“Ultra-wideband Radios with Transmitted Reference Methods”

by

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CSI-06-04-02
Ultra-wideband Radios with Transmitted Reference Methods

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A Dissertation Presented to the
FACULTY OF THE GRADUATE SCHOOL
UNIVERSITY OF SOUTHERN CALIFORNIA
In Partial Fulfillment of the
Requirements for the Degree
DOCTOR OF PHILOSOPHY
(ELECTRICAL ENGINEERING)

May 2005

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Yi-Ling Chao
Dedication

To my dear parents, brothers, sister-in-law, nephew and niece - *may your lives be full of happiness*
Acknowledgements

I would like to express my deep gratitude to my thesis advisor, Professor Robert A. Scholtz. He gave me a lot of support, encouragement, and guidance so I can finish my thesis. From him, I have learned not only the professional knowledge but also the attitude towards doing research. It was truly my honor to work with him. I would also like to thank Professor Won Namgoong, Urbashi Mitra and Charles L. Weber at the Department of Electrical Engineering for their valuable suggestions on my research. I am also thankful to Professor Fengzhu Sun at the Department of Mathematics for his statistics courses and being the outside member of my committee.

My gratitude goes to the staff at the Communication Sciences Institute, especially Mrs. Milly Montenegro, Ms. Mayumi Thrasher and Ms. Gerrielyn Ramos whose administrative help made my life at USC easier. I also thank my colleagues at the UltRa Lab. I have learned a lot from the discussion with them. I have my special thanks to Mrs. Lolly Scholtz whose hospitality made me feel warm while my families were not around, and my friends who gave me a joyful life during my studies in the United States.
Last but not the least, I sincerely thank my families whose unconditioned love, support, and letting me do anything I like without putting any pressure on me have helped me accomplish what I have today.

The financial support for my studies was provided in part by the Army Research Office under MURI Grant No. DAAD19-01-1-0477.
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Abstract

Ultra-wideband radio, conveying data by transmitting narrow pulses without carrier, is a promising communication technology. Transmitted reference method can work with cross-correlation receivers, which have simple channel estimation and multipath diversity acquirement as well as low sampling frequency, to make a low cost application available. Optimal and suboptimal single user receivers for ultra-wideband transmitted reference (UWB TR) systems in multipath environments are derived, based on both the average likelihood ratio test and generalized likelihood ratio test. These theoretically derived receivers are compared to the ad hoc cross-correlation receivers in structures and bit error probability (BEP) performance.

A weighted cross-correlation receiver is obtained by applying a weighting function which has a priori channel information to a conventional cross-correlation receiver to improve the BEP performance with restrictive receiver complexity. How to construct of a theoretical-weighting function and a rectangular-weighting function as well as the performance improvement by using weighted cross-correlation receivers are discussed in detail.

This thesis also proposes a generalized UWB TR signal model which combines the traditional TR and differential TR (uses the previous data-modulated pulse as a reference
by utilizing the differential encoding method) techniques to increase power efficiency and improve BEP. In addition, this generalized TR scheme can transmit data by using either binary or $M$-ary modulation. In the binary system, transmitted signals are designed so that the noise level in a correlator template can be reduced within a restrictive complexity. The $M$-ary modulation approach with a conventional or weighted cross-correlation receiver can enhance the BEP performance by transmitting data bits through block codes instead of repetition codes.

The multiple access performance of TR and differential TR systems with rectangular-weighted cross-correlation receivers is evaluated in multipath environments. The structure and probability distribution of the multiple access interference are studied, and the closed-form solution of BEP is theoretically analyzed under the Gaussian multiple access interference assumption.
Chapter 1

Introduction

1.1 Motivation

A ultra-Wideband (UWB) radio is a communication system that transmits signals whose 10dB bandwidth is greater than 20% of its center frequency or exceeds 500MHz [2]. Traditional UWB impulse radio communication systems transmit data by modulation of sub-nanosecond pulses. These narrow pulses are distorted by the channel, but often can resolve many distinct propagation paths (multipath) because of their fine time-resolution capability [3]. However, a Rake receiver that implements tens or even hundreds of correlation operations may be required to take full advantage of the available signal energy [4]. On the other hand, a receiver using a single correlator matched to one transmission path may be operating at a 10 - 15dB signal energy disadvantage relative to a full Rake receiver.

Because of the uncertainties of channels, some portion of transmitted energy is used to probe the channel in order to get the full knowledge for the Rake reception. How much energy should be put in sounding channels and how to arrange these training symbols depend on how fast channels vary and the transceiver complexity. In order to reduce the
stringent synchronization requirement, channel estimation, and Rake reception, Hoctor and Tomlinson in the GE Research and Development Center proposed a delay-hopped transmitted-reference (DHTR) system with a simple receiver structure to capture all of the energy available in a UWB multipath channel [6]. In this DHTR system, a reference waveform is transmitted before each data-modulated waveform for the purpose of determining the current multipath channel response. Since the reference pulse and data-modulated pulse are transmitted within the coherence time of the channel, it is assumed that the channel responses to these two pulses are the same. The proposed receiver correlates the data signal with the reference signal to use all the energy of the data signal without requiring additional channel estimation and Rake reception. This transmitted reference (TR) modulation combined with the conventional cross-correlation receiver can work in rapidly varying environments. In addition, the correlator, implemented by a delay line and an integrator in the analog portion of the receiver, avoids the difficulty and cost caused by a high-sampling-frequency analog-to-digital converter (ADC) that an all-digital receiver needs.

1.2 History of Transmitted Reference Methods

It is worth noting that the TR approach is not new, but dates back to the early days of communication theory [8]-[12]. All these early papers discuss transmitting signals in unknown channels which have multiplication and addition effects on signals. The output of the multiplication process as well as the additive noise are both assumed Gaussian distributed with known mean vectors and covariance matrices. Thus the received signals have
a Gaussian distribution. The mean vectors and covariance matrices are \textit{a priori} knowledge of the propagation channel. This Gaussian channel assumption is partially practical, and makes finding an optimal receiver structure possible due to the nice properties that Gaussian probability density functions possess.

The materials in [9]-[12] can be divided into two groups. Optimal receiver structures are discussed in [9] and [12] based on a fixed transmission strategy which sends one reference waveform before each data waveform. In [9], a maximum likelihood (ML) one-shot receiver of a binary system was found. In [12], the optimal one-shot receiver structure of an \( M \)-ary system was derived, and the probability of error for a binary phase-shift-keying (BPSK) system using both optimal and suboptimal receivers was analyzed. On the other hand, [10] and [11], without fixing signal models, investigate the structures of maximum \textit{a posteriori} (MAP) receivers in an \( M \)-ary system based on the prior information of the channels. In [10], a code which could perform best under the optimal receiver structure was also designed, and adaptive receivers which can learn the channel information from previous received signals were discussed. Although these authors have some variety in their transmitted signal structures and assumptions, results show that the optimal receiver structures based on MAP or ML criterion are matched filter receivers, which match the received signal with a weighted sum of the prior mean and the received (reference) signal. The weights of the received reference signal and the prior mean depend on the covariance matrices of the multiplicative channel and the additive noise.
1.3 Organization

Although a UWB radio with TR modulation combined with the conventional cross-correlation receiver is an easy system to implement, the bit error probability performance is restricted by two major drawbacks: (1) the transmitted reference waveform used as a correlator template is noisy, and (2) a fraction of the transmitted energy is not data bearing. Averaging multiple reference pulses to produce a cleaner template can improve the receiver performance [13, 25], but these cross-correlation receivers are still ad hoc receivers, and how well a more general UWB TR system can perform is still a complicated function of channel descriptions/statistics and channel stability, as well as complexity constraints on the receiver. When complexity constraints are removed or relaxed, more exotic channel estimation techniques and Rake receivers design are possible that provide better performance (at a higher complexity cost) than ad hoc conventional receivers, and in this case the utility of devoting energy to the reference signal is questionable.

In Section 2.1, the model of a multiple access UWB system with the conventional TR method is described. With the tapped delay line channel model described in Section 2.2.2, optimal and suboptimal receivers based on the average likelihood ratio test (ALRT) and on the generalized likelihood ratio test (GLRT) without any complexity constraints are derived in Chapter 3. The bit error probability (BEP) of conventional and average correlation receivers are discussed in detail in Section 4.1 and 4.2. A weighted cross-correlation receiver, which is an improvement of the conventional cross-correlation receiver with a complexity constraint, is investigated in Section 4.4. These theoretical optimal and suboptimal receivers are now compared to the ad hoc cross-correlation receiver in
Section 4.3 and their BEPs are compared in Section 4.5. Besides the tapped delay line channel model, a UWB channel model which is proposed by IEEE P802.15 working group is also described in Section 2.2.1. This clustered random path-arrival model with lognormal amplitude distribution is not tractable in theoretical analysis, but will be used in numerical examples later in this thesis. It is worth noting that all the receiver structures in Chapter 3 and 4 are contributed by the author except for the conventional and average cross-correlation receivers.

The DHTR system [6] is not exactly the same as the system described in Section 2.1, but both of them provide multiple access (MA) capability. The multiple access performance of the DHTR system with cross-correlation receiver is evaluated in [7] through numerical simulations. The MA performance of UWB systems with TR modulation and conventional cross-correlation reception was not theoretically analyzed before. In Chapter 5, the theoretical analysis is conducted by modelling the multiple access interference (MAI) as Gaussian distributed, and this Gaussian assumption is examined by simulations.

For the non-TR modulated UWB systems with slightly different structures which all involve time hopping and/or direct sequences in modulation, their multiple access performance has been evaluated by many researchers. Model the MAI by a Gaussian random variable is always disputable. Special orthogonal waveforms like hermite functions [26], as well as orthogonal sequences like Walsh-Hadamard codes [27], are exploited to prevent the MAI. In order to use these characteristics, all the users have to synchronize to each other, i.e., network synchronization has to be provided, because orthogonality does not exist if those waveforms or sequences are not time aligned. In addition, additive white Gaussian noise environments, long chip times, or guard intervals are also required to maintain the
orthogonality. Some other papers [28]-[30] discussed asynchronous systems and tried to find out the exact probability density functions of the MAI in order to obtain an accurate evaluation of the BEP. The assumption that the interferences coming from different users are independent and identically distributed (i.i.d.) is adopted in the derivation. With $N_u - 1$ undesired active transmitters in the environment, the probability density function (pdf) of the MAI is the convolution of $N_u - 1$ identical pdfs. When $N_u$ gets large, the computation becomes more and more untractable.

The TR-modulated UWB systems investigated in this dissertation are asynchronous systems. In addition, the i.i.d. assumption does not apply to a transmitted reference system because the MAI includes the correlation of signals from each undesired transmitter with the desired transmitter’s signal, as well as the correlation of signals from any two undesired transmitters which arrive during some specific intervals decided by the desired transmitter. Therefore, these two different MAI sources are not independent. Based on this reason, the exact distribution of MAI is untractable in a UWB TR system with conventional cross-correlation receivers. In [5], a Gaussian assumption is made for the MAI which is accurate when the number of users is large. With the transmitted reference technique and a conventional cross-correlation receiver structure, the MAI levels are greater than in a non-TR modulated system. Based on the consideration of the possibility of obtaining the distribution of the MAI and a simple closed-from solution of the BEP, the Gaussian assumption is adopted in the performance analysis in a TR multiple access system. The structure of the MAI and the accuracy of this assumption is also discussed in Chapter 5.
The low energy/power efficiency of a TR modulated system due to the fact that half of the transmitted energy is not data bearing can be improved by utilizing a differential encoder in the transmitter. This differential encoder, embedding information bits in the phase difference of two adjacent pulses, means that data-modulated pulses can also be a correlator template. The signal model of this differential transmitted reference (DTR) method is described in Chapter 6. Its multiple access performance with a cross-correlation receiver is also analyzed and compared to a conventional TR system.

Although averaging several reference pulses can clean up the correlator template and DTR methods can increase the energy/power efficiency, they also complicate the receiver. Because of the structure of the signals and hopping sequences in the conventional TR scheme, delays with variable length, which are difficult to implement using analog devices, are needed for these two performance improvement approaches. In order to make a feasible receiver as well as improve two drawbacks of the conventional TR method, transmitted signals are rearranged in Chapter 7 with a generalized UWB TR scheme. This novel TR scheme can transmit data using either binary or \( M \)-ary modulation. In the binary system, transmitted signals are designed so that the noise level in a correlator template can be reduced within a restrictive receiver complexity. The use of \( M \)-ary modulation with a conventional cross-correlation receiver can enhance the BEP performance by transmitting data bits through block codes other than simple repetition codes.
Chapter 2

System and Channel Models

2.1 Conventional UWB TR Modulation

A conventional direct-sequence time-hopping spread-spectrum (DS-TH-SS) multiple-access UWB system with transmitted reference modulation transmits one reference pulse before every data-modulated pulse, and the modulation scheme is binary antipodal modulation.

The transmitted signal of user \( n \) is

\[
s_{\text{tr}}^{(n)}(t) = \sum_{i=-\infty}^{\infty} d_{i}^{(n)}[g_{\text{tr}}(t - iT - c_{i}^{(n)}T_c) + b_{i/N_h}^{(n)}g_{\text{tr}}(t - iT - c_{i}^{(n)}T_c - T_d^{(n)})]. \tag{2.1}
\]

Here \( g_{\text{tr}}(t) \) is a transmitted monocycle pulse that is non-zero only for \( t \in (0, T_p) \), \( T_f \) is the frame time, and \( T_c \) is the duration of each time slot. The hopping sequence of user \( n \), \( \{c_{i}^{(n)}\} \), is a periodic code with period \( N_{hs} \), i.e., \( c_{i+jN_{hs}}^{(n)} = c_{i}^{(n)} \) for all integers \( i \) and \( j \). This hopping sequence with each code element in \( \{0, 1, ..., N_{h}^{(n)} - 1\} \) is a pulse shift pattern. The other pseudo-random sequence, \( \{d_{i}^{(n)}\} \), is a direct sequence with period \( N_{ds} \) and elements in \( \{+1, -1\} \). The hopping and direct sequence can not only provide multiple access capability
by preventing catastrophic collisions, but also smooth the spikes in the power spectrum by increasing the period of the transmitted signals. Each frame contains two monocycle pulses. The first is a reference and the second, $T_d^{(n)}$ seconds later, is a data-modulated pulse. In order to prevent the interpulse interference, $T_d^{(n)}$ should be at least equal to the channel delay spread $T_{mds}$. The data bits $b_{\lfloor i/N_s \rfloor}^{(n)}$ is in $\{+1, -1\}$, and the index $\lfloor i/N_s \rfloor$, i.e., the integer part of $i/N_s$, represents the index of the data bit modulating the data pulse in the $i^{th}$ frame. Hence each bit is transmitted in $N_s$ successive frames to achieve an adequate bit energy in the receiver. The frame time $T_f = (N_s - 1)T_c + T_p + T_d^{(n)} + T_{mds}$, so no interframe interference exists. An example of transmitted and received signals through a multipath environment from transmitter $n$ is plotted in Figure 2.1.

![Transmitted Signal](image1)

![Received Signal](image2)

**Figure 2.1**: An example of transmitted and received signals through a multipath environment from transmitter $n$ with $b_0^{(n)} = -1$, $d_0^{(n)} = 1$, $d_1^{(n)} = -1$, $d_{N_s-1}^{(n)} = 1$, $c_0^{(n)} = 0$, $c_1^{(n)} = 6$, and $c_{N_s-1}^{(n)} = 3$. A pulse with letter R or D represents a reference or data-modulated pulse.

