<th>Attenuation (in degree)</th>
<th>DOA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>-</td>
<td>1</td>
<td>0.90</td>
<td>-25</td>
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<tr>
<td></td>
<td>2</td>
<td>-</td>
<td>1</td>
<td>0.97</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>-</td>
<td>1</td>
<td>0.92</td>
<td>-45</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>-</td>
<td>1</td>
<td>0.88</td>
<td>-75</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
<td>-</td>
<td>1</td>
<td>0.75</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>155</td>
<td>-</td>
<td>1</td>
<td>0.67</td>
<td>-35</td>
</tr>
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<td></td>
<td>160</td>
<td>-</td>
<td>1</td>
<td>0.82</td>
<td>85</td>
</tr>
<tr>
<td>5</td>
<td>162</td>
<td>1</td>
<td>2</td>
<td>0.99</td>
<td>-15</td>
</tr>
<tr>
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<td>75</td>
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<tr>
<td>7</td>
<td>324</td>
<td>1</td>
<td>3</td>
<td>0.94</td>
<td>25</td>
</tr>
</tbody>
</table>

= sync. code sequence 1 = sync. code sequence 2 = sync. code sequence 3

Figure 5.5: IS-136 Scenario II with channel Parameters.

5.5.2 IS-136 Scenario II

Scenario II and its channel parameters are shown in Figure 5.5. The number of antenna elements is increased from four to eight and simultaneously the cochannel signals are up to seven. Compared with the previous scenario, the multipath environment is stressed in Scenario II. There are 13 TDMA bursts from $K = 7$ users in a frame, and the number of paths is $K_1 = 3$, $K_4 = 2$, and $K_2 = K_3 = K_5 = K_6 = K_7 = 1$, i.e., users 1 and 4 generate the multipath bursts caused by the multipath reflections in this scenario. Note that users 1, 5, and 7 are synchronous within the local cell, while users 2, 3, 4, and 6 from nearby cochannel cells are asynchronous. The SINR is even up to -10dB in severe interfered time period while the SNR remains the same as in Scenario I. The attenuation factors for the multiple paths are also considered. The frame synchronization result is shown
in Figure 5.9(a). A zoom-in picture of 5.9(a) is presented in 5.9(b) for the x-coordinates between 10 to 190 and for the y-coordinates between 0 to 250. Both figures show that all locations of the synchronization code sequences are identified accurately, even under the severe interference period as shown in Figure 5.9(b).

## 5.6 Conclusions

A smart antenna together with adaptive spatial-temporal processing technique is presented to mitigate CCI, time dispersion, and multipath fading that occur in TDMA-based cellular network. The results show that the timings and the DOAs of the synchronization code sequences in an overlapping TDMA bursts scenario are perfectly identified by two alternative approaches, the adaptive GLRT using the two MLEs of DOA or using the constrained MLE of DOA along with the ESPRIT. The approach of the constrained MLE of DOA along with the ESPRIT can release an exhaustive search in 1-dimensional space and also achieve up to an order of computational savings from two MLEs using adaptive GLRT while a large antenna array is employed. In order to use the spatial diversity of the receiver antenna array, the beamformer weights at any time instant are computed by means of the estimated DOA of each cochannel burst. This makes precise beamforming available and improve the output SINR simultaneously. A linear equalizer with temporal diversity is adopted to remove the ISI induced by the transmit filter for the CCI-free output of the beamformer.

Compared with the SQ and MSB methods, no matter the number of antenna elements, the proposed algorithm not only substantially reduces the complexity of data management, but also achieves much better performance due to the derivation of the information of DOAs for cochannel bursts. Although results for the IS-136 signal format are presented in the computer simulations, the antenna algorithm can be modified easily for GSM signals and also for the TDMA-based 3G.
cellular network, UWC-136. Moreover, it is evident that the acquiring frame synchronization algorithm presented here is also capable to be modified and applied easily to asynchronous wideband code-division multiple access (W-CDMA) scenario for detecting the multipath timings of the spreading code sequence assigned to the desired user. For a practical realization, a reduced-rank modification of the proposed algorithm in order to reduce the computational complexity presently is being investigated.
Figure 5.6: Frame synchronization results of (a): the sample cross-correlation algorithm, (b): the SQ algorithm, and (c): the adaptive GLRT algorithm for IS-136 Scenario I.
Figure 5.7: Signal constellation results of the adaptive GLRT algorithm using
(a-c): the non-constrained MLE of the DOA, (d-f): the two MLEs of the DOA,
and (g-i): the constrained MLE of the DOA along with the ESPRIT for bursts
1, 5, and 7 in IS-136 Scenario I.
Figure 5.8: The acquisition results of the impinging angles of (a): burst 1, (b): burst 5, and (c): burst 7 acquired by the constrained MLE of the DOA for IS-136 Scenario I. (d): Beampatterns (Gain in dB) of the two MLEs of the DOA for burst 5 over the interval of $[t_a, t_b]$ in IS-136 Scenario I. Burst 5 is the SOI whereas bursts 3, 4, and 6 are the cochannel interferers.
Figure 5.9: (a): Frame synchronization results of the adaptive GLRT algorithm for IS-136 Scenario II. (b): Zoom-in of (a): x-axis(10 190) and y-axis(0 250).
Chapter 6

Conclusions and Future Studies

6.1 Conclusions

The important results achieved in this dissertation are summarized as follows:

1. A new adaptive self-synchronizing receiver with a $J$-antenna array is presented in Chapter 3 for asynchronous DS-CDMA systems. The receiver can work well with only the desired signature code sequence. No knowledge of each active user’s propagation delay is required. The capability of the proposed receiver to acquire the multipath-timing structure of the desired user is emphasized. Also the proposed algorithm can be extended to multiuser detection by forming all of the single-user detectors in parallel if the other users’ signature sequences are available. Multiple-access interference is suppressed significantly by placing nulls that are formed by the adaptive coefficients of the $J$-element array receiver, in the directions of the interferers. Therefore, the performance of acquisition and demodulation is shown experimentally to achieve significant improvement when a larger antenna array is employed and to have potential against channel fading and near-far problems.

2. In Chapter 4 a low complexity version of the self-synchronizing DS-CDMA antenna-array receiver is developed that utilizes the concept of the multistage
reduced-rank Wiener filter. This technique releases the explicit estimation of the covariance matrix and obviates the task of a covariance matrix inversion as well. By projecting the observed data vector onto a reduced-rank subspace, the number of filter tap-weights is substantially reduced thereby improving the convergence and tracking performance. These important features result in a substantial reduction of the computational burden. As a consequence the computational complexity of the proposed system is reduced substantially from $O((JNS)^3)$ to $O(M(JNS))$ for each computing cycle of clock time. This achieved complexity approximates the complexity $O(JNS)$ of the MMSE CDMA receiver that uses the adaptive least mean squares (LMS) coefficients update algorithm and is much less than that of the adaptive recursive least squares (RLS) algorithm for use in the linear MMSE CDMA receiver (with $O((JNS)^2)$ operations). Also this reduced-rank multistage receiver achieves a rapid convergence for the adaption of the filter coefficients. The amount of the training samples, needed to initially estimate the filter coefficients, can be considerably decreased. These facts can directly translate into an enhancement of the bandwidth efficiency of the communication system or the potential performance gain when the amount of sample support is limited. Furthermore, the dimension of the projected subspace can be much less than the dimension of the signal subspace without compromising performance. When compared the performance to MMSE-type receiver, the proposed receiver can achieve a similar performance level without the need for prior information about synchronization or known propagation delay. Also due to its near-far resistant property, this new DS-CDMA receiver has no demand of the stringent power control mechanism that is needed in the conventional receiver.

Experimental results show that the proposed receiver obtains a better performance than that of the conventional detector in all simulations. Moreover, a 2-element antenna receiver has a performance that is comparable to a larger antenna array. It also achieves a significant improvement in acquisition and
demodulation in comparison with a single-element antenna receiver. These important facts make the new receiver meet the requirements of a lower-complexity, small-size, and light-weight receiver that mobile users demand today.