In this system, $T_d^{(n)}$, $\{c_i^{(n)}\}$, and $\{d_i^{(n)}\}$ are different for each user in order to provide MA capability. For a received pulse $g_{rx}(t)$, a good choice of a set of $\{T_d^{(n)}\}$ is to make
\[ \int_{-\infty}^{\infty} g_{rx}(t) g_{rx}(t + T_d^{(n)} - T_d^{(m)}) \, dt \] as close to zero as possible for \( n \neq m \) which indicates the multiuser interference is minimized. Besides, we also want \( N_h^{(n)} \) as large as possible to provide a better capability to avoid collisions with other transmitters. Therefore, for a specified frame time \( T_f \), \( T_d^{(n)} \) should be as small as possible. In this case, the number of hopping time slots \( N_h^{(n)} \) is different for each user if all the users have the same frame time. By using a second order derivative Gaussian received pulse as an example,

\[
g_{rx}(t) = \begin{cases} 
\frac{1}{c} [1 - 4\pi (\frac{t - \tau_1}{\tau})^2] \exp[-2\pi (\frac{t - \tau_1}{\tau})^2] & t \in [0, 0.7\text{ns}] \\
0 & \text{elsewhere}
\end{cases}, \tag{2.2}
\]

where \( \tau = 0.2877 \text{ ns}, \tau_1 = 0.35 \text{ ns}, \) and the constant \( c \) normalizes the energy in \( g_{rx}(t) \) to 1, an example rule to assign \( T_d^{(n)} \) and \( N_h^{(n)} \) are given as

\[
T_d^{(n)} = T_{mds} + \frac{(n-1)T_p}{2}, \tag{2.3}
\]

\[
N_h^{(n)} = N_h^{(N_u)} + \left\lfloor \frac{N_u - n}{2} \right\rfloor, \tag{2.4}
\]

where \( N_u \) is the number of active transmitters. As long as \( |m - n| \geq 2 \), \( \int_{-\infty}^{\infty} g_{rx}(t) g_{rx}(t + T_d^{(n)} - T_d^{(m)}) \, dt = 0 \). For \( |m - n| = 1 \), \( \int_{-\infty}^{\infty} g_{rx}(t) g_{rx}(t + T_d^{(n)} - T_d^{(m)}) \, dt \) is equal to 0.07 which is close to zero. The waveform \( g_{rx}(t) \) and its autocorrelation function are plotted in Figure 2.2. Note that a fixed time separation for each transmitter is not the only choice. For each transmitter, the value of the time separation can change according to a pseudo-random sequence, i.e, \( T_d^{(n)} \) in (2.1) is replaced by \( T_d^{(n)}_{d,i} \). This can enhance the multiple access capability of the UWB TR system at the expense of receiver complexity.
2.2 Channel Models

Unlike narrow band systems, UWB systems do not have an accepted statistical channel model. Different researchers have proposed different channel models from their experimental measurements [16]-[22]. For both the theoretical and numerical reasons, two multipath models are used in this thesis. One is the clustered channel model which is adopted by the IEEE P802.15 working group for wireless personal area networks [16]. This channel model, which can be used to evaluate the performance of a designed UWB system, is not suitable for theoretical analyses. The other one is the non-clustered tapped delay line (TDL) channel model which can simplify theoretical analyses and make closed-form solutions possible. Shadowing effects are not considered in this thesis.
2.2.1 Clustered Channel Models

The impulse response of the clustered channel model is

\[ h(t) = \sum_{l=0}^{L-1} \sum_{k=0}^{K-1} \alpha_{k,l} \delta_{D}(t - T_l - \tau_{k,l}), \quad (2.5) \]

where \( \delta_{D}(t) \) is the delta function, \( T_l \) is the arrival time of the first path in the \( l \)th cluster with \( T_0 \leq T_1 \cdots \leq T_{L-1} \), \( \tau_{k,l} \) is the arrival time of the \( k \)th path in the \( l \)th cluster relative to \( T_l \) with \( 0 = \tau_{0,l} \leq \tau_{1,l} \cdots \leq \tau_{K-1,l} \), and \( \alpha_{k,l} \) is the amplitude of the \( k \)th path of the \( l \)th cluster. The amplitude \( \alpha_{k,l} = p_{k,l} \xi_l \beta_{k,l} \), \( p_{k,l} \in \{+1, -1\} \) with equal probability, and \( \xi_l \beta_{k,l} \) has log-normal distribution in (2.6)

\[ 20 \log_{10}(\xi_l \beta_{k,l}) \sim \text{Normal}(\mu_{k,l}, \sigma_1^2 + \sigma_2^2). \quad (2.6) \]

Equation (2.6) is equivalent to \( \xi_l \beta_{k,l} = 10^{(\mu_{k,l} + n_1 + n_2)/20} \) where \( n_1 \sim \text{Normal}(0, \sigma_1^2) \) and \( n_2 \sim \text{Normal}(0, \sigma_2^2) \) are independent and correspond to the effect of the \( l \)th cluster and \( k \)th ray in the cluster, respectively. Units of the mean \( \mu_{k,l} \) and standard deviation \( \sqrt{\sigma_1^2 + \sigma_2^2} \) are decibels, and \( \sigma_1^2 \) as well as \( \sigma_2^2 \) are variances of the cluster and ray strength model. The clusters and intra-cluster paths have exponentially distributed inter-arrival time

\[ p(T_l|T_{l-1}) = \Lambda \exp[-\Lambda(T_l - T_{l-1})] \quad l > 0 \quad (2.7) \]

\[ p(\tau_{k,l}|\tau_{k,l-1}) = \lambda \exp[-\lambda(\tau_{k,l} - \tau_{k,l-1})] \quad k > 0, \quad (2.8) \]
Table 2.1: Clustered channel model parameters [16].

<table>
<thead>
<tr>
<th>Parameter / Model</th>
<th>CM1</th>
<th>CM2</th>
<th>CM3</th>
<th>CM4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Λ (1/ns)</td>
<td>0.0233</td>
<td>0.4</td>
<td>0.0667</td>
<td>0.0667</td>
</tr>
<tr>
<td>λ (1/ns)</td>
<td>2.5</td>
<td>0.5</td>
<td>2.1</td>
<td>2.1</td>
</tr>
<tr>
<td>Γ (ns)</td>
<td>7.1</td>
<td>5.5</td>
<td>14.0</td>
<td>24.0</td>
</tr>
<tr>
<td>γ (ns)</td>
<td>4.3</td>
<td>6.7</td>
<td>7.9</td>
<td>12.0</td>
</tr>
<tr>
<td>σ₁ (dB)</td>
<td>3.3941</td>
<td>3.3941</td>
<td>3.3941</td>
<td>3.3941</td>
</tr>
<tr>
<td>σ₂ (dB)</td>
<td>3.3941</td>
<td>3.3941</td>
<td>3.3941</td>
<td>3.3941</td>
</tr>
<tr>
<td>Mean access delay spread (ns)</td>
<td>4.9</td>
<td>9.4</td>
<td>13.8</td>
<td>26.8</td>
</tr>
<tr>
<td>RMS delay spread (ns)</td>
<td>5</td>
<td>8</td>
<td>14</td>
<td>26</td>
</tr>
<tr>
<td>Number of paths (10dB)</td>
<td>13.3</td>
<td>18.2</td>
<td>25.3</td>
<td>41.4</td>
</tr>
<tr>
<td>Number of paths (85% energy)</td>
<td>21.4</td>
<td>37.2</td>
<td>62.7</td>
<td>122.8</td>
</tr>
</tbody>
</table>

where Λ and λ are the arrival rates of clusters and rays in each cluster. The mean value of the amplitude of the $k^{th}$ path in the $l^{th}$ cluster is decided by

$$\mathbb{E}\{|\xi_l\beta_{k,l}|^2\} = \Omega_0 \exp\left\{-\frac{T_l}{\Gamma} - \frac{\tau_{k,l}}{\gamma}\right\},$$

where $\Omega_0$ is the mean power of the first path in the first cluster, and $\Gamma$ as well as $\gamma$ are power decay constants of clusters and rays. Independent strength variables are assumed for each cluster and each ray within the cluster.

Four sets of parameters are suggested by IEEE P802.15 working group to model small indoor environments with different sizes and with/without line-of-sight (LOS). Model CM1 is a statistical model based on 0-4 m transmitter-receiver distance measurement with LOS propagation, CM2 is based on 0-4 m transmitter-receiver distance non-line-of-sight (NLOS) channel measurement, CM3 is based on 4-10 m transmitter-receiver distance NLOS channel measurement, and CM4 is generated to model a 25 nsec root-mean-squared (RMS) delay spread NLOS environments. Their parameters are listed in Table 2.1.
2.2.2 Non-clustered Tapped Delay Line Channel Models

It is verified that clustered channel models can be replaced by their non-clustered equivalents [31], which makes the theoretical analysis tractable. The non-clustered channel model can be separated again into two categories depending on how to model the arrival of each path. One adopts Poisson distributed arrivals like those in (2.7) or (2.8). The other has the tapped delay line (TDL) model, and uses an additional random variable to model the probability of arrival in each time slot. The latter is described here

$$h(t) = \sum_{k=0}^{K-1} p_k \alpha_k \delta_D(t - k\Delta).$$

(2.10)

The multipath delay spread is uniformly upper bounded by $K\Delta$. Any two paths are assumed independent. Channel statistics in the $k^{th}$ time slot $[(k-1)\Delta, k\Delta)$ are described as follows. The polarity of the path $p_k$ is in $\{+1, -1\}$ with equal probability, and the amplitude $\alpha_k$ has a mixture distribution so $\alpha_k$ is equal to zero with probability $1 - a$ and is Rayleigh or lognormal distributed with probability $a$. So in each time slot, the probability to have an arrival path is $a$, and the probability that no path arrives is $1 - a$. Rayleigh path strength model is often assumed in wireless environments, and lognormal path strength is a good fit in some measurement of UWB channels.

The mathematical description of the probability density function of $p_k$ is

$$f(p_k) = \left[ \frac{1}{2} \delta_D(p_k - 1) + \frac{1}{2} \delta_D(p_k + 1) \right].$$

(2.11)
The amplitude $\alpha_k$, which is independent of $p_k$, has probability density function

$$f(\alpha_k) = a \times f(\alpha_k|\text{slot } k \text{ occupied}) + (1 - a)\delta_D(\alpha_k).$$

(2.12)

With the Rayleigh path strength model,

$$f(\alpha_k|\text{slot } k \text{ occupied}) = \frac{\alpha_k}{\sigma_k^2} \exp\left(-\frac{\alpha_k^2}{2\sigma_k^2}\right), \quad \alpha_k > 0,$$

(2.13)

where $2\sigma_k^2 = \Omega_0 \exp\left(-\frac{k\Delta}{\Gamma}\right)$ is the average power of the arrival in $[(k - 1)\Delta, k\Delta)$, $\Omega_0$ is the average power of the arrival in $[0, \Delta)$, and $\Gamma$ is the power decay time constant. With the lognormal path strength model,

$$f(\alpha_k|\text{slot } k \text{ occupied}) = \frac{20}{\ln 10 \sqrt{2\pi\sigma_k^2} \alpha_k} \exp\left\{-\frac{(20 \log \alpha_k - \mu_k)^2}{2\sigma_k^2}\right\}, \quad \alpha_k > 0,$$

(2.14)

where $20 \log \alpha_k \sim \text{Normal}(\mu_k, \sigma^2)$. The function $\log()$ is a 10-based logarithm (common logarithm), $\mu_k$ (decibels) and $\sigma$ (decibels) are the mean and standard deviation of $20 \log \alpha_k$.

In lognormal environments, $\mathbb{E}[\alpha_k^2] = \Omega_0 \exp\left(-\frac{k\Delta}{\Gamma}\right)$, and the mean of $20 \log(\alpha_k)$ can be computed as $\mu_k = \frac{10 \ln \Omega_0 - 10 \ln \Omega_k}{\ln 10} - \frac{\sigma_k^2 \ln 10}{20}$. By defining $p = [p_0, p_1, \ldots, p_{K-1}]$ and $\alpha = [\alpha_0, \alpha_1, \ldots, \alpha_{K-1}]$, the multipath channel is described by the probability density function

$$f(p, \alpha) = \prod_{k=0}^{K-1} f(p_k) f(\alpha_k).$$
Chapter 3

Theoretical Optimal and Suboptimal Single User Receivers for Conventional UWB TR Modulation

This chapter derives optimal and related suboptimal receiver structures using ALRT with either a Rayleigh or a lognormal path strength model, along with a suboptimal receiver without any specific statistical models for the path strength. The GLRT optimal receiver is also derived, and shown as one of the ALRT suboptimal receivers. In these analyses, a simplifying resolvable multipath assumption is employed which is valid in channel environments in which the time difference between every two multipath signal components is greater than a direct-path pulse width. When the transmitted pulse width in a UWB system is less than a nanosecond, this resolvable multipath assumption applies when differential propagation path lengths are always greater than one foot. In reality, because of its short duration, a pulse received over a single propagation path may only overlap in time and correlate with few other multipath component pulses. Therefore, even if the resolvable multipath assumption is not exactly true in some environments, it may still provide a reasonable approximation to real channel models. The nonlinear operations of
these optimal and suboptimal receivers make theoretical BEP evaluation difficult, and the
performance of only two ALRT suboptimal receivers is analyzed in this chapter.

For analytical purposes, the data bit stream is assumed to be composed of independent,
identically distributed binary random variables $b_{[i/N_s]} \in \{1,-1\}$, taking on either value
with probability 1/2. The channel is assumed invariant over one bit time. For simplicity
but without loss of generality, the time-hopping or spreading sequence modulation which
is used to reduce multiuser interference is not considered here because only the single user
case is discussed in Chapter 3 and 4. The superscript $(n)$ which indicates users is also
eliminated. The transmitted signal is now

$$s_{tr}(t) = \sum_{i=\infty}^{i=-\infty} g_{tr}(t - iT_f) + b_{[i/N_s]} g_{tr}(t - iT_f - T_d),$$

(3.1)

and the received signal of bit $b_0$ is

$$r(t) = r_s(t) + n_t(u, t),$$

(3.2)

where $n_t(u, t)$ represents Gaussian receiver noise with two-sided power spectral density

$\frac{N_0}{2}$, and

$$r_s(t) = \sum_{i=0}^{N_s-1} [g(t - iT_f) + b_{[i/N_s]} g(t - iT_f - T_d)].$$

(3.3)

Here a received waveform $g(t)$ is modelled as the output of a non-clustered TDL channel

$$g(t) = \sum_{k=0}^{K-1} p_k \alpha_k g_{tx}(t - k\Delta).$$

(3.4)
The received monocycle waveform $g_{rx}(t)$ of a single multipath component, normalized to unit energy, i.e., $\int_{-\infty}^{\infty} g_{rx}^2(t)dt = 1$, will differ in shape from the transmitted waveform, and its shape may vary for different multipath components [14, 17, 34]. In the design and analysis of ALRT and GLRT receivers, we assume that $g_{rx}(t)$ is known and the same for all multipath components, and can be used as a template in a correlator. The average energy in the $k^{th}$ path component is

$$2N_0\mathbb{E}\{\alpha_k^2\} = 2N_s\alpha\mathbb{E}\{\alpha_k^2|\text{slot } k \text{ occupied}\}$$

due to the normalization of $g_{rx}(t)$, and the average path signal-energy-to-noise-power-density-ratio (ASNR) as well as the realized path signal-energy-to-noise-power-density-ratio (RSNR) in the $k^{th}$ path component are defined as $\text{ASNR}_k = \frac{2N_0\mathbb{E}\{\alpha_k^2\}}{N_0}$ and $\text{RSNR}_k = \frac{2N_s\alpha^2}{N_0}$.

### 3.1 ALRT Optimal Receivers

We now detect the bit $b_0$ based on the observation $\tilde{r}$ of $r(t)$, $t \in (0, N_sT_f)$. Using the average likelihood ratio test which minimizes the average bit error probability, the decision rule is of the form

$$\frac{p(\tilde{r}|b_0 = 1)}{p(\tilde{r}|b_0 = -1)} \overset{1}{\gtrless} \frac{1}{-1},$$

(3.5)

and $p(\tilde{r}|b_0 = 1)$ and $p(\tilde{r}|b_0 = -1)$ are averages of

$$p(\tilde{r}|b_0 = \pm 1, p, \alpha) \equiv \exp \left\{ -\frac{1}{N_0} \int_0^{N_sT_f} [r(t) - r_s(t)]^2 dt \right\}$$

(3.6)
over $\alpha$ and $p$ which along with $b_0$ are imbedded in $r_s(t)$. The equivalence ($\equiv$) indicates that irrelevant constants have been dropped.

By applying the resolvable multipath assumption to eliminate the cross-correlation of any two pulses in (3.6) in the averaging process, and taking natural logarithm in both sides of (3.5), the ALRT rule can be reduced to

$$\sum_{k=0}^{K-1} L_k(C(k)) \underset{\geq}{\geq} \frac{1}{\sum_{k=0}^{K-1} L_k(D(k))}, \quad (3.7)$$

in which the log-likelihood function $L_k(x)$ is defined as

$$L_k(x) = \ln \left[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\alpha_k) f(p_k) \exp \left( p_k \alpha_k x - \frac{2 N_s}{N_0} \alpha_k^2 \right) d\alpha_k dp_k \right], \quad (3.8)$$

and the quantities $C(k)$ and $D(k)$ are

$$C(k) \triangleq \frac{2}{N_0} [C_R(k) + C_D(k)],$$

$$D(k) \triangleq \frac{2}{N_0} [C_R(k) - C_D(k)],$$

where

$$C_R(k) \triangleq \sum_{i=0}^{N_s-1} \int_0^{N_s T_i} r(t) g_{tx}(t - iT_i - k\Delta) dt, \quad (3.9)$$

$$C_D(k) \triangleq \sum_{i=0}^{N_s-1} \int_0^{N_s T_i} r(t) g_{tx}(t - iT_i - T_d - k\Delta) dt. \quad (3.10)$$

In (3.7), the received signal information is embedded in $C(k)$ and $D(k)$ for $k = 0, 1, \ldots, K - 1$. The optimal receiver structure always has signal processors with $C(k)$
and $D(k)$ as inputs no matter what the distributions of $p$ and $\alpha$ are, and is shown in Figure 3.1.

Figure 3.1: The block diagram of the ALRT optimal receiver in which $X = \sum_{k=0}^{K-1} L_k(C(k))$ and $Y = \sum_{k=0}^{K-1} L_k(D(k))$. 

\[ \hat{b}_0 \text{ if } X > Y \]
3.1.1 Rayleigh Path Strength Models

By using (2.11), (2.12), and (2.13), the log-likelihood function in (3.8) in the Rayleigh case is computed in Appendix A.1. This computation contains the positive parameter

\[ w_R(k) \triangleq \frac{\sigma_k^2}{1 + SNR_k}, \]  

(3.11)

where \( SNR_k = \frac{4N_s\sigma_k^2}{N_0} \) is twice the ratio of the average energy in the occupied \( k^{th} \) time slot to the noise power density. By eliminating those terms in (A.5) which are same for both the \( b_0 = 1 \) and \( b_0 = -1 \) cases and have no effect on the decision, the log-likelihood function becomes

\[
L_k(x) = \frac{w_R(k)x^2}{2} + \ln \left\{ \exp \left( -\frac{w_R(k)x^2}{2} \right) + \sqrt{\pi w_R(k)x^2} \left( 1 - 2Q\left( \sqrt{w_R(k)x^2} \right) \right) + \left( \frac{1-a}{a} \right) (1 + SNR_k) \exp \left( -\frac{w_R(k)x^2}{2} \right) \right\},
\]  

(3.12)

where \( Q(x) = \int_x^\infty (2\pi)^{-\frac{1}{2}} e^{-z^2/2} dz \) is a Gaussian Q-function. This function is monotone increasing in \( x \).
3.1.2 Lognormal Path Strength Models

For the lognormal path strength model, (2.12), (2.14), (3.8), and change variables by letting \( y_k = 20 \log \alpha_k \) and \( \beta_k = \frac{y_k - \mu_k}{\sqrt{2}\sigma^2} \) are applied. The nuisance parameter \( \alpha_k \) can be eliminated to give

\[
L_k(x) = \ln \left\{ \int_{-\infty}^{\infty} f(p_k) \left[ (1 - a) + a \int_{-\infty}^{\infty} \exp(-\beta_k^2) \right. \right. \\
\times \exp \left( x p_k 10^{\frac{\sqrt{2}\sigma \beta_k + \mu_k}{20}} - \frac{2N_s}{N_0} \cdot 10^{\frac{2(\sqrt{2}\sigma \beta_k + \mu_k)}{20}} \right) \left. d\beta \right) dp_k \right\}.
\]

There is no closed-form simplification when a lognormal distribution is involved in an integral. But we can use the Hermite-Gauss integral [37] to simplify the inner integral in (3.13). The Hermite-Gauss integral is (see [36], equation 25.4.46 and Table 25.10)

\[
\int_{-\infty}^{\infty} e^{-x^2} f(x) dx = \sum_{i=1}^{N} \omega_i f(x_i) + R_N,
\]

where \( x_i \) is the \( i \)th zero of \( H_N(x) \), a Hermite polynomial. The weights \( \omega_i \) and remainder \( R_N \) are defined as

\[
\omega_i \triangleq \frac{2^{n-1} N! \sqrt{\pi}}{N^2 |H_{N-1}(x_i)|^2},
\]

\[
R_N \triangleq \frac{N! \sqrt{\pi}}{2^n (2N)!} f^{(2N)}(\xi), \quad -\infty < \xi < \infty.
\]

If the value of \( N \) in (3.14) is even, we will have \( N/2 \) different positive numbers and their negatives as \( \{x_i\}_{i=1}^{N} \). If \( N \) is odd, we will have the same situation as in the even-\( N \) case for
\( \{x_i\}_{i=1}^{N-1} \), and \( x_N = 0 \). The smaller the absolute value of \( x_i \) is, the larger the corresponding weight \( w_i \) is. Usually \( R_N \) becomes very small when \( N \geq 20 \), and can be ignored.

By applying (3.14), (2.11), and the definitions

\[
\begin{align*}
    w_L(k) & \triangleq 10^{\frac{\mu_k}{20}}, \\
    h(i) & \triangleq 10^{\frac{\sqrt{2} \sigma x_i}{20}},
\end{align*}
\]

(3.13) is simplified further

\[
L_k(x) = \ln \left\{ \int_{-\infty}^{\infty} \left[ \sum_{i=1}^{N} \omega_i \exp \left( 10^{\frac{\sqrt{2} \sigma x_i + \mu_k}{20}} \cdot x p_k - \frac{2 N_s}{N_0} 10^{\frac{2(\sqrt{2} \sigma x_i + \mu_k)}{20}} \right) \right] f(p_k) dp_k \right\}.
\]

Discarding the remainder \( R_N \) and constant \( \ln(a) \) in (3.15), the log-likelihood function with lognormal path strength is

\[
L_k(x) = \ln \left\{ \sum_{i=1}^{N} \omega_i \exp \left[ -\frac{2 N_s}{N_0} h^2(i) w_L^2(k) \right] \cosh[h(i) w_L(k) x] + 1 - a + a R_N \right\}. \quad (3.16)
\]

This function is monotone increasing in \( x \). The effect of eliminating \( R_N \) on the BEP performance is tested for different values of \( N \) in Section 4.5 by simulation.
3.2 ALRT Suboptimal Receivers

In the beginning of this section, a delta function approximation is applied in a general sense without any specific statistical models of path amplitudes to obtain a suboptimal receiver structure. The suboptimal receivers, which approximate the ALRT optimal receivers with Rayleigh and lognormal path strength models, will be derived in next two subsections.

Suboptimal receivers use simple structures to approximate the log-likelihood function $L_k(\cdot)$ in (3.8). The integration over $p_k$ can be carried out first due to the equal probability of the positive and negative which results in

$$L_k(x) = \ln \left\{ \frac{1}{2} \int_0^\infty f(\alpha_k) \left[ \exp \left( \alpha_k x - \frac{2 N_s}{N_0} \alpha_k^2 \right) + \exp \left( -\alpha_k x - \frac{2 N_s}{N_0} \alpha_k^2 \right) \right] d\alpha_k \right\}$$

$$= \ln \left\{ \frac{k'(x)}{2} \int_0^\infty f(\varrho_k) \left[ N_{\varrho_k}(m'_+(x), \rho'_k^2) + N_{\varrho_k}(m'_-(x), \rho'_k^2) \right] d\varrho_k \right\}$$  \hspace{1cm} (3.17)

where $\alpha_k = \rho_k \varrho_k$ with $\varrho_k$ being a unit second moment random variable and $\rho_k^2$ being the possibly unknown second moment of $\alpha_k$, $f(\varrho_k)$ is the density function of $\varrho_k$ which might include a dirac delta function in it and can be written as $f(\varrho_k) = f'(\varrho_k) + \varpi \delta_D(\varrho_k - \psi)$ with $0 \leq \varpi \leq 1$, $N_{\varrho_k}(\cdot)$ represents a normal density function in the variable $\varrho_k$ with mean $m'_\pm(x)$ or $m'_-(x)$ and variance $\rho'_k^2$, and

$$k'(x) = \sqrt{\frac{\pi N_0}{2 N_s \rho_k^2}} \exp \left\{ \frac{x^2 N_0}{8 N_s} \right\},$$

$$m'_\pm(x) = \pm \frac{x N_0}{4 N_s \rho_k^2},$$

$$\rho'_k^2 = \frac{N_0}{4 N_s \rho_k^2},$$
From the definition in the beginning of Chapter 3, \( \text{ASNR}_k = \frac{2N_s\rho_k^2}{N_0} \). If the \( \text{ASNR}_k \) is high enough, \( N_{\theta_k}(m'_{\pm}(x), \rho'_k) \) in (3.17) with a narrow shape behaves like a delta function. If the \( \text{ASNR}_k \) is low, \( f(\theta_k) \) with a relatively narrow shape behaves like a delta function at the mean value of \( \theta_k \). Therefore, the integral in (3.17) is approximated asymptotically by

\[
 k'(x) \int_0^\infty N_{\theta_k}(m'_{\pm}(x), \rho'_k) f(\theta_k) d\theta_k \\
 \approx \begin{cases} 
 k'(x) \cdot [f'(m'_{\pm}(x)) + \varpi N_\psi(m'_{\pm}(x), \rho'_k)] & \text{high } \text{ASNR}_k \\
 k'(x) \cdot N_{\xi(\theta_k)}(m'_{\pm}(x), \rho'_k^2) & \text{low } \text{ASNR}_k.
\end{cases}
\]

(3.18)

Note that either \( f'(m'_+(x)) \) or \( f'(m'_-(x)) \) equals zero because \( \theta_k \geq 0 \). In addition,

\[
f'(|m'_+(x)|) \leq \max_{\theta_k} N_{\theta_k}(|m'_+(x)|, \rho'_k) = \sqrt{2N_s\rho_k^2/\pi N_0}
\]

is true for any \( |m'_+(x)| \) when \( \text{ASNR}_k \) is high because of the delta function behavior of \( N_{\theta_k}(m'_{\pm}(x), \rho'_k) \), and \( \varpi N_\psi(m'_{\pm}(x), \rho'_k^2) \leq \varpi \max_\psi N_\psi(m'_\pm(x), \rho'_k) = \varpi \sqrt{2N_s\rho_k^2/\pi N_0} \).

For the large \( \text{ASNR}_k \) situation, either \( \frac{C_2(k)N_0}{N_s} \) or \( \frac{D_2(k)N_0}{N_s} \) is large, and the other is close to zero, depending on the transmitted bit. Suppose \( \frac{C_2(k)N_0}{N_s} \) is the large one whose mean is \( 4 + \frac{16N_s\rho_k^2}{N_0} \), then

\[
k'(C(k)) \gg f'(|m'_\pm(C(k))|) + \varpi N_\psi(m'_\pm(C(k)), \rho'_k^2),
\]
in which \( C(k) \) is substituted for \( x \). By applying the above inequality to the high \( \text{ASNR}_k \) case in (3.18) and (3.17), the suboptimal rule approximating (3.7) is

\[
\sum_{k=0}^{K-1} \ln\{k'(C(k))\} \gtrapprox \sum_{k=0}^{K-1} \ln\{k'(D(k))\},
\]

which is equivalent to

\[
\sum_{k=0}^{K-1} C_R(k) C_D(k) \gtrapprox 0.
\] (3.19)

For the low \( \text{ASNR}_k \) case, by substituting (3.18) into (3.17) and eliminating terms which are not related to \( x \) and have no effect on the decision, the suboptimal rule approximating (3.7) is

\[
\sum_{k=0}^{K-1} \ln\{\cosh(\mathbb{E}\{\alpha_k\} C(k))\} \gtrapprox \sum_{k=0}^{K-1} \ln\{\cosh(\mathbb{E}\{\alpha_k\} D(k))\}. \quad (3.20)
\]

Both \( \ln(\cdot) \) and \( \cosh(\cdot) \) are nonlinear strictly monotonic increasing functions. Because \( \cosh(\cdot) \) is even, the function \( \ln(\cosh(\cdot)) \) is also even, and

\[
\ln\{\cosh(y)\} = \ln \left( \frac{e^y + e^{-y}}{2} \right) = \ln\{e^{|y|} + e^{-|y|}\} - \ln 2 \\
\approx \ln\{e^{|y|}\} - \ln 2 = |y| - \ln 2. \quad (3.21)
\]

Equation (3.20) is now be simplified by applying (3.21)

\[
\sum_{k=0}^{K-1} \mathbb{E}\{\alpha_k\} |C_R(k)| + C_D(k) \gtrapprox \sum_{k=0}^{K-1} \mathbb{E}\{\alpha_k\} |C_R(k) - C_D(k)|, \quad (3.22)
\]
where \(|C_R(k) + C_D(k)|\) and \(|C_R(k) - C_D(k)|\) are received information of bit \(b_0\) which are weighted by the mean value of the amplitude of arrivals. This suboptimal decision rule can be simplified further to

\[
\sum_{k=0}^{K-1} |C_R(k)) + C_D(k)| \geq \frac{1}{2} \sum_{k=0}^{K-1} |C_R(k) - C_D(k)|
\]

(3.23)

because multiplying the prior information \(\mathbb{E}\{\alpha_k\}\) at low ASNR does not improve the bit error probability much which will be seen in Section 4.5.

For the performance analysis, a closed-form evaluation of the suboptimal receiver in (3.19) can be obtained by using Appendix 9A in [24]. With the assumption that \(K\) is even, and by defining \(A = B = 0, C = 1/2, L = K/2, X_n = C_R(n) + jC_R(n + K/2), Y_n = C_D(n) + jC_D(n + K/2)\) for \(n = 0, 1, \ldots, L - 1\), the left-hand side of (3.19), which is equal to \(\frac{1}{2} \sum_{n=0}^{L-1} (X_n^H Y_n + Y_n^H X_n)\), can be represented in the same form as (9A.1) in [24]. The means of \(C_R(k)\) and \(C_D(k)\) are \(N_s p_k \alpha_k\) and \(b_0 N_s p_k \alpha_k\), their variances are both \(N_s N_0\) which are independent of \(k\), and any two of \(\{C_R(k), C_D(k)\}\) are uncorrelated which can be computed by using (3.9) and (3.10). Assuming the transmitted bit \(b_0 = 1\), the values of parameters \(a\) and \(b\) defined in [24] are computed as \(a = 0\) and \(b = \sqrt{\frac{2N_s \sum_{k=0}^{K-1} \alpha_k^2}{N_0}} = \sqrt{\frac{E_b}{N_0}}\)

where \(E_b\), the realized bit energy, is a random variable. The BEP conditioned on \(\frac{E_b}{N_0}\) is acquired by substituting \(a\) and \(b\) into (9A.15) in [24]

\[
P_{\text{bit}}(\alpha_0, \ldots, \alpha_{K-1}) = \frac{1}{2} + \frac{1}{2^{K-1}} \sum_{l=1}^{K/2} \left( \frac{K}{2} - l \right) \left[ Q_l \left( 0, \sqrt{\frac{E_b}{N_0}} \right) - Q_l \left( \sqrt{\frac{E_b}{N_0}}, 0 \right) \right]
\]

(3.24)
where $Q_l(\zeta, \varsigma)$ is a Marcum $Q$-function [24]. Because $Q_l(b, 0) = 1$ for all $l, b,$ and

$$Q_l(0, b) = \sum_{n=0}^{l-1} \exp \left(-\frac{b^2}{2}\right) \frac{(b^2/2)^n}{n!}$$

(3.25)

if $l$ is an integer, (3.24) can be simplified further to

$$P_{\text{bit}}(\alpha_0, \ldots, \alpha_{K-1}) = \frac{1}{2^{K-1}} \sum_{l=1}^{K/2} \left(\frac{K-1}{K/2-l}\right) \sum_{n=0}^{l-1} \frac{1}{n!} \exp \left(-\frac{E_b}{2N_0}\right) \left(\frac{E_b}{2N_0}\right)^n.$$  

(3.26)

In Section 4.5, Figure 4.4 and 4.6-4.9 show the fit of these analytical results and simulations.

The closed-form solution of the average BEP can be acquired if $E\left\{\exp \left(-\frac{E_b}{2N_0}\right) \left(\frac{E_b}{2N_0}\right)^n\right\}$ can be calculated, i.e., the moment generating function of $\frac{E_b}{2N_0}$ exists.

### 3.2.1 Rayleigh Path Strength Models

The log-likelihood function in (3.12) is a function of $w_R(k)x^2,$ and $x$ could be $C(k)$ or $D(k).$ Using $x = C(k)$ as an example, without the receiver noise, $C(k)$ is equal to zero when $b_0 = -1.$ Therefore, with moderate or high RSNR$k = 2N_s\alpha^2_k/N_0,$ $C(k)$ and also $w_R(k)C^2(k)$ should be close to zero for $b_0 = -1,$ and should have significant positive values for $b_0 = 1.$ When $a \approx 1$ and the value of $w_R(k)C^2(k)$ is large which indicates both large RSNR$k$ and $b_0 = 1,$ $L_k(C(k))$ can be approximated by

$$L_k(C(k)) \approx \frac{w_R(k)C^2(k)}{2}$$

(3.27)

(see Appendix A.2). This approximation is also verified by Figure 3.2 with $a = 1$ which shows that $L_k(C(k))$ is close to $w_R(k)C^2(k)/2$ when $w_R(k)C^2(k)$ is large. For small RSNR$k$
or $b_0 = -1$, this approximation is not so precise. It is worth noting that when the value of $a$ decreases, the probability that $\text{RSNR}_k$ is small even with high $\text{ASNR}_k$ and $b_0 = 1$ increases. Therefore, high $\text{ASNR}_k$ (or $\text{SNR}_k$) with small $a$ can make the approximation in (3.27) deviate more from $L_k(C(k))$ because of the term $(\frac{1-a}{a}) (1 + \text{SNR}_k) \exp \left(-\frac{w_R(k)C^2(k)}{2}\right)$, and this is shown in Figure 3.2. Similar arguments and conclusions apply to (3.12) for $x = D(k)$.

By using the approximation in (3.27), the logarithm term in (3.12) is abandoned, and the suboptimal receiver is

$$
\sum_{k=0}^{K-1} w_R(k) C_R(k) C_D(k) \geq 0
$$

(3.28)

by expanding $C^2(k)$ and $D^2(k)$, and eliminating common terms on both sides. For a large $\text{SNR}_k$, approximation for $w_R(k)$ also exists

$$
w_R(k) = \frac{\sigma_k^2}{1 + \text{SNR}_k} \approx \frac{\sigma_k^2}{\text{SNR}_k} \approx \frac{N_0}{4N_s},
$$

which is a constant and does not have any effect. Then the decision rule in (3.28) is further reduced to

$$
\sum_{k=0}^{K-1} C_R(k) C_D(k) \geq 0
$$

(3.29)

which is the same as (3.19).

The transmitted bit is the phase difference of the reference pulse and data pulse. Without receiver noise, the polarity of $C_R(k)C_D(k)$ for all $k$ should be the same and is the transmitted bit. It is possible that the polarity of any $C_R(k)C_D(k)$ could differ from the transmitted bit because of the receiver noise, but the error probability decreases as
Figure 3.2: $\frac{w_R(k)x^2}{2}$ versus $L_k(x)$ for $k = 0, 1, \ldots, K - 1$.

The ASNR$_k$ and RSNR$_k$ increase. Therefore, we should give $C_R(k)C_D(k)$ different weights for different $k$ according to both the a priori information and received information in the $k^{th}$ path. For the weighted suboptimal detection in (3.28), we discard the information of RSNR and a part of ASNR, which is included in the logarithm term in (3.12), in order to reduce the receiver complexity. And from Figure 3.2, this should have a small effect upon the BEP performance for $a$ close to 1, or moderate and small $a$ along with small ASNR.