3. The proposed cochannel TDMA array receiver in Chapter 5 does not require known propagation delay for each user. Also the proposed algorithm not only substantially reduces the complexity of data management and also achieves a much better performance when compared with the SQ and MSB methods.

6.2 Future Studies

Some research directions that may be made more efforts in the near future are summarized as follows:

- Extend the proposed low-complexity multistage DS-CDMA array-antenna receiver to the multipath case.

- Further studies are needed to improve the convergence rate of the iterative version of the multistage reduced-rank filter.

- Include the effects of a fading channel on performance into the reduced-rank multistage DS-CDMA array receiver.
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Appendix A

Gold Sequence Generator

Consider the generation of the Gold sequences [10] of length \( N = 31 = 2^5 - 1 \) with the pair of preferred sequences that are obtained from the book [31] by Peterson and Weldon (1972), described by the polynomials,

\[
\begin{align*}
g_1(D) & = D^5 + D^2 + 1, \\
g_2(D) & = D^5 + D^4 + D^3 + D^2 + 1.
\end{align*}
\]

Let \( b \) and \( b' \) represent two maximum-length shift-register sequences (\( m \)-sequences) having period \( N = 31 \) derived from \( g_1(D) \) and \( g_2(D) \). The Gold family consists of \( 2^m + 1 = 2^5 + 1 = 33 \) sequences of length 31 which are given in [31] by

\[
G = \{b\} \cup \{b'\} \cup \{b + D^\nu b'\} \mid 0 \leq \nu \leq N - 1, \tag{A.1}
\]

where \( m \) denotes an \( m \)-stage shift register and the term \( D^\nu b' \) represents a phase shift of the \( m \)-sequence \( b' \) by \( \nu \) units. \( \cup \) represents set union. The Gold sequences used in computer simulations in Chapter 3 are derived by setting \( \nu \) in Eq.(A.1) equal to 0-5, respectively.
Appendix B

ESPRIT

This appendix briefly explains the estimation of signal parameters via rotational invariance techniques (ESPRIT) and its application to uniform linear array (ULA) for the DOAs estimation. In late 1990, Roy et al. [40] developed the ESPRIT algorithm which is a subspace-based DOA estimation technique. The ESPRIT algorithm dramatically reduces the computational and storage requirements of MUSIC and excludes an exhaustive search through all possible steering vectors by exploiting the rotational invariance of the underlying signal subspace induced by the translational invariance of two identical sensor subarrays. In the case of a ULA, the $J$-element antenna array is assumed to be composed of two identical $(J-1)$-element subarrays separated by a fixed distance $d$. If $Q$ signals impinge on the antenna array from distinct DOAs denoted by $\theta_1, \theta_2, \ldots, \theta_Q$, the ULA steering matrix takes the form

\[
B = \begin{bmatrix} b(\theta_1) & b(\theta_2) & \cdots & b(\theta_Q) \end{bmatrix}
= \begin{bmatrix} 1 & 1 & \cdots & 1 \\
1 & e^{j\phi_1} & \cdots & e^{j\phi_Q} \\
\vdots & \vdots & \ddots & \vdots \\
e^{j(J-1)\phi_1} & e^{j(J-1)\phi_2} & \cdots & e^{j(J-1)\phi_Q} \end{bmatrix}.
\]

146
Define two subarrays $\mathbf{B}_1$ and $\mathbf{B}_2$ by deleting the first and last rows from $\mathbf{B}$ respectively, i.e.,

$$
\mathbf{B} = \begin{bmatrix}
\mathbf{B}_1 \\
\text{last row}
\end{bmatrix} = \begin{bmatrix}
\text{first row} \\
\mathbf{B}_2
\end{bmatrix}.
$$  \hfill (B.1)

$\mathbf{B}_1$ and $\mathbf{B}_2$ in Eq.(B.1) are given by

$$
\mathbf{B}_1 = \mathbf{J}_1 \mathbf{B} \quad \text{and} \quad \mathbf{B}_2 = \mathbf{J}_2 \mathbf{B},
$$  \hfill (B.2)

where

$$
\mathbf{J}_1 = \begin{bmatrix}
1 & \cdots & 1 \\
0 & \cdots & 0
\end{bmatrix} \quad \text{and} \quad \mathbf{J}_2 = \begin{bmatrix}
0 & \cdots & 0 \\
1 & \cdots & 1
\end{bmatrix}.
$$