The weights for $\{C_R(k)C_D(k)\}_{k=0}^{K-1}$, which are $\{w_R(k)\}_{k=0}^{K-1}$, can be generated according to the a priori knowledge in advance, and stored in the receiver. Since $w_R(k)$ is an increasing function in $\sigma_k^2$, $C_R(k)C_D(k)$ gets a large weight if the average energy of an arrival in the $k^{th}$ path is large.
In (3.29), no a priori information is used, and the receiver makes decision only according to the received information. It will be seen that (3.29) is the same as the optimal decision rule based on the GLRT in Section 3.3. The block diagram of the suboptimal receiver in (3.28) is shown in Figure 3.3, and the structure described in (3.29) is also drawn in the same figure except the multiplications by weights \( \{w_R(k)\}_{k=0}^{K-1} \) in the dotted box are removed.

The BEP of using the suboptimal decision rule in (3.28) conditioned on a channel realization can be computed by utilizing the result in [38]. Define

\[
X_k = X_{Rk} + jX_{Ik} \triangleq \sqrt{w_R(2k-2)C_R(2k-2)} + j\sqrt{w_R(2k-1)C_R(2k-1)},
\]

\[
Y_k = Y_{Rk} + jY_{Ik} \triangleq \sqrt{w_R(2k-2)C_D(2k-2)} + j\sqrt{w_R(2k-1)C_D(2k-1)},
\]

for \( k = 1, 2, \ldots, \frac{K}{2} \) assuming \( K \) is even which can be easily produced by adding an additional path with zero magnitude if necessary. The variance and covariance of \( X_{Rk}, X_{Ik}, Y_{Rk}, \) and \( Y_{Ik} \) can therefore be calculated which satisfy (1) and (2) in [38] except that

\[
\text{Var}\{X_{Rk}\} \neq \text{Var}\{X_{Ik}\}
\]

\[
\text{Var}\{Y_{Rk}\} \neq \text{Var}\{Y_{Ik}\},
\]

where \( \text{Var}\{X_{Rk}\} = \text{Var}\{Y_{Rk}\} = \frac{1}{2}N_sN_0w_R(2k-2) \) and \( \text{Var}\{X_{Ik}\} = \text{Var}\{Y_{Ik}\} = \frac{1}{2}N_sN_0 \times w_R(2k-1). \) Because of the dense UWB environment which makes \( w_R(2k-2) \approx w_R(2k-1), \) the relationships in (1) and (2) in [38] are still approximately true. The effect of this
approximation on evaluating the suboptimal receiver structure (3.13) is little which will be seen in Section 4.5. Let \( \mathbf{v} = [X_1, Y_1, X_2, Y_2, \ldots, X_{K/2}, Y_{K/2}]^t \), and

\[
\mathbf{Q} = \frac{1}{2} \begin{bmatrix}
H & 0 & \cdots & 0 \\
0 & H & 0 & \cdots \\
\vdots & \ddots & \ddots & 0 \\
0 & \cdots & 0 & H
\end{bmatrix}
\]

where

\[
\mathbf{H} = \begin{bmatrix}
0 & 1 \\
1 & 0
\end{bmatrix},
\]

then

\[
\Theta = \mathbf{v}^h \mathbf{Q} \mathbf{v} \sim \frac{1}{-1} \mathbf{0}
\]

is equal to (3.28) with \( \mathbf{v}^h \) representing the complex conjugate transpose of \( \mathbf{v} \).

Let \( \bar{\mathbf{v}} \) and \( \mathbf{L} \) denote the conditional mean vector and covariance matrix of \( \mathbf{v} \), then

\[
\bar{\mathbf{v}} = N_s \begin{bmatrix}
\sqrt{w_R(0)p_0\alpha_0} + j\sqrt{w_R(1)p_1\alpha_1} \\
b\sqrt{w_R(0)p_0\alpha_0} + jb\sqrt{w_R(1)p_1\alpha_1} \\
\vdots \\
\sqrt{w_R(K-2)p_{K-2}\alpha_{K-2}} + j\sqrt{w_R(K-1)p_{K-1}\alpha_{K-1}} \\
b\sqrt{w_R(K-2)p_{K-2}\alpha_{K-2}} + jb\sqrt{w_R(K-1)p_{K-1}\alpha_{K-1}}
\end{bmatrix},
\]
and

\[
L = \mathbb{E}\{(v - \bar{v})(v - \bar{v})^h\} = \frac{N_s N_0}{2} \begin{bmatrix}
L_1 & 0 & \cdots & 0 \\
0 & L_2 & 0 & \cdots \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & L_K
\end{bmatrix},
\]

where

\[
L_k = \begin{bmatrix}
w_R(2k - 2) + w_R(2k - 1) & 0 \\
0 & w_R(2k - 2) + w_R(2k - 1)
\end{bmatrix}.
\]

Assuming \( b_0 = 1 \), the characteristic function of \( y \) is given by

\[
\phi(jt) = |I - jtlQ|^{-1} \exp\{-\bar{v}^hL^{-1}[I - (I - jtlQ)^{-1}]\bar{v}\},
\]

where \( I \) is the identity matrix. The quadratic form in the exponent can be diagonalized by use the transformation \( \bar{v} = U_1M^{-1}U_2d \), where \( U_1 \) is the normalized modal matrix of \( L^{-1} \), \( M^2 = U_1^hL^{-1}U_1 \), and \( U_2 \) is the normalized modal matrix of \( M^{-1}U_1^hQU_1M^{-1} \). Then

\[
\phi(jt|b_0 = 1) = \prod_{k=1}^{K} \frac{\exp\{j\lambda_k |d_k|^2\}}{1 - j\lambda_k},
\]

(3.30)

where \( \lambda_k \) are the eigenvalues of \( LQ \), and

\[
d_k = \begin{cases}
\sqrt{\frac{4N_s N_0}{N_0}} \sqrt{w(k-1)p_{k-1}\alpha_{k-1} + j\sqrt{w(k)p_k\alpha_k}} & k \text{ is odd} \\
\sqrt{w(k-1) + w(k)} & k \text{ is even}
\end{cases}
\]
are computed using the methods in [38]. By defining \( \xi_m = \frac{N_kN_0}{4}[w_R(2m - 2) + w_R(2m - 1)] \geq 0 \) for \( m = 1, 2, \ldots, \frac{K}{2} \), the eigenvalues are

\[
\lambda_l = \xi_{l+\frac{1}{2}}, \quad \lambda_{l+1} = -\xi_{l+\frac{1}{2}}
\]

for \( l = 1, 3, 5, \ldots, K - 1 \). The characteristic function of \( \Theta \) by substituting \( \lambda_k \) and \( d_k \) into (3.30) and setting \( z = jt \) is

\[
\phi(z|b_0 = 1) = \left[ \prod_{k=1}^{K/2} \frac{-1}{\xi_k} \exp\left(\frac{z_0|d_{2k-1}|^2}{1-z_0^2}\right) \right] \left[ \prod_{k=1}^{K/2} \frac{1}{\xi_k} z + \frac{1}{\xi_k} \right].
\]

Knowing the characteristic function \( \phi(z|b_0 = 1) \), the probability density function of \( \Theta \) given \( b_0 = 1 \) is

\[
f(\theta|b_0 = 1) = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} G(z) dz,
\]

where

\[
G(z) = \exp(-z\theta)\phi(z|b_0 = 1)
\]

\[
= \exp(-z\theta) \times \left[ \prod_{k=1}^{K/2} \frac{-1}{\xi_k} \exp\left(\frac{z_0|d_{2k-1}|^2}{1-z_0^2}\right) \right] .
\]

By defining \( z_k = \frac{1}{\xi_k} \) for \( k = 1, 2, \ldots, \frac{K}{2} \), the poles of \( G(z) \) are \( \pm z_k \) which are all simple poles. The bit error probability is

\[
P_b = \int_{-\infty}^{0} f(\theta|b_0 = 1) d\theta = \sum_{k=1}^{K/2} \int_{-\infty}^{0} \text{Res } G(z) d\theta
\]
due to the symmetry of the transmitted bit and receiver noise, as well as the claim in Appendix A.3. The residue of $G(z)$ at $-z_k$ is

$$
\text{Res}_{z=-z_k} G(z) = G(z)(z + z_k)|_{z=-z_k}.
$$

(3.33)

And the BEP conditioned on one channel realization by substituting (3.33) into (3.32) is

$$
P_b = \frac{1}{2} \sum_{k=1}^{K/2} \exp\left\{-\frac{1}{2} |d_{2k-1}|^2 \right\} \frac{K/2}{\prod_{n=1 \atop n \neq k}^{K/2} \exp\left\{\frac{-\xi_n|d_{2n-1}|^2}{(\xi_k + \xi_n)^2}\right\}} \cdot \left(1 - \left(\frac{\xi_n}{\xi_k}\right)^2\right),
$$

(3.34)

where $d_l$ are functions of $\alpha_0, \alpha_1, \ldots, \alpha_{K-1}$.

### 3.2.2 Lognormal Path Strength Models

Suppose that $N = 1$ in the Hermite-Gauss approximation (3.14) is adopted for which $x_1 = 0$, $h(1) = 1$, and $w_1 = 1$. Then the log-likelihood function in (3.16) is reduced and equivalent to

$$
L_k(x) \equiv \ln \left\{ \exp \left( -\frac{2N_s}{N_0} w_L^2(k) \right) \cosh(w_L(k)x) + \frac{1-a}{a} \right\},
$$

(3.35)

where $w_L(k)$ is the prior information, i.e., the mean value of the amplitude of an arrival in the $k^{\text{th}}$ path component, and $x = C(k)$ or $x = D(k)$ is the received information of the transmitted bit and the channel. From the definition of $w_L(k)$, $C(k)$ and $D(k)$, it can be shown that without receiver noise, $w_L(k)C(k)$ is proportional to the ratio of the energy of the $k^{\text{th}}$ multipath signal component to the noise power spectral density when $b_0 = 1$, and it should be equal to zero when $b_0 = -1$. A similar argument can be applied to $w_L(k)D(k)$.
The second term in the braces in (3.35) is equal or close to zero when \( a \) is equal or close to one, then

\[
L_k(x) \cong -\frac{2N_s}{N_0} w_L^2(k) + \ln \{ \cosh(w_L(k)x) \}
\]

\[
\cong \ln \{ \cosh(w_L(k)x) \} .
\]  
(3.36)

For those cases with small \( a \), not only \( \frac{1-a}{a} \) is large, the probability of \( \cosh(w_L(k)x) \) in (3.35) being small for a specific ASNR\( _k \) is also large. So the approximation in (3.36) could deviate from (3.35). By assuming a dense environment (\( a \) is close to 1) and adopting the approximation in (3.21), a suboptimal decision rule which is simplified from (3.35) is

\[
\sum_{k=0}^{K-1} w_L(k)|C_R(k) + C_D(k)| \geq \sum_{k=0}^{K-1} w_L(k)|C_R(k) - C_D(k)|
\]  
(3.37)

which is the same as (3.22).

If the weights in (3.37) are eliminated to simplify the receiver structure further, the second suboptimal receiver structure becomes

\[
\sum_{k=0}^{K-1} |C_R(k) + C_D(k)| \geq \sum_{k=0}^{K-1} |C_R(k) - C_D(k)|
\]  
(3.38)

which uses the received information only to make a decision, and is the same as (3.23). The block diagram of the suboptimal receiver in (3.37) is shown in Figure 3.4, and the structure described in (3.38) is also drawn in the same figure, except that the multiplications of weights \( \{ w_L(k) \}_{k=0}^{K-1} \) in the dotted box are removed. If there is only one path, i.e., \( K = 1 \), the decision regions of rules (3.7), (3.28), (3.29), (3.38) and (3.37) are all the same. Let
the horizontal axis represent \( C_R(0) \) and the vertical axis represent \( C_D(0) \), we say that the transmitted bit is equal to 1 if the value of \((C_R(0), C_D(0))\) falls in quadrant one and three, and the transmitted bit is equal to -1 otherwise.

### 3.3 GLRT Optimal Receiver

Another method to derive a receiver structure is to use the generalized likelihood ratio test. Instead of averaging out the nuisance parameters, GLRT receiver estimates two sets of channel parameters which maximize the likelihood functions under two hypotheses, i.e., \( b_0 = 1 \) and \( b_0 = -1 \), respectively, and substitutes them into the likelihood functions to make a decision by choosing the larger one. The decision rule of a GLRT receiver is of the form

\[
\frac{\max_{\beta} p(\tilde{r}|b_0 = 1, \beta)}{\max_{\beta} p(\tilde{r}|b_0 = -1, \beta)} \geq 1
\]

where

\[
\beta \triangleq [\beta_0, \beta_1, ..., \beta_{K-1}]^t = [p_0 \alpha_0, p_1 \alpha_1, ..., p_{K-1} \alpha_{K-1}]^t,
\]

and

\[
p(\tilde{r}|b_0 = 1, \beta) = \prod_{k=0}^{K-1} \exp\{\beta_k C(k) - \frac{2N_s}{N_0} \beta_k^2\} = \exp\left\{ \frac{2}{N_0} (\beta^t X_1 - N_s \beta^t \beta) \right\},
\]

in which \( X_1 = [C_R(0) + C_D(0), C_R(1) + C_D(1), ..., C_R(K-1) + C_D(K-1)]^t \). It can be observed immediately from (3.40) that

\[
\arg\max_{\beta} p(\tilde{r}|b_0 = 1, \beta) = \arg\min_{\beta} N_s \beta^t \beta - \beta^t X_1.
\]
The choice for $\beta$ which minimizes $N_s\beta'\beta - \beta'X_1$ in (3.41) is [25]

$$\hat{\beta}^{(1)} = \frac{X_1}{2N_s}.$$  

(3.42)

The same computation can be applied to estimate $\beta^{(-1)}$ which minimizes

$$p(\tilde{r}|b_0 = -1, \beta) \equiv \exp\left\{\frac{2}{N_0}(\beta'X_2 - N_s\beta')\right\}$$  

(3.43)

by letting $\beta = \beta^{(-1)}$, and

$$\hat{\beta}^{(-1)} = \frac{X_2}{2N_s}$$  

(3.44)

where $X_2 = [C_R(0) - C_D(0), C_R(1) - C_D(1), \ldots, C_R(K - 1) - C_D(K - 1)]^t$. The GLRT decision rule can be reduced to

$$\sum_{k=0}^{K-1} C_R(k)C_D(k) \begin{cases} 1 & \geq 0 \\ -1 & < 0 \end{cases}$$  

(3.45)

by substituting $\beta^{(1)}$ and $\beta^{(-1)}$ into (3.39), (3.40) and (3.43), taking natural logarithm $\ln(\cdot)$ on both sides of (3.39), expanding $X_1^tX_1$ and $X_2^tX_2$, and eliminating the common terms.

It is observed that (3.45) is the same as (3.19) and (3.29).
Figure 3.3: The block diagram of ALRT suboptimal receivers described in (3.28) and (3.29) with Rayleigh path strength models.
Figure 3.4: The block diagram of ALRT suboptimal receivers described in (3.37) and (3.38) with lognormal path strength models.
Chapter 4

Cross-correlation Receivers for Conventional UWB TR Modulation

Although those decision rules derived in Chapter 3 have different optimal and suboptimal criteria, they all need a correlation operation for each multipath component to generate sufficient statistics \( \{C_R(k)\}_{k=0}^{K-1} \) and \( \{C_D(k)\}_{k=0}^{K-1} \) for detection. The more paths we have, the more correlation operations we need before the ADC. Because of the fine-time resolution capability a UWB signal has, it can resolve many multipath components, i.e., the number \( K \) is large. Therefore, the main reason that a transmitted reference method was proposed to use with a UWB system is to simplify receiver structures instead of achieving optimal performance. The concept was to improve the performance with limited receiver complexity.

4.1 Conventional Cross-correlation Receivers

An ad hoc conventional cross-correlation receiver has been discussed widely with a UWB TR system [6, 13, 25]. This receiver correlates the received data-modulated waveform with
Figure 4.1: A conventional cross-correlation receiver.

the reference waveform, which is received $T_d$ seconds earlier, to capture all the energy in the received signal, and sums the $N_s$ correlator outputs that are affected by a single data bit to be the decision statistic. Since the time separation of the reference and data pulses are fixed to $T_d$, the delay mechanism in the correlator can be implemented by a transmission line, a passive device, or an active device. In addition, because the correlation operation is done before the ADC, the sampling frequency of the ADC can also be reduced. For a cross-correlation receiver, received signals are passed through a bandpass filter before the correlation operation to reduce the incoming noise power, and the filtered signals and noises are indicated by a tilde to prevent confusion. The decision rule of this conventional cross-correlation receiver is

$$D_c = \sum_{j=0}^{N_s-1} \int_{jT_T+T_d}^{(j+1)T_T+T_d+T_{mds}} \tilde{r}(t-T_d)\tilde{r}(t)dt \quad \left\{ \begin{array}{ll} 1 & z_j \geq 0 \\ -1 & \text{otherwise} \end{array} \right.$$  (4.1)

where $T_{mds}$ is the integration time of the correlator as well as the channel delay spread.

The receiver structure of the weighted cross-correlation receiver is shown in Figure 4.1.

With an ideal bandpass filter having the one-sided bandwidth $B_w$, the BEP can be evaluated by applying the orthogonal expansion technique as well as Appendix 9A in [24]. Without actually implementing it in the conventional receiver, we use the concept that both the bandpass signal and noise have complex lowpass equivalents. The energy in the
The lowpass equivalent of the filtered received waveform \( \hat{g}(t) \) is twice the energy in \( \tilde{g}(t) \). The lowpass equivalent of the bandlimited noise \( \hat{n}(u, t) \) has power spectral density \( 2N_0 \) from \(-B_w/2\) to \(B_w/2\), and zero elsewhere. In addition to the lowpass equivalent, another useful theorem states that a time-limited \( (T_{\text{mds}}) \) band-limited \( (B_w) \) signal has dimension \( B_wT_{\text{mds}} \), and can be represented by \( \{b_k(t)\}_{k=1}^{B_wT_{\text{mds}}} \), an orthonormal set with

\[
\int_0^{T_{\text{mds}}} b_j(t)b_k^*(t)dt = \begin{cases} 
0 & j \neq k \\
1 & j = k,
\end{cases}
\tag{4.2}
\]

where \( b_k^*(t) \) denotes the complex conjugate of \( b_k(t) \). Thus the lowpass equivalent of the filtered signal and noise in time duration \( T_{\text{mds}} \), can be represented by the set \( \{b_k(t)\}_{k=1}^{B_wT_{\text{mds}}} \).

The complex lowpass equivalent of \( \tilde{g}(t) \) for \( t \in [0, T_{\text{mds}}] \) can be written as

\[
\tilde{g}(t) = \sum_{k=1}^{B_wT_{\text{mds}}} \hat{g}_k b_k(t),
\tag{4.3}
\]

where the weights are

\[
\hat{g}_k = \int_0^{T_{\text{mds}}} \tilde{g}(t)b_k^*(t)dt,
\]

and

\[
\tilde{E}_p = \int_0^{T_{\text{mds}}} \tilde{g}^2(t)dt = \frac{1}{2} \sum_{k=1}^{B_wT_{\text{mds}}} \hat{g}_k^2
\tag{4.4}
\]

is the total energy in a filtered receiver waveform. The noise covariance function of \( \hat{n}(u, t) \), \( K_{\hat{n}\hat{n}^*}(t_1, t_2) \), satisfies

\[
2N_0b_k(t_1) = \int_0^{T_{\text{mds}}} K_{\hat{n}\hat{n}^*}(t_1, t_2)b_k(t_2)dt_2
\tag{4.5}
for \( k = 1, 2, \ldots, B_w T_{mds} \) and \( t_1 \in [0, T_{mds}] \), i.e., \( \{b_k(t)\}_{k=1}^{B_w T_{mds}} \) are eigenfunctions of \( \hat{n}(u,t) \) with same eigenvalues \( 2N_0 \). The noise for \( t \in [0, T_{mds}] \) is represented as

\[
\hat{n}(u, t + iT) = \sum_{k=1}^{B_w T_{mds}} \hat{n}_{r,i,k} b_k(t),
\]

(4.6)

\[
\hat{n}(u, t + iT + T_d) = \sum_{k=1}^{B_w T_{mds}} \hat{n}_{d,i,k} b_k(t),
\]

(4.7)

for \( i = 0, 1, \ldots, N_s - 1 \), in which

\[
\hat{n}_{r,i,k} = \int_0^{T_{mds}} \hat{n}(u, t + iT)b_k^*(t)dt,
\]

\[
\hat{n}_{d,i,k} = \int_0^{T_{mds}} \hat{n}(u, t + iT + T_d)b_k^*(t)dt.
\]

It can be computed from (4.2) and (4.5) that

\[
\mathbb{E}\{\hat{n}_{r,i,k}\} = \mathbb{E}\{\hat{n}_{d,i,k}\} = 0, \quad \mathbb{E}\{\hat{n}_{r,i,k}\hat{n}_{r,i,k}^*\} = 2N_0 \text{ for any } i, k, \quad \mathbb{E}\{\hat{n}_{r,i,k}\hat{n}_{r,i',k'}\} = 0 \text{ for any } i, k, i', k', \quad \text{and } \mathbb{E}\{\hat{n}_{d,i,k}\hat{n}_{d,i',k'}\} = 0 \text{ if } i \neq i' \text{ or } k \neq k'.
\]

By defining

\[
\hat{g} \triangleq [\hat{g}_1, \hat{g}_2, \ldots, \hat{g}_{B_w T_{mds}}]^t,
\]

the received signal for the 0th bit can be represented as

\[
\hat{r} = \hat{r}_s + \hat{n} = [\hat{g}^t, b_0 \hat{g}_1^t, \hat{g}_1^t, b_0 \hat{g}_2^t, \ldots, \hat{g}_t^t, b_0 \hat{g}_{N_s}^t]^t + [\hat{n}_{r,0}^t, \hat{n}_{d,0}^t, \hat{n}_{r,1}^t, \hat{n}_{d,1}^t, \ldots, \hat{n}_{r,N_s-1}^t, \hat{n}_{d,N_s-1}^t]^t,
\]

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where

\[
\hat{n}_{r,i} = \begin{bmatrix} \hat{n}_{r,i,1}, & \hat{n}_{r,i,2}, & \hat{n}_{r,i,3}, & \ldots, & \hat{n}_{r,i,B_wT_{mds}} \end{bmatrix}^t,
\]

\[
\hat{n}_{d,i} = \begin{bmatrix} \hat{n}_{d,i,1}, & \hat{n}_{d,i,2}, & \hat{n}_{d,i,3}, & \ldots, & \hat{n}_{d,i,B_wT_{mds}} \end{bmatrix}^t.
\]

By denoting

\[
X = [X_0^t, X_1^t, \ldots, X_{N_s-1}^t]^t \quad \text{with} \quad X_i = \hat{g} + \hat{n}_{r,i},
\]

\[
Y = [Y_0^t, Y_1^t, \ldots, Y_{N_s-1}^t]^t \quad \text{with} \quad Y_i = b_0\hat{g} + \hat{n}_{d,i},
\]

the decision rule in (4.1) is equivalent to

\[
D_c = \frac{1}{2}(X^H Y + Y^H X) \begin{array}{c} \underset{\sim}{=} \frac{1}{2} \\ \approx \end{array} 0,
\]

where \(X^H\) denotes the complex conjugate transpose of \(X\). It is obvious now that the orthogonal expansion only helps to calculate the precise BEP without really implementing it in the receiver.

Now, (4.10) can be equated to (9A.1) and (9A.2) in Appendix 9A in [24] by letting \(A = 0, B = 0, C = \frac{1}{2}\) and \(L = N_sB_wT_{mds}\). The BEP of this conventional cross-correlation receiver is \(Pr\{D_c < 0 | b_0 = 1\}\) due to the symmetry of the transmitted data and the receiver noise, and can be computed utilizing (9A.15) in [24]. The value of \(a\) and \(b\) needed in computing the BEP can be calculated using (4.4) as well as (9A.4) and (9A.5) in [24],
which results in $a = 0$ and $b = \sqrt{\frac{E_b}{N_0}}$ with $\tilde{E}_b = 2N_s \tilde{E}_p$. The BEP of a conventional cross-correlation receiver conditioned on the filtered channel response $\tilde{g}(t)$ is

$$P_{\text{bit}}^c = \frac{1}{2} + \frac{1}{2^{2N_s B_w T_{\text{mds}}} - 1} \sum_{l=1}^{N_s B_w T_{\text{mds}}} \left( \frac{2N_s B_w T_{\text{mds}} - 1}{N_s B_w T_{\text{mds}} - l} \right) \times \left[ Q_l \left( 0, \sqrt{\frac{\tilde{E}_b}{N_0}} \right) - Q_l \left( \sqrt{\frac{\tilde{E}_b}{N_0}}, 0 \right) \right]$$

(4.11)

where $\tilde{g}(t)$ is implicitly imbedded in $\tilde{E}_b$. By utilizing (3.25) and the fact that $Q_l(b, 0) = 1$ for all $l, b$, (4.11) is further simplified to

$$P_{\text{bit}}^c = \frac{1}{2^{2N_s B_w T_{\text{mds}}} - 1} \sum_{l=1}^{N_s B_w T_{\text{mds}}} \left( \frac{2N_s B_w T_{\text{mds}} - 1}{N_s B_w T_{\text{mds}} - l} \right) \sum_{n=0}^{l-1} \frac{1}{n!} \exp \left( -\frac{\tilde{E}_b}{2N_0} \right) \left( \frac{\tilde{E}_b}{2N_0} \right)^n.$$  

(4.12)

The average BEP over all channel response functions of the conventional cross-correlation receivers can be obtained if $E\{\exp(-\frac{\tilde{E}_b}{2N_0})(\frac{\tilde{E}_b}{2N_0})^n)\}$ exists for all $n$, i.e., the moment generating function of $\frac{\tilde{E}_b}{2N_0}$ exists.

In addition to using the orthogonal function expansion, the BEP of a conventional cross-correlation receiver can also be well predicted by using the central limit theorem. The decision statistic $D_c$ in (4.1) is written in detail

$$D_c = b_0 N_s \tilde{E}_p + b_0 \tilde{N}_d + \tilde{N}_r + \tilde{N},$$

(4.13)

where

$$\tilde{N}_d = \sum_{j=0}^{N_s-1} \int_0^{T_{\text{mds}}} \tilde{n}(u, t + jT_f) \tilde{g}(t) dt,$$
\[ \tilde{N}_r = \sum_{j=0}^{N_s-1} \int_0^{T_{mds}} \tilde{n}(u, t + jT_I + T_d) \tilde{g}(t) dt, \]
\[ \tilde{N} \triangleq \sum_{j=0}^{N_s-1} \int_{jT_I + T_d}^{jT_I + T_{mds}} \tilde{n}(u, t - T_d) \tilde{n}(u, t) dt. \]

It is immediately seen that \( \tilde{N}_d \) and \( \tilde{N}_r \) are independent Gaussian random variables with zero mean. The noise term \( \tilde{N} \) is generally non-Gaussian, and its probability density can be computed precisely (see [15], chapter 2, section 3). But the probability density of \( \tilde{N}_d + \tilde{N}_r + \tilde{N} \) is still complicated and difficult to obtain. In addition to the ultra-wide bandwidth of the transmitted and received signals, often a UWB application environment in which the TR method is preferred has a large number of paths, namely the channel delay spread \( T_{mds} \) is not short. Therefore, the noise dimension \( 2N_sB_wT_{mds} \) is large enough to conclude that this sum of integrals of the product of two Gaussian random processes is itself approximately Gaussian by central limit theorem arguments. Only first and second moments under this Gaussian assumption are required to evaluate the BEP. The mean and variance of \( D_c \) conditioned on the transmitted bit \( b_0 \) and the filtered channel realization \( \tilde{g}(t) \) are \( m = \mathbb{E}\{D_c | b_0, \tilde{g}(t)\} = b_0 N_s \tilde{E}_p \) and \( \sigma^2 = \mathbb{V}\{D_c | \tilde{g}(t)\} = N_s N_0 \tilde{E}_p + \frac{N_s B_w T_{mds} N_0^2}{2} \) which are derived in Appendix A.4, and the BEP conditioned on the channel response function is

\[ P_{\text{bit}}^c \cong Q \left( \frac{|m|}{\sqrt{\sigma^2}} \right) = Q \left( \left[ \frac{2N_0}{\tilde{E}_b} + 2N_s B_w T_{mds} \left( \frac{N_0}{\tilde{E}_b} \right)^2 \right]^{-\frac{1}{2}} \right). \] (4.14)
4.2 Average Cross-correlation Receivers

One method to improve the BEP performance of the conventional cross-correlation receiver is to average the $N_s$ reference waveforms in one bit time to reduce the noise in the correlator template, then data detection proceeds with this noise reduced template. The decision statistic of this average correlation receiver is

$$D_a = \sum_{j=0}^{N_s-1} \int_{-T_d+T_f}^{jT_f+T_d+T_{mds}} \tilde{r}(t) \times \left[ \frac{1}{N_s} \sum_{i=-j}^{N_s-1-j} \tilde{r}(t+iT_f-T_d) \right] dt \quad 1 \gg 0, \quad (4.15)$$

and the BEP can be computed by employing the same method as for the conventional cross-correlation receiver, but now $X = \sum_{i=0}^{N_s-1} X_i$ and $Y = \frac{1}{N_s} \sum_{i=0}^{N_s-1} Y_i$ instead of the definitions in (4.8) and (4.9). Equation (9A.1) and (9A.2) in [24] are equated to $D_a$ by letting $A = 0$, $B = 0$, $C = \frac{1}{2}$, and $L = B_w T_{mds}$. The BEP of this average cross-correlation receiver conditioned on the filtered channel realization $\tilde{g}(t)$ by substituting $a = 0$, $b = \sqrt{\frac{\tilde{E}_b}{N_0}}$ into (9A.15) in [24] after simplification is

$$P_{\text{bit}}^a = \frac{1}{2^{2B_w T_{mds}-1}} \sum_{l=1}^{B_w T_{mds}} \left( \frac{2B_w T_{mds} - 1}{B_w T_{mds} - l} \right) \sum_{n=0}^{l-1} \frac{1}{n!} \exp \left( -\frac{\tilde{E}_b}{2N_0} \right) \left( \frac{\tilde{E}_b}{2N_0} \right)^n. \quad (4.16)$$

The average BEP of this average cross-correlation receivers can be obtained if the moment generating function of $\frac{\tilde{E}_b}{2N_0}$ exists. Arguing again by the central limit theorem, the pdf of the noise $\times$ noise in $D_a$ also approaches the Gaussian distribution if $B_w T_{mds}$, which is the noise dimension, is large. The conditional BEP using this Gaussian assumption is

$$P_{\text{bit}}^a \approx Q \left( \left[ \left( \frac{2N_0}{\tilde{E}_b} \right) + 2B_w T_{mds} \left( \frac{N_0}{\tilde{E}_b} \right)^2 \right]^{-\frac{1}{2}} \right). \quad (4.17)$$
It is worth mentioning that the resolvable multipath assumption is not required in the above analysis of conventional and average cross-correlation receivers.

Comparing (4.14) and (4.17), an average cross-correlation receiver can reduce the noise power in the noise × noise term by a factor of $N_s$. The problem is the averaging process might not be easily implemented because of restricted receiver complexity, namely be implemented using analog devices before the correlator. In a multiple access environment, a hopping sequence is applied to each user to avoid catastrophic collisions. The time separation of the reference and data pulses remains as $T_d$, but their position in different frames varies with the hopping sequence (See (2.1)). In order to average $N_s$ received reference waveforms, we need a delay mechanism to put those $N_s$ references together. If a single user case is considered, without the hopping sequence, we can have delay lines with $N_s$ different values \{$T_d, T_f, 2T_f, \ldots, (N_s - 1)T_f$\} to implement the average process. Even so, when $N_s$ is large, the number of delay lines and the length of delay make the implementation impossible. If the average process is implemented using digital techniques, then we need an ADC with high-sampling-frequency to sample and quantize the received signal. In this case, since the signals have already been digitized, there is no need to restrict the receiver structure as a cross-correlation receiver, and all kinds of digital signal processing schemes can be used to improved the BEP performance. Either in a single user or a multiple access environment, the time separation is fixed to $T_d$. This means the structure of the conventional cross-correlation receiver remains the same in both cases.

For both the conventional and average cross-correlation receivers, BEPs in (4.12) and (4.16) are only precise if the receivers have an ideal front-end bandpass filter. In reality, this ideal filter is impossible to implement which makes (4.12) and (4.16) still approximations.
for a real system. For the application of the central limit theorem, the variances of $D_c$ and $D_a$ can be computed for any kind of front-end bandpass filter. Therefore in real systems with large noise time×bandwidth, (4.12) and (4.16) do not necessarily evaluate the BEPs better than (4.14) and (4.17).

4.3 Structure Comparisons of Theoretical Receivers and Ad Hoc Cross-correlation Receivers

It is important to discuss the similarities between ad hoc cross-correlation receivers and the ALRT suboptimal receiver in (3.19). Under the resolvable multipath assumption, the set including the correlator templates $\{g_{rx}(t - k\Delta)\}_{k=0}^{K-1}$ is a complete set of orthonormal basis functions for the received signal $g(t)$. The suboptimal receiver in (3.19) therefore obtains all the energy in the received signal, as does the average cross-correlation receiver (which can be shown by expressing $g(t)$ using the orthogonal function expansion). For the average cross-correlation receiver, the incoming receiver noise has dimension $2B_wT_{mds}$. On the other hand, the suboptimal receiver (3.19) has noise dimension $K < 2B_wT_{mds}$ under the resolvable multipath conditions. Therefore, the receiver in (3.19) performs better than the average cross-correlation receiver by capturing the same signal energy with less incoming receiver noise. Suppose now additional $2B_wT_{mds} - K$ orthonormal functions, which along with the original $K$ basis functions expanding the filtered noise space, are put in the orthonormal basis set. The receiver in (3.19) with these $2B_wT_{mds}$ orthonormal functions as correlator templates is equivalent to the average cross-correlation receiver. But note that the additional $2B_wT_{mds} - K$ orthonormal functions only capture noise.
The receiver in (3.19) and the average cross-correlation receiver acquire the multipath
diversity with pre-detection combining. The conventional cross-correlation receiver ac-
quires the multipath diversity through a post-detection combination. Investigating (4.12)
and (4.14) or (4.16) and (4.17) gives us an interesting observation that for a specified
$\tilde{E}_b/N_0$, the BEP of an average cross-correlation receiver is irrelevant to $N_s$ while the BEP
of a conventional cross-correlation receiver varies with $N_s$. The effect of the noise $\times$ noise
term depends on the energy in the template to the noise power ratio $\frac{N_t \tilde{E}_b}{2N_s N_0}$, where $N_t$ is the
number of reference waveforms accumulated in the correlator template, and becomes more
serious when this value is low. This value for the average cross-correlation receiver is fixed
for a specified $\tilde{E}_b/N_0$ because the signal energy in the reference waveforms in $N_s$ frames
is accumulated first before the correlation operation. For a conventional cross-correlation
receiver and a specified $\tilde{E}_b/N_0$, because only one reference waveform serves as a template,
the larger the $N_s$ is, the smaller the energy in one reference pulse is. Therefore, the BEP
performance becomes worse when $N_s$ becomes larger. Pre-detection and post-detection
combinations perform differently in \textit{ad hoc} cross-correlation receivers. And the subopti-
mal receiver in (3.19), even with an extended orthonomal set to expand the noise space,
is still not equivalent to the conventional cross-correlation receiver.