$I_{J-1}$ denotes an identity matrix with dimension $J-1$. Both $\mathbf{B}_1$ and $\mathbf{B}_2$ are $(J-1) \times Q$ matrices which sustain the same rank as $\mathbf{B}$. There exists several choices to define these two subarrays for ESPRIT, such as maximum overlap, interleaved and mixed model etc.[42]. Two key properties of ESPRIT are each matched pair in two subarrays must have the fixed distance and subarrays are under overdetermined condition, i.e., $(J-1) \geq Q$ (measurements exceed parameters). The maximum overlap model is discussed in this Appendix. Of course, the flexibility on the model is relied on the number of antenna elements. According to Eqs.(B.1) and (B.2), $\mathbf{B}_1$ and $\mathbf{B}_2$ are related by the formula

$$
\mathbf{J}_2 \mathbf{B} = \mathbf{J}_1 \mathbf{B} \Phi.
$$  \hfill (B.3)

The diagonal matrix $\Phi$ in Eqs.(B.3) is defined as

$$
\Phi = \text{diag}\{e^{j\phi_1}, e^{j\phi_2}, \ldots, e^{j\phi_Q}\},
$$
where $\phi_i$ and the corresponding impinging angle $\theta_i$ for $i = 1, 2, \ldots, Q$ have the relationship as defined in Eq.(3.57).

The signal received at the antenna array is modeled as

$$\mathbf{x}(t) = \sum_{i=1}^{Q} \mathbf{b}(\theta_i) s_i(t) + n(t) = [\mathbf{b}(\theta_1)|\mathbf{b}(\theta_2)|\cdots|\mathbf{b}(\theta_Q)]\mathbf{s}(t) + \mathbf{n}(t) = \mathbf{B}\mathbf{s}(t) + \mathbf{n}(t),$$

where $\mathbf{s}(t) = [s_1(t), s_2(t), \ldots, s_Q(t)]^T$ denotes the vector of signal amplitudes and phases and $\mathbf{n}(t)$ is AWGN. Therefore, the autocorrelation matrix of antenna array output is given by

$$\mathbf{R} = E\{\mathbf{x}(t)\mathbf{x}^\dagger(t)\} = \mathbf{B}\mathbf{R}_{ss}\mathbf{B}^\dagger + \sigma^2\mathbf{I} = \mathbf{VAV}^\dagger = \begin{bmatrix} \mathbf{V}_s & \mathbf{V}_n \end{bmatrix} \begin{bmatrix} \mathbf{A}_s & 0 \\ 0 & \mathbf{A}_n \end{bmatrix} \begin{bmatrix} \mathbf{V}_s^\dagger \\ \mathbf{V}_n^\dagger \end{bmatrix},$$

where $\mathbf{R}_{ss} = E\{\mathbf{s}(t)\mathbf{s}^\dagger(t)\}$ stands for the signal correlation matrix and is assumed to be full rank $Q$ (no unity correlated signals); $\sigma^2$ represents the noise variance at each array element. Denote $\mathbf{V} = [\mathbf{V}_s \mathbf{V}_n]$ and $\mathbf{A} = \text{diag}(\mathbf{A}_s, \mathbf{A}_n)$: Also let $\mathbf{A}_s = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_Q)$ which contains the $Q$ largest eigenvalues of $\mathbf{R}$ in descending order and $\mathbf{V}_s = [\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_Q]$ contains the corresponding orthonormal eigenvectors; $\mathbf{A}_n = \sigma^2\mathbf{I}_{J-Q}$ and $\mathbf{V}_n = [\mathbf{v}_{Q+1}, \mathbf{v}_{Q+2}, \ldots, \mathbf{v}_J]$ contains the $(J - Q)$ orthonormal eigenvectors that have the eigenvalues $\sigma^2$. The range space of $\mathbf{V}_s$ is called the signal subspace, and its orthogonal complement (the null space of $\mathbf{V}_s^\dagger$), the noise subspace, is spanned by $\mathbf{V}_n$. 