4.4 Weighted Cross-correlation Receivers

The performance of the suboptimal receiver in (3.28) is better than the one in (3.19) be-
cause \textit{a priori} information is utilized. According to the similarities of the cross-correlation
receivers and the suboptimal receiver in (3.19) which are discussed in the previous section,
the performance of cross-correlation receivers can also be improved by applying a continuous weighting function $w(t)$ which contains prior information. Due to the impracticality of averaging using analog devices, only applying weighting functions to the conventional cross-correlation receiver is discussed in this section. The decision statistic of the weighted cross-correlation receiver is

$$D_w = \sum_{j=0}^{N_s-1} \int_{jT_t+T_d}^{jT_t+T_d + T_{mds}} \tilde{r}(t-T_d) \tilde{r}(t) w(t-jT_t-T_d) dt,$$  \hspace{1cm} (4.18)

and we say the transmitted bit is 1 if $D_w > 0$, otherwise $-1$. The receiver structure of the weighted cross-correlation receiver is in Figure 4.2. The BEP conditioned on the filtered received waveform $\tilde{g}(t)$ also can be derived using the central limited theorem argument, and it is

$$P_{bit} \approx Q \left( \frac{\sqrt{N_0} \int_0^{T_{mds}} w(t) \tilde{g}^2(t) dt}{\sqrt{N_0} \int_0^{T_{mds}} w^2(t) \tilde{g}^2(t) dt + \frac{B_w N_s^2}{2} \int_0^{T_{mds}} w^2(t) dt} \right).$$  \hspace{1cm} (4.19)
4.4.1 Theoretical-weighted Cross-correlation Receivers

The first choice of \( w(t) \) is \( w(t) = w_R(t) \), and \( w_R(t) \), which is called a theoretical weighting function, is an interpolation of \( \{ w_R(k) \}_k \) in (3.11). The parameter \( SNR_k \) in (3.11) can be written as

\[
SNR_k = \frac{4N_s \sigma_k^2}{N_0} = \frac{4a N_s \sum_{k=0}^{K-1} \sigma_k^2}{N_0} \times \frac{\sigma_k^2}{\kappa \sum_{k=0}^{K-1} \sigma_k^2} = \frac{A-SNR'}{\kappa} \sigma_k^2,
\]

where \( A-SNR' \) denotes the average energy per bit to the noise power spectral density ratio, and \( \kappa = a \sum_{k=0}^{K-1} \sigma_k^2 \). Suppose the average power profile of the channel can be described by a function \( f_p(t) \), the weighting function \( w_R(t) \) based on the interpolation is

\[
w'_R(t) = \frac{f_p(t)}{1 + (A-SNR') \times \frac{1}{\kappa} f_p(t)}.
\] (4.20)

The function in (4.20) must be adjusted before applying to the conventional correlation receivers. Conventional cross-correlation receivers have post-detection combinations, and the weighting function, which is interpolated from a pre-detection combination receiver, is applied before the combinations. This makes the \( A-SNR' \) in (4.20) inappropriate when \( N_s > 1 \) because the energy in the \( N_s \) frames has not been gathered yet. On the other hand, we can not replace \( A-SNR' \) in (4.20) by the average energy per waveform to the noise power spectral density ratio either because the effect of the post-detection combination on the BEP is not linear. Therefore, the weighting function is adjusted as

\[
w_R(t) = \frac{f_p(t)}{1 + (A-SNR) \times \frac{1}{\kappa} f_p(t)},
\] (4.21)
$w_R(t)$ with different values of A-SNR and $\Gamma$ is shown in Figure 4.3. We can see that increasing $\Gamma$ lengthens the tail of $w_R(t)$, while increasing A-SNR increases the height.

4.4.2 Rectangular-Weighted Cross-correlation Receivers

Note that $w_R(t)$ is not the only choice for a weighting function. If we investigate (4.1) and (4.14) in detail, the length of the integration interval can also affect the BEP. The integration time, which is defined as $T_{corr}$ here, should follow the constraint that $T_{corr} \leq T_{mds}$. Under the constraint, increasing $T_{corr}$ can increase the incoming signal energy, but
we also get more extra noise power through the noise × noise term. Since the tail of the channel impulse response is usually small, it might not be good to use \( T_{\text{corr}} = T_{\text{mds}} \) as in (4.1). We should adjust \( T_{\text{corr}} \) according to the channels to minimize the BEP, and this adjustment is equivalent to applying a rectangular weighting function

\[
w_s(t) = \begin{cases} 
1, & 0 \leq t \leq T_{\text{corr}} \\
0, & \text{otherwise.}
\end{cases}
\]

The decision rule in (4.1) is then modified to

\[
\sum_{j=0}^{N_s-1} \int_{jT_f+T_d+T_{\text{mds}}}^{jT_f+T_d+T_{\text{corr}}} \tilde{r}(t-T_d)\tilde{r}(t)w_s(t-jT_f-T_d) dt \equiv \sum_{j=0}^{N_s-1} \int_{jT_f+T_d+T_{\text{corr}}}^{jT_f+T_d+T_{\text{mds}}} \tilde{r}(t-T_d)\tilde{r}(t) dt \overset{\text{1}}{\sim} 0,
\]

(4.22)

and the BEP conditioned on a channel realization can be modified from (4.12) and (4.14) with \( T_{\text{mds}} \) replaced by \( T_{\text{corr}} \) as well as \( \tilde{E}_b \) replaced by \( \eta \tilde{E}_b \)

\[
P_{\text{bit}} = \frac{1}{2^{2N_sB_wT_{\text{corr}}}} \sum_{l=1}^{N_sB_wT_{\text{corr}}} \left( \frac{2N_sB_wT_{\text{corr}} - l}{N_sB_wT_{\text{corr}} - l} \right) \sum_{n=0}^{l-1} \frac{1}{n!} \exp \left( -\frac{\eta \tilde{E}_b}{2N_0} \right) \left( \frac{\eta \tilde{E}_b}{2N_0} \right)^n
\]

(4.23)

\[
Q \left( \frac{|m|}{\sqrt{\sigma^2}} \right) = Q \left( \frac{2N_0}{\eta \tilde{E}_b} + 2N_sB_wT_{\text{corr}} \left( \frac{N_0}{\eta \tilde{E}_b} \right)^{2} \right)^{-\frac{1}{2}}.
\]

(4.24)

The parameter \( \eta = \frac{\int_{0}^{T_{\text{corr}}} \tilde{g}^2(t) dt}{\int_{0}^{T_{\text{mds}}} \tilde{g}^2(t) dt} \) is the efficiency factor which denotes the portion of the signal energy a rectangular-weighted cross-correlation receiver can capture. This rectangular weighting function can also apply to a average cross-correlation receiver for the same
reason, and the BEP of the rectangular-weighted average cross-correlation receiver conditioned on a channel response is as in (4.16) and (4.17) with \( T_{mds} \) replaced by \( T_{corr} \) as well as \( \tilde{E}_b \) replaced by \( \eta \tilde{E}_b \).

The channel power profile, the ratio of the average signal energy to the noise power spectral density (ASNR), and receiver bandwidth all affect the selection of \( T_{corr} \). For minimizing the average BEP, the optimal value of \( T_{corr} \) increases as ASNR increases, and/or the decay time constant of the average power profile increases, and/or the receiver bandwidth decreases. The integration time analysis is detailed in Appendix B.

### 4.5 Numerical Results

#### 4.5.1 BEP Comparisons of Theoretical Receivers and Ad Hoc Cross-correlation Receivers

The receiver structures we discussed above have nonlinear operations which make the theoretical BEP analysis difficult. Except for the receivers in (3.19), (3.28), (4.1) and (4.15), the BEPs of other receiver structures are evaluated by Monte Carlo simulations. In this section, we generate both Rayleigh and lognormal path strength models to test different receiver structures.

The single received pulse is a second derivative Gaussian pulse described in (2.2), and 97\% of the received pulse’s power is contained in 1-5GHz. Therefore, an ideal bandpass filter with 1-5GHz pass band is used in cross-correlation receivers, and \( B_w = 4\text{GHz} \). This 4GHz bandwidth might not be the optimal choice because decreasing the receiver bandwidth will reduce the signal energy but also reduce the incoming noise power. For
both Rayleigh and lognormal environments, $\Delta = 0.7$ ns, $a = 1, 0.7$, and 0.3, $K = 84$, and the average power decay profile is assumed exponential with decay time constant $\Gamma = 8.5$ ns. The average power of the first multipath signal component $\Omega_0$ is chosen such that the average $E_b/N_0 (\bar{E}_b/N_0)$ can achieve different values. For lognormal cases, 

$$2N_s a \sum_{k=0}^{K-1} E[\alpha_k^2] = \bar{E}_b, \text{ and } E[\alpha_k^2] = \Omega_0 \exp(\frac{-k\Delta}{\Gamma}).$$

In addition, the standard deviation of $20 \log (\alpha_k)$, $\sigma$, is equal to $4.8/\sqrt{2}$ for all $k$, and the mean of $20 \log (\alpha_k)$ can be computed as $\mu_k = \frac{10 \ln \Omega_0 - 10k\Delta/\Gamma}{\ln 10} - \frac{\sigma^2 \ln 10}{20}$. For Rayleigh models, $4N_s a \sum_{k=0}^{K-1} \sigma_k^2 = \bar{E}_b$, and $2\sigma_k^2 = \Omega_0 \exp(\frac{-k\Delta}{\Gamma})$. These Rayleigh and lognormal channel models with root mean square (RMS) delay spread around 7 ns represent small indoor environments.

In order to explain the BEP performance clearly, we name the decision rules in

- (3.7) with (3.12): ALRT Rayleigh receiver,
- (3.28): Rayleigh suboptimal receiver 1,
- (3.29): Rayleigh suboptimal receiver 2,
- (3.7) with (3.16): ALRT lognormal receiver with $N$,
- (3.37): lognormal suboptimal receiver 1,
- (3.38): lognormal suboptimal receiver 2,
- (4.1): conventional cross-correlation receiver,
- (4.15): average cross-correlation receiver.

In the simulations of the ALRT lognormal receiver, we tried $N = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 16,$ and 20, and the simulated BEP results remain unchanged for $N \geq 8$. Besides, results from $N = 2$ to $N = 8$ are close. Therefore in performance figures, we only show the cases of $N = 1, 2,$ and 8.
The differences of the BEP of conventional and average cross-correlation receivers evaluated by the precise formulas ((4.12) and (4.16)) and Gaussian approximation formulas ((4.14) and (4.17)) can not be distinguished in our simulation environments, thus only one result line for each kind of cross-correlation receivers is displayed in performance figures. This also indicates that the Gaussian approximation is good if the noise time-bandwidth product is large enough. Performance figures showing that theoretical analyses and simulation results of the Rayleigh suboptimal receiver 1 and 2 agree with each other verify the programs.

The following figures show the bit error probability averaged over channel statistics, i.e., channel realizations, versus the average $E_b/N_0$. Thus the value of the error probability is the observation in time varying channels over a long time. Suppose the channel is stationary during the observation, the BEP obtained could deviate from the curves shown in the figures, and should depend on that specific channel.

4.5.1.1 Rayleigh Environments versus Lognormal Environments

Figure 4.4 shows that the four best performing receivers in Rayleigh environments with $a = 1$ are the ALRT Rayleigh receiver, Rayleigh suboptimal receiver 1, and ALRT lognormal receivers with $N = 8$ and 2, which have nearly identical BEP performance. Hence the Rayleigh suboptimal receiver 1 is a better choice among these four receivers because of its simpler structure. ALRT lognormal receiver with $N = 1$ and lognormal suboptimal receiver 1 have slightly worse but competitive performance than the best, and the degradation occurs because only one term is used in the Hermite-Gauss integral (see (3.35)-(3.37)). It is expected that the performance of the Rayleigh suboptimal receiver 2 and lognormal
suboptimal receiver 2 are worse because they discard *a priori* information about random channels. The lack of terms in the Hermite-Gauss integral also causes performance degradation in the lognormal suboptimal receiver 2 because it is derived by approximating the ALRT lognormal receiver with \( N = 1 \).

Figure 4.5 shows the BEP performance for different receivers in lognormal environments with \( a = 1 \). The five best performed receivers in turn are the ALRT lognormal receivers with \( N = 8 \) and 2, ALRT Rayleigh receiver, ALRT lognormal receivers with \( N = 1 \), and Rayleigh suboptimal receiver 1, and their BEPs are close. The performance of the lognormal suboptimal receiver 1 is only slightly worse than the best. Again, the Rayleigh and lognormal suboptimal receiver 2 perform worse than other receivers because of lack of *a priori* information about channels. The average and conventional cross-correlation receivers have 5dB and 10dB performance degradation respectively compared to the best performance for \( N_s = 10 \) at BEP=1e-3.

Figure 4.4, 4.5, 4.6, and 4.8 show that the Rayleigh suboptimal receiver 1 with an easier-to-implement structure performs close to the optimal receiver in both Rayleigh and lognormal environments with normal to high multipath component arrival probability \( a \). Figure 4.4-4.9 show that optimal ALRT receivers derived from Rayleigh and lognormal path strength models perform equally well in each other’s environments. Different amplitude distributions have been proposed by different authors for UWB systems [16, 17], but these figures show that the BEP performance of an ALRT optimal receiver is irrelevant to path strength models. Because an integration involving the lognormal distribution does not have a closed form simplification, 4.4-4.9 indicate that we can derive a receiver structure for Rayleigh environments and apply it successfully to lognormal environments.
4.5.1.2 Effects of Multipath Component Arrival Probability $a$

Figure 4.6 and 4.7 also show different receivers performing in Rayleigh environments but with $a = 0.7$ and $0.3$. For $a=0.7$ and in the interested BEP range, we can see again that the ALRT Rayleigh receiver, ALRT lognormal receiver, and Rayleigh suboptimal receiver 1 almost perform equally well. The performance of other receivers is on the same order as in the $a = 1$ case. But when $a$ becomes small, this situation changes. In Figure 4.7, ALRT Rayleigh and ALRT lognormal receivers still perform best, but the BEP of the Rayleigh suboptimal receiver 1 departs from them. As $\bar{E}_b/N_0$ increases, the performance of the Rayleigh and lognormal suboptimal receiver 1 approach that of the Rayleigh and lognormal suboptimal receiver 2, respectively. We can see from the figure that as $\bar{E}_b/N_0$ is large enough, the suboptimal receiver 2 even performs better than the suboptimal receiver 1.

The Rayleigh and lognormal suboptimal receiver 1 give each possible multipath signal component a weight according to a priori information which does not include the path existence probability $a$. As the value of $a$ decreases, the probability of having an arrival in each time slot becomes small. It might happen that large weights are given to some time slots without arrivals. This weighting strategy can make the performance worse than without weighting. This observation is similar to the situation happening in the Rake reception. Maximal ratio combining (MRC) is optimal in maximum likelihood sense only if channel states are completely known. In reality, we have to estimate the channel and some estimation error can happen. When the estimation error increases, the performance of MRC and equal gain combining (EGC) become close, and EGC can eventually perform better than MRC when the estimation error is large enough. Figure 4.4, 4.6, and 4.7 also
show that for a specified $E_b/N_0$, all the receivers perform worse in an environment with smaller $a$ because of the increased path existence uncertainty.

Figure 4.8 and 4.9 show the performance of different receivers in lognormal environments with $a = 0.7$ and $0.3$. The observation we made for Rayleigh environments with $a = 0.7$ and $0.3$ also applies here.

### 4.5.1.3 Effects of $N_s$

In Figure 4.4-4.9, one bit is transmitted in $N_s = 10$ consecutive frames. For all ALRT optimal and suboptimal receivers as well as the average cross-correlation receiver, the value of $N_s$ does not affect their BEPs because of the pre-detection combination. The effect of $N_s$ on the conventional cross-correlation receiver is shown in these figures and Figure 4.10 which illustrate that increasing $N_s$ degrades the BEP. For $N_s = 10$ and $100$, the receiver has 5dB and 10dB performance degradation compared to the $N_s = 1$ case at BEP=1e-3. And the conventional and average cross-correlation receivers are the same when $N_s = 1$.

The signal energy to noise power ratio in a conventional cross-correlation receiver output is defined as

$$d_{\text{SNR}}^c = \left[ \frac{2N_0}{E_b} + 2B_w N_s T_{\text{mds}} \left( \frac{N_0}{E_b} \right)^2 \right]^{-1},$$

and the BEP using Gaussian approximation in (4.14) is equal to $Q(\sqrt{d_{\text{SNR}}^c})$. For a given BEP, we can find a value $y$ so that the BEP is achieved if $d_{\text{SNR}}^c = y$. By solving $d_{\text{SNR}}^c = y$, the required $\frac{E_b}{N_0}$ is

$$\frac{E_b}{N_0} = \frac{2B_w N_s T_{\text{mds}}}{\sqrt{1 + \frac{2B_w N_s T_{\text{mds}}}{y}}} \approx \sqrt{2B_w N_s T_{\text{mds}}} y.$$  (4.25)
We can now see the effect of increasing $N_s$. For a specific BEP (or $y$), the required $\tilde{E}_b/N_0$ approximately increases $10\log(\sqrt{n_2/n_1})$ dB for $N_s$ increasing from $n_1$ to $n_2$. Figure 4.4-4.10 agree with this result.

### 4.5.1.4 Rayleigh Suboptimal Receiver 2 versus Average Cross-correlation Receiver

In Figure 4.4-4.9, the average and conventional cross-correlation receivers have roughly 5dB and 10dB performance degradation respectively compared to the best performance at BEP=1e-3. The amounts of separation between curves for the Rayleigh suboptimal receiver 2 and average cross-correlation receiver can be predicted theoretically. As in the discussion of conventional cross-correlation receivers in the previous paragraph, the signal energy to noise power ratio in the decision statistic for the Rayleigh suboptimal receiver 2 is

$$
\frac{d_{\text{SNR}}^R}{\text{SNR}} = \left[ \frac{2N_0}{E_b} + K \left( \frac{N_0}{E_b} \right)^2 \right]^{-1}.
$$

By using the same method, the required $\frac{E_b}{N_0}$ to achieve $d_{\text{SNR}}^R = y$ is

$$
\frac{E_b}{N_0} \approx \sqrt{K y}.
$$

By comparing (4.25) and (4.26) with $N_s = 1$, $T_{\text{mds}} = 7K \times 10^{-10}$ and $\tilde{E}_b \approx E_b$, the performance difference is roughly $10\log(\sqrt{1.4B_w \times 10^{-9}}) = 3.7$ dB which is verified in the figures.
Figure 4.4: Average bit error probabilities of different receiver structures for $N_s = 10$ in Rayleigh environments with $a = 1$.

Figure 4.5: Average bit error probabilities of different receiver structures for $N_s = 10$ in lognormal environments with $a = 1$. 
Figure 4.6: Average bit error probabilities of different receiver structures for $N_s = 10$ in Rayleigh environments with $a = 0.7$.

Figure 4.7: Average bit error probabilities of different receiver structures for $N_s = 10$ in Rayleigh environments with $a = 0.3$. 

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Figure 4.8: Average bit error probabilities of different receiver structures for $N_s = 10$ in lognormal environments with $a = 0.7$.

Figure 4.9: Average bit error probabilities of different receiver structures for $N_s = 10$ in lognormal environments with $a = 0.3$. 
4.5.2 BEP Comparisons of Theoretical-weighted and Rectangular-weighted Cross-correlation Receivers

The weighted cross-correlation receivers are tested by using IEEE 802.15.3a UWB channel models CM1, CM2, CM3 and CM4 [16]. The single received pulse $g_{rx}(t)$, $B_w$ and $\Delta$ are the same as in Section 4.5.1, and 1000 equal power channel realizations for each model are generated for average BEP evaluation. For the rectangular-weighted cross-correlation receiver, performance is evaluated by (4.24). The optimal values of $T_{corr}$ are those which make the rectangular-weighted cross-correlation receiver achieve the average BEP=1e-4 with minimum $E_b/N_0$, and are listed in Table 4.1. The resolution of $T_{corr}$ is 1 ns.
Performance evaluation of theoretical-weighted cross-correlation receivers is obtained by using (4.19). Although paths arrive in clusters in IEEE 802.15.3a, equivalent non-clustered models exist for performance analysis and evaluation [31]. And the equivalent $\Gamma$s have to be chosen first. Due to the two uncertainties A-SNR and $\Gamma$, $N_s$ is set to one first so A-SNR is equal to the average energy per bit to noise power spectral density ratio. Then iterative methods are adopted to acquire the equivalent $\Gamma$s for four models. The initial values of A-SNRs are set to those minimum values of $\bar{E}_b/N_0$ that achieve the average BEP=1e-4 using rectangular-weighted cross-correlation receivers. Then $\Gamma$s, which achieve the average BEP=1e-4 with minimum $\bar{E}_b/N_0$s can be found. The A-SNRs are set to the new $\bar{E}_b/N_0$s, and the equivalent $\Gamma$s are searched again. This process is continued until that the A-SNRs and $\Gamma$s converge. For a known $\Gamma$, A-SNR is the value which minimizes the average BEP at a given value for a given $N_s$. The values of $\Gamma$s and A-SNRs for four models are listed in Table 4.1. The resolution of $\Gamma$ and A-SNR are 0.5 ns and 0.1 dB.

Using those values of A-SNR, $\Gamma$ and $T_{\text{corr}}$ in Table 4.1 to produce $w_R(t)$ and $w_s(t)$, BEP curves in four environments are plotted in Figure 4.11. Curves show that BEPs depend on application environments, and using the theoretical weighting function outperforms using the rectangular weighting function. But the differences are not large.

<table>
<thead>
<tr>
<th></th>
<th>CM1</th>
<th>CM2</th>
<th>CM3</th>
<th>CM4</th>
</tr>
</thead>
<tbody>
<tr>
<td>theoretical WF, $w_R(t)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A-SNR (dB)</td>
<td>16.7</td>
<td>18.6</td>
<td>19.2</td>
<td>20.7</td>
</tr>
<tr>
<td>$\Gamma$ (ns)</td>
<td>7.5</td>
<td>10.0</td>
<td>20.5</td>
<td>39.5</td>
</tr>
<tr>
<td>rectangular WF, $w_s(t)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{\text{corr}}$ (ns)</td>
<td>21</td>
<td>22</td>
<td>48</td>
<td>77</td>
</tr>
</tbody>
</table>

Table 4.1: Optimal parameter values of theoretical and rectangular weighting functions at BEP=1e-4 with $N_s = 100$. 
Figure 4.12 shows the performance degradation of a theoretical-weighted cross-correlation receiver in four environments when the value of A-SNR or $\Gamma$ is not optimized. The x-axis is A-SNR (upper figure) or $\Gamma$ (lower figure), and the y-axis is the required $E_b/N_0$ to achieve the average BEP=1e-4. The circle on each line marks the position of the optimal value of A-SNR or $\Gamma$. Results show the receiver is not sensitive to the A-SNR deviation. As for $\Gamma$, an underestimate degrades the performance more than an overestimate. Since $\Gamma$ is related to application environments, we should consider adopting large values of $\Gamma$ if the weighting function is not adjustable in the receiver.

Figure 4.13 compares the performance degradation in CM1 (upper figure) and CM4 (lower figure) environments using theoretical and rectangular weighting functions while parameters are not optimized. Because $w_R(t)$ and $w_s(t)$ have different parameters, we have to find a fair way to compare them. Table 4.1 shows the optimal values of A-SNR and $\Gamma$ for $w_R(t)$, and the optimal values of $T_{corr}$ for $w_s(t)$ in four environments. For the theoretical and rectangular weighting functions, 1, 2, 3 and 4 at the x-axis in Figure 4.13 represent the optimal values of $(A$-SNR, $\Gamma)$ and $T_{corr}$ in environments CM1, CM2, CM3, and CM4, respectively. The y-axis is the required $E_b/N_0$ to achieve the average BEP=1e-4. Under this definition of the x-axis, the comparison is fair. In the upper figure, we can see that the best performance happens while $x=1$ because parameters in both weighting functions are optimized. For the same reason, the best performance happens while $x=4$ in the lower figure. Both the upper and lower figures show that the performance degradation of using a theoretical weighting function is smaller than that of using a rectangular weighting function when the values of parameters are not optimized. Using theoretical weighting functions makes the receiver more flexible in different environments.
Figure 4.11: BEP performance of using theoretical and rectangular weighting functions in four environments with $N_s = 100$.

Figure 4.12: $\bar{E}_b/N_0$ required to achieve the average BEP=$1e^{-4}$ for different values of A-SNR and $\Gamma$ with $N_s = 100$. 
Figure 4.13: $E_b/N_0$ required to achieve BEP=1e-4 of using theoretical and rectangular weighting functions with non-optimal parameter values and $N_s = 100$. For the theoretical weighting function, 1, 2, 3, and 4 at the x-axis represent that $(A-SNR \text{ (dB)} , \Gamma \text{ (ns)})=(16.7, 7.5), (18.6, 10.0), (19.2, 20.5), \text{ and } (20.7, 39.5)$. For the rectangular weighting function, 1, 2, 3, and 4 at the x-axis represent that $T_{corr} \text{ (ns)}=21, 22, 48, \text{ and } 77$. 
Chapter 5

Multiple Access Performance of Conventional UWB TR Systems

5.1 Received Signal Structure

The received signal of any receiver in an asynchronous UWB system with conventional TR modulation and \( N_u \) active transmitters is

\[
r(t) = N_u \sum_{n=1}^{\infty} \sum_{i=\infty}^{\lambda} \xi_i^{(n)}(t) + n_t(u,t),
\]

(5.1)

where

\[
\xi_i^{(n)}(t) = d_i^{(n)} g_i^{(n)}(t - iT_i - c_i^{(n)}T_c - \tau_n) + b_{i/N_i}^{(n)} g_i^{(n)}(t - iT_i - c_i^{(n)}T_c - T_d^{(n)} - \tau_n).
\]

(5.2)

Here \( g_i^{(n)}(t) \) is the received waveform of transmitter \( n \) in the \( i \)-th frame, and \( \tau_n \) is the relative asynchronous delay of user \( n \) to the receiver. The rectangular-weighted cross-correlation receiver is perfectly synchronized to the desired transmitter which is assumed transmitter...
1 in this chapter without loss of generality, i.e., \( \tau_1 \) and \( \{c_i^{(1)}\} \) are perfectly known. In addition, only the signal from the transmitter 1 can have reference and data-modulated waveform alignment for detection because all the users have different time separation \( T_{d(n)} \).

In the derivation of the MA performance of a rectangular-weighted cross-correlation receiver, some reasonable assumptions are made:

1. Data bits \( b_i^{(n)} \) for \( n = 1, \ldots, N_u \) are independent and uniformly distributed in \( \{+1, -1\} \) so that \( \mathbb{E}\{b_i^{(n)}\} = 0 \).

2. The hopping code elements \( c_i^{(n)}, i = 1, \ldots, N_{hs}, n = 1, \ldots, N_u \) are independent and uniformly distributed in \( \{0, 1, \ldots, N_h^{(n)} - 1\} \).

3. The direct sequence elements \( d_i^{(n)}, i = 1, \ldots, N_{ds}, n = 1, \ldots, N_u \) are independent and uniformly distributed in \( \{+1, -1\} \) so that \( \mathbb{E}\{d_i^{(n)}\} = 0 \). The random spreading codes assumptions in (2) and (3) are conservative because better performance is expected by reducing the multiple access interference through code designs.

4. Without any network synchronization, \( \tau_m \) is uniformly distributed in \( [0, N_u T_f] \), and \( \tau_n \) and \( \tau_m \) are independent for \( n \neq m \).

5. Transmitted data bits, hopping sequences, direct sequences, and asynchronous delays are mutually independent. In addition, \( T_f \geq 3T_{mds} \) is assumed in a later derivation.

### 5.2 MAI and Gaussian Assumption

The received signal in (5.1) can be divided into three elements which are the desired transmitter’s signal \( s(t) = \sum_{i=-\infty}^{\infty} \xi_i^{(1)}(t) \), receiver noise \( n_t(u, t) \), and interfering transmitters’ signals \( n_m(u, t) = \sum_{n=2}^{N_u} \sum_{i=-\infty}^{\infty} \xi_i^{(n)}(t) \), and \( \tau_1 \) is assumed 0 without loss of generality because
Table 5.1: Signals and noise/interference contained in $D_s(i)$.

<table>
<thead>
<tr>
<th>$s_{ir}(t)$</th>
<th>$s_i$</th>
<th>$n_i(3)$</th>
<th>$n_i(6)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_{m(ir)}(u,t)$</td>
<td>$n_i(1)$</td>
<td>$n_i(4)$</td>
<td>$n_i(7)$</td>
</tr>
<tr>
<td>$n_{t(ir)}(u,t)$</td>
<td>$n_i(2)$</td>
<td>$n_i(5)$</td>
<td>$n_i(8)$</td>
</tr>
</tbody>
</table>

of the perfect synchronization to transmitter 1. Let $b^{(1)}_l$ and $D_s$ be the desired bit and the decision statistic, then

$$D_s = \sum_{i=1}^{(l+1)N_s-1} D_s(i) = \sum_{i=1}^{(l+1)N_s-1} \int_{T_l+c_t^{(1)}}^{T_l+c_t^{(1)}+T_{corr}} \tilde{r}(t) \times \tilde{\tau}(t-T^{(1)}_d) dt$$

$$= \sum_{i=1}^{(l+1)N_s-1} \left[ s_i + \sum_{j=1}^{8} n_i(j) \right] \frac{1}{\xi_i^{(1)}(t)} 0, \quad (5.3)$$

where $T_{corr}$ is the correlator integration time, and the signal $s_i$ and noise/interference $n_i(j)$ are explained in follows. By defining $I_{ir} = [iT_l + c_t^{(1)} T_c + T^{(1)}_{d} + T_{corr}]$ and $I_{id} = [iT_l + c_t^{(1)} T_c + T^{(1)}_{d} + T^{(1)}_f + T_{corr}]$, it is clear that the receiver noise and interfering transmitters’ signals can affect $D_s(i)$ if any portions of them are in $I_{ir}$ or $I_{id}$. Let $s_{ia}(t) = \xi_i^{(1)}(t)$, $n_{m(ia)}(u,t) = n_m(u,t)$ and $n_{t(ia)}(u,t) = n_t(u,t)$ for $t \in I_{ia}$ with $a=r, d$, and 0 elsewhere, then noise/interference $n_i(j), j = 1, \ldots, 8$, can be defined in Table 5.1. The first column and row in Table 5.1 contain the sources which can cause interference/noise in $D_s(i)$. The entry $t_{l,k}$, which is in the $l^{th}$ row and the $k^{th}$ column, is the result of the correlation operation of $t_{l,1}$ and $t_{1,k}$ for $l, k \geq 2$.

The transmitted bits from the interfering transmitter $n$ which may interfere $b^{(1)}_l$ are $b^{(n)}_{l-1}$ and $b^{(n)}_l$ because $\tau_n \in [0, N_s T_f)$. To simplify notation, the subscript of $g^{(n)}_k(t)$ is dropped in
the following analysis. This implies that the channel response for each user is essentially unchanged in the two-bit interval. Let $l$ and $i$ be zero without loss of generality and define

$$f_{n,m}(\alpha, \beta) \triangleq \int_0^{T_{\text{corr}}} \hat{g}^{(n)}(t + T^{(1)}_d + c^{(1)}_0 T - \tau_n + \alpha) \hat{g}^{(m)}(t + c^{(1)}_0 T - \tau_m + \beta) dt,$$

$$N_m(\alpha, \beta) \triangleq \int_0^{T_{\text{corr}}} \hat{g}^{(m)}(t + \alpha) \hat{n}(u, t + \beta) dt,$$

$$R_{1m}(\nu) \triangleq \int_0^{T_{\text{corr}}} \hat{g}^{(1)}(t) \hat{g}^{(m)}(t - \nu) dt.$$

The noise/interference variables in Table 5.1 are tabulated in Appendix C.1 in terms of the above quantities. We can see that $R_{1m}(\nu)$ is similar to a crosscorrelation function of $\hat{g}^{(1)}(t)$ and $\hat{g}^{(m)}(t)$, but the integration interval is $[0, T_{\text{corr}}]$ instead of $(-\infty$ to $\infty$). In addition, $R_{1m}(\nu) \neq 0$ for $\nu \in [-T_{\text{mds}}, T_{\text{corr}}] \in [-T_{\text{fl}}, T_{\text{fl}}]$ because $T_{\text{corr}}$ is less than or equal to the channel delay spread $T_{\text{mds}}$.

The channel responses for all transmitted signals are implicitly embedded in the received waveforms $g^{(n)}(t)$ for $1 \leq n \leq N_u$. Let $g^{(n)}$, a row vector, represent channel parameters determining $g^{(n)}(t)$, $h^{(n)} = [c^{(n)}_{-1}, c^{(n)}_0, \tau_n, g^{(n)}]$, and $h = [h^{(1)}, h^{(2)}, \ldots, h^{(N_u)}]$, it is easy to verify that the distributions of $n_0(2)$, $n_0(5)$, $n_0(6)$, and $n_0(7)$ conditioned on $h$ are Gaussian. By using the central limit theorem arguments, $n_0(8)$ is also approximately Gaussian distributed. A UWB signal can be modelled as a shot noise whose amplitude becomes Gaussian distributed as the path arrival rate goes to infinity (proof in [23]). Then the correlation of two shot noises under this circumstance again approaches a Gaussian random variable by using the central limit theorem. The arrival rate $\alpha'$ is proportional to the number of paths in the integration interval $N_{\text{hs}}$, $N_u$ and $N_s$, and is inverse proportional to $T_f$ with possibly different powers. The quantity $x = n_0(1) + n_0(3) + n_0(4)$ approaches
Figure 5.1: The pdf of $x = n_0(1) + n_0(3) + n_0(4)$ with $T_f = 1800.4$ nsec, $N_s = 1$, and $N_u = 50$.