148
$V_s$ is partitioned conformably with $B$ into two subarrays $V_1$ and $V_2$ corresponding to $B_1$ and $B_2$. There must exist a unique nonsingular $Q \times Q$ matrix $T$ that satisfies

$$V_s = BT, \quad V_1 = J_1 BT, \quad and \quad V_2 = J_2 BT. \quad (B.4)$$

Combining Eqs.(B.3) and (B.4) yields

$$V_2 = V_1 \Psi, \quad (B.5)$$

where

$$\Psi \triangleq T^{-1} \Phi T. \quad (B.6)$$

Note that $\Psi$ and $\Phi$ are related by a similarity transformation, and hence have the same eigenvalues. Thus, the DOAs estimation is analogously converted to find the eigenvalues of $\Psi$. $\Psi$ in Eq.(B.6) can be solved by either in the Least-Squares sense (LS-ESPRIT) or in the Total Least-Squares sense (TLS-ESPRIT) [40, 42].
Appendix C

Analysis of the Parameter \((\kappa^{-1}[i] \Delta_1[i])\)

The verification that the quantity \(\omega_1[i] = (\kappa^{-1}_1[i] \Delta_1[i])\) in (4.35) is a positive scalar is the same as showing that \(\kappa_1[i]\) is positive due to the fact that the norm of \(u\), namely \(\Delta_1[i]\), is positive. To show this note first that the matrix \(M[i] = T_1[i] R_v[i] T_1^*[i]\) in (4.27) can be re-expressed as follows:

\[
M[i] = \begin{bmatrix}
M_{11}[i] & M_{12}[i] \\
M_{21}[i] & M_{22}[i]
\end{bmatrix},
\]

where

\[
\begin{align*}
M_{11}[i] &= \sigma^2_\delta[i] - \Delta^2_1[i] \text{ (under } H_1) \text{ or } M_{11}[i] = \sigma^2_\delta[i] \text{ (under } H_0), \\
M_{12}[i] &= r_{x, \delta_1[i]}, \quad M_{21}[i] = r_{x, \delta_1[i]}, \quad M_{22}[i] = R_{x_1[i]}.
\end{align*}
\]

Thus the determinant of matrix \(M[i]\) ([45], pp.294) is given by

\[
\det(M[i]) = \det \left( \begin{bmatrix}
M_{11}[i] - M_{12}[i] M_{22}^{-1}[i] M_{21}[i] & 0 \\
M_{21}[i] & M_{22}[i]
\end{bmatrix} \right) = \Sigma[i] \cdot \det(M_{22}[i]), \quad (C.1)
\]
where $\Sigma[i]$ in (C.1), called the Schur complement of $M_{11}[i]$, is given by

$$
\Sigma[i] = M_{11}[i] - M_{12}[i] M_{22}^{-1}[i] M_{21}[i].
$$

(E.2)

Evidently $\Sigma[i]$ equals $\kappa_1[i]$ in (4.37).

Because of the invertibility of the linear transformation $T_1[i]$ and the positive definiteness of the signal-free correlation matrix $R_v[i]$, the matrix $M[i]$ and its inverse (i.e., $M^{-1}[i]$ or $(T_1[i]^\dagger R_v[i] T_1[i])^{-1}$) are also positive definite. If one introduces an arbitrary non-zero $JNS$-vector $q$, then the positive definiteness implies (see [12], pp. 141)

$$
q^\dagger M[i] q = (T_1[i]^\dagger q)^\dagger R_v[i] (T_1[i]^\dagger q) > 0.
$$

This property in turn implies that the determinant of $M[i]$ is positive, i.e.,

$$
\det(M[i]) > 0.
$$

Also $\det(R_x[i]) > 0$ is satisfied because of the positive definiteness of $R_x[i]$. Thus, the inequality, $\kappa_1[i] > 0$ is verified.