As $\alpha'$ increases without doubt, but it might not be close to Gaussian when $\alpha'$ is not large enough.

The distributions of $x$ for different arrival rates are simulated with IEEE P802.15.3a channel model CM1, which is described in Section 2.2. Figure 5.1-5.3 show the simulated pdf of $x$ along with Laplace, Logistic [32, 33], and Gaussian pdfs with the arrival rate going from small to large. The simulation adopts equal energy channel realizations, $g_{rx}(t)$ in (2.2), $T_{corr} = 18$ ns, $T_{mds} = 60$ ns, $B_w = 4$ GHz, and the rule to assign $T_d^{(n)}$ and $N_h^{(n)}$ in (2.3) and (2.4). Both Laplace and Logistic distributions decay exponentially with $x$, but the Laplace distribution has a heavier tail than the Logistic. Both of them have a heavier tail than the Gaussian distribution which decays exponentially with $x^2$. Figures show that when the arrival rate increases, the tail of the pdf of $x$ becomes thinner, and the pdf does
Figure 5.2: The pdf of $x = n_0(1) + n_0(3) + n_0(4)$ with $T_f = 900.2$ nsec, $N_s = 2$, and $N_u = 50$.

Figure 5.3: The pdf of $x = n_0(1) + n_0(3) + n_0(4)$ with $T_f = 180.6$ nsec, $N_s = 10$, and $N_u = 50$. 

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approach a Gaussian distribution. This also means that for a fixed large enough signal-to-noise power ratio so that only tail responses are considered, the bit error probabilities from small to large are produced in turn by Gaussian, Logistic and Laplace distributions. Although it is not fair to model $x$ as a Gaussian random variable in all situations, there are some legitimate reasons to treat the total noise/interference as a Gaussian random variable in following two sections. One is that the desired BEP might not need a large signal-to-noise power ratio (SNR) to be achieved. When the SNR is not large enough, the effect of $x$ is smaller than other noises/interference which have Gaussian distributions. Another important reason being accepted by many researchers is that a useful closed-form performance analysis is possible under the Gaussian assumption.

5.3 MA performance analysis for $N_s = 1$

Even with the Gaussian MAI assumption which makes second-order statistics enough for the BEP analysis, the computation for the $N_s > 1$ case is still intractable. Therefore, the BEP of modulation with $N_s = 1$ is considered first in this section, then the result is generalized to the system with $N_s > 1$ by some arguments without further computation details in section 5.4. The BEP for $N_s = 1$ is investigated with the assumption that the detection is based on the hypothesis test of $b_0^{(1)}$. It can be verified that $\mathbb{E}\{n_0(j)\} = 0$ and $\mathbb{E}\{n_0(i)n_0(j)\} = 0$ for $i \neq j$ and $i, j \in \{1, \ldots, 8\}$ by using assumption (1) and (3) in section 5.1 and the fact that $\mathbb{E}\{n_t(u, t)\} = 0$. By defining $g = [g^{(1)}, g^{(2)}, \ldots, g^{(N_u)}]$ which includes all the channel information,

$$\mathbb{E}\{D_s(0)|b_0^{(1)}, g^{(1)}\} = s_0 = b_0^{(1)} \int_0^{T_{corr}} |\tilde{g}^{(1)}(t)|^2 dt,$$

(5.5)
\[ \text{Var}\{D_6(0)|g\} = \sum_{j=1}^{8} \text{Var}\{n_0(j)|g\}. \]  \\
(5.6)

And the BEP given \( g \) is

\[ P_{\text{bit}} = Q\left( \frac{|s_0|}{\sqrt{\text{Var}\{D_6(0)|g\}}} \right) \]  \\
(5.7)

due to the symmetry of the information bit and noise densities.

5.3.1 Variances of \( n_0(1), n_0(2), n_0(3), n_0(5), n_0(6), n_0(7) \) and \( n_0(8) \)

Table 5.1 shows that two noise/interference variables in symmetric positions of the diagonal happen for the same reason. Both \( n_i(2) \) and \( n_i(6) \) are the correlation result of the signal from transmitter 1 and the receiver noise. Signals from transmitter 1 and interfering transmitters’ signals produce \( n_i(1) \) and \( n_i(3) \). The interfering transmitters’ signals and the receiver noise generate \( n_i(5) \) and \( n_i(7) \). Thus the symmetric noise/interference variables have the same variance. One important observation that can simplify the variance computation by making integrals zero is that the only regions, \([0, T_{mds})\) and \([-T_{mds}, T_{corr})\), in which \( \tilde{g}^{(n)}(t) \) and \( R_{1n}(\tau) \) have nonzero values are not covered by the integration limits.

The details of obtaining \( \text{Var}\{n_0(1)\} \) and \( \text{Var}\{n_0(5)\} \) are written in Appendix C.2 and C.4, and the results are

\[ \text{Var}\{n_0(1)\} = \text{Var}\{n_0(3)\} = \frac{2}{T_1} \sum_{n=2}^{N_u} \int_{-\infty}^{\infty} R_{1n}(\tau_n) d\tau_n, \]  \\
(5.8)

\[ \text{Var}\{n_0(5)\} = \text{Var}\{n_0(7)\} = \frac{N_0 T_{\text{corr}}}{T_1} \sum_{n=2}^{N_u} \int_{-\infty}^{\infty} [\tilde{g}^{(n)}(\tau_n)]^2 d\tau_n. \]  \\
(5.9)
From now on, the condition $g$ of the variance is omitted to keep the notation simple. The variance of $n_0(1)$ and $n_0(3)$ is a function of $R_{1n}(\tau)$ because these variables are generated by the correlation of the signals from transmitter 1 and interfering transmitters’ signals.

The variance of $n_0(5)$ and $n_0(7)$ is a function of $\{\tilde{g}^{(n)}(t)\}_{n=2}^{N_0}$ because these variables are correlation results of signals from interfering transmitters and the receiver noise. The computation of $\text{Var}\{n_0(2)\}$ and $\text{Var}\{n_0(8)\}$ have been similarly detailed in Appendix A.4, and the results are

\[
\text{Var}\{n_0(2)\} = \text{Var}\{n_0(6)\} = \frac{N_0 \eta \tilde{E}_p}{2}, \quad (5.10)
\]

\[
\text{Var}\{n_0(8)\} = \frac{T_{\text{corr}} N_0^2 B_w}{2}. \quad (5.11)
\]

### 5.3.2 Variance of $n_0(4)$

The only variance left to be calculated is $\text{Var}\{n_0(4)\}$ which, as can be seen from the structure of $n_0(4)$ in the table in Appendix C.1, is the most complex one. Before the computation, one useful proposition is proposed here.

**Proposition 1** The contribution to $n_0(4)$ from an interfering transmitter does not change with or without using uniformly distributed hopping sequences in the system.

**Proof:** Suppose that the $i^{\text{th}}$ frame of user $n$ can produce interference to the detection of $b_0^{(1)}$, this $i$ actually depends on $[\tau_n/T_f]$, and is not specified here. To prove this proposition, it is enough to show that the probability $p_n$ such that the first non-zero point of a noiseless signal waveform in the $i^{\text{th}}$ frame from user $n$ falls into $[c_0^{(1)} T_c - T_{\text{mds}}, c_0^{(1)} T_c + T_{\text{corr}}]$ or $[c_0^{(1)} T_c + T_d^{(1)} - T_{\text{mds}}, c_0^{(1)} T_c + T_d^{(1)} + T_{\text{corr}}]$ does not change with or without using uniformly
distributed hopping sequences in the system. Without hopping sequences, the received
signal from transmitter $n$ in the $i^{th}$ frame is

$$s_{rx}^{(n)}(t) = d_{i}^{(n)}g_{i}^{(n)}(t - iT_{f} - \tau_{n}) + b_{i}^{(n)}\tilde{g}_{i}^{(n)}(t - iT_{f} - T_{d}^{(n)} - \tau_{n}),$$

which indicates that $p_{n}$ depends only on $\tau_{n}$, and

$$p_{n} = \frac{T_{mds} + T_{corr}}{T_{f}}$$

(5.12)

because $\tau_{n}$ modulo $T_{f}$ is uniformly distributed in $[0, T_{f})$. For the uniformly distributed
hopping sequence case, $s_{rx}^{(n)}(t) = \xi_{i}^{(n)}(t)$ in (5.2), we can assume that $c_{0}^{(1)} = N_{h}^{(1)} - 1$
without loss of generality. For $c_{i}^{(n)}$ taking any possible value with probability $\frac{1}{N_{h}^{(1)}}$, $\tau_{n}$ has
probability $\frac{T_{mds} + T_{corr}}{T_{f}}$ of being in $[c_{0}^{(1)}T_{c} - T_{mds}, c_{0}^{(1)}T_{c} + T_{corr}]$ or $[c_{0}^{(1)}T_{c} - T_{mds} + T_{d}^{(1)}, c_{0}^{(1)}T_{c} + T_{d}^{(1)} + T_{corr})$. Thus

$$p_{n} = \frac{T_{mds} + T_{corr}}{T_{f}} \times \frac{1}{N_{h}^{(n)}} \times N_{h}^{(n)} = \frac{T_{mds} + T_{corr}}{T_{f}}.$$  

(5.13)

Since (5.12) and (5.13) are equal, the proposition is proved.

In the remaining computation of $\text{Var}\{n_{0}(4)\}$, Proposition 1 is applied so $c_{j}^{(m)} = 0$ for all
$j, m$.

From the definition of $f_{n,m}(\alpha, \beta)$ in (5.4), the region for $t$ so that $f_{n,m}(\alpha, \beta) \neq 0$ and
the region for $t$ so that $f_{n,m}(\alpha + a, \beta + b) \neq 0$ do not overlap if $a \geq T_{mds}$ or $b \geq T_{mds}$. In
other words, $f_{n,m}(\alpha, \beta)f_{n,m}(\alpha + a, \beta + b) = 0$ for $a \geq T_{mds}$ or $b \geq T_{mds}$. Using this property
\[ \begin{align*}
\xi_1 & = f_{n,m,n',m'}(T_t, T_t, T_t, T_t) + b^{(n)} \cdot b^{(m)} \cdot b^{(n')} \cdot b^{(m')} f_{n,m,n',m'}(T_t - T_{d}^{(m)}, T_t, T_{d}^{(n)}, T_t - T_{d}^{(n')}),
& \quad + b^{(n)} \cdot b^{(m)} f_{n,m,n',m'}(T_t, T_t - T_d^{(m)}, T_t - T_d^{(n)}, T_t - T_d^{(n')}),
& \quad + b^{(n)} \cdot b^{(m)} f_{n,m,n',m'}(T_t, T_t, T_t - T_d^{(n)}, T_t - T_d^{(n')}),
& \quad + b^{(n)} \cdot b^{(m)} f_{n,m,n',m'}(T_t - T_d^{(n)}, T_t, T_t - T_d^{(n')}).
\end{align*} \]

\[ \begin{align*}
\xi_2 & = f_{n,m,n',m'}(T_t, 0, T_t, 0) + b^{(n)} b^{(m)} f_{n,m,n',m'}(T_t - T_d^{(m)}, -T_d^{(m)}, T_t, -T_d^{(m)}),
& \quad + b^{(n)} b^{(m)} f_{n,m,n',m'}(0, T_t, -T_d^{(m)}, -T_d^{(m)}),
\end{align*} \]

\[ \begin{align*}
\xi_3 & = f_{n,m,n',m'}(0, 0, 0, 0) + b^{(n)} b^{(m)} f_{n,m,n',m'}(-T_d^{(n)}, -T_d^{(n)}, -T_d^{(m)}),
\end{align*} \]

\[ \begin{align*}
\xi_4 & = f_{n,m,n',m'}(0, 0, 0, 0) + b^{(n)} b^{(m)} f_{n,m,n',m'}(0, -T_d^{(m)}, 0, -T_d^{(m)}),
\end{align*} \]

<table>
<thead>
<tr>
<th>Table 5.2: Definitions of ( \xi_1, \xi_2, \xi_3, ) and ( \xi_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \xi_1 = \sum_{n=2}^{N_u} \sum_{m=2}^{N_0} \sum_{n'=2}^{N_u} \sum_{m'=2}^{N_0} \psi_{d-1}^{(n)} d_{d-1}^{(m)} d_{d-1}^{(n')} d_{d-1}^{(m')} \xi_1 + d_{d-1}^{(n)} d_{d-1}^{(m)} d_{d-1}^{(n')} d_{d-1}^{(m')} \xi_2 ) (5.14)</td>
</tr>
<tr>
<td>( \xi_2 = \sum_{n=2}^{N_u} \sum_{m=2}^{N_0} \sum_{n'=2}^{N_u} \sum_{m'=2}^{N_0} \psi_{d-1}^{(n)} d_{d-1}^{(m)} d_{d-1}^{(n')} d_{d-1}^{(m')} \xi_3 + d_{d-1}^{(n)} d_{d-1}^{(m)} d_{d-1}^{(n')} d_{d-1}^{(m')} \xi_4 )</td>
</tr>
</tbody>
</table>

where \( \xi_1, \xi_2, \xi_3, \) and \( \xi_4 \) are defined in Table 5.2, and

\[ f_{n,m,n',m'}(\alpha, \beta, \gamma, \lambda) = f_n m(\alpha, \beta) f_{n',m'}(\gamma, \lambda). \]

Substituting \( c_0^{(1)} = 0 \) into (5.4), \( f_{n,m}(\alpha, \beta) = 0 \) for \( \alpha \geq T_t \) or \( \beta \geq T_t + T_{mds} \) by considering the nonzero regions of \( \tilde{g}^{(n)}(t) \) and \( \tilde{g}^{(m)}(t) \). Therefore, \( f_{n,m,n',m'}(\alpha, \beta, \gamma, \lambda) = 0 \) if \( \alpha \geq T_t \) or \( \beta \geq T_t + T_{mds} \) or \( \gamma \geq T_t \) or \( \lambda \geq T_t + T_{mds} \). Because of the independence of each direct sequence (see assumption (3) in section 5.1), \( \text{Var} \{ n_0(4) \} \neq 0 \) only happens when one of the following four conditions occurs.

**Case 1: \( n = m = n' = m' \)**

It is true that \( f_{n,n}(\alpha, \beta) = f_{n,n,n}(\alpha, \beta, \gamma, \lambda) = 0 \) for \( |T_{d}^{(1)} + \alpha - \beta| \geq T_{mds} \) because the
regions in which \( \tilde{g}^{(n)}(t + T^{(1)}_{d} - \tau_{n} + \alpha) \neq 0 \) and \( \tilde{g}^{(n)}(t - \tau_{n} + \beta) \neq 0 \) do not overlap. Thus \( \text{Var}\{n_0(4)\} \) is reduced to (5.15) where the expectation is implicitly over \( \tau_n \). Using a change of variables which sets \( \tau'_n \) to \( T_l - \tau_n \) and \( -\tau_n \) in the first and second terms in the braces in (5.15) as well as the equality \( \int_{a}^{b} f(t)dt = \int_{a}^{c} f(t)dt + \int_{c}^{b} f(t)dt \), the variance reduces to (5.16), which is equivalent to (5.17) because \( \tilde{g}^{(n)}(t) \neq 0 \) only for \( 0 \leq t \leq T_{mds} \).

\[
\text{Var}\{n_0(4)\} = \sum_{n=2}^{N_0} E \left\{ f_{n,n,n,n}(T_l - T^{(n)}_{d}, T_l, T_l - T^{(n)}_{d}, T_l) \right. \\
+ f_{n,n,n,n}(-T^{(n)}_{d}, 0, -T^{(n)}_{d}, 0) \right\} \\
= \frac{1}{T_l} \sum_{n=2}^{N_0} \int_{-T_l}^{T_l} \int_{-T_l}^{T_l} \left[ \int_{-T_l}^{T_l} \tilde{g}^{(n)}(t + T^{(1)}_{d} - T^{(n)}_{d} + \tau'_n)\tilde{g}^{(n)}(t + \tau'_n)dt \right]^2 d\tau'_n (5.15) \\
= \frac{1}{T_l} \sum_{n=2}^{N_0} \int_{-T_{mds}}^{T_{mds}} \left[ \int_{-T_{mds}}^{T_{mds}} \tilde{g}^{(n)}(t)\tilde{g}^{(n)}(t + T^{(1)}_{d} - T^{(n)}_{d})dt \right]^2 d\tau'_n. (5.16)
\]

**Case 2: \( n = n' \) and \( m = m' \), but \( n \neq m \)**

Substituting \( n \) and \( m \) for \( n' \) and \( m' \), using a change of variables to remove \( T_l \) as well as invert the sign of \( \tau_n \) and \( \tau_m \) in \( f_{n,m,n,m}(\cdot) \), and again utilizing \( \int_{a}^{b} f(t)dt = \int_{a}^{c} f(t)dt + \int_{c}^{b} f(t)dt \), the variance of \( n_4(0) \) is now derived from (5.14) to (5.18). Then by interchanging integrals to integrate over \( \tau_n \) and \( \tau_m \) first, and noticing that these integrals cover whole regions in which \( \tilde{g}^{(n)}(\cdot) \) and \( \tilde{g}^{(m)}(\cdot) \) are not zero, (5.18) is simplified to (5.19).

\[
\text{Var}\{n_0(4)\} = \frac{1}{T_l} \sum_{n=2}^{N_0} \sum_{m=2, m \neq n}^{N_0} \int_{-T_l}^{T_l} \int_{-T_l}^{T_l} \int_{-T_{mds}}^{T_{mds}} \left[ \vartheta_1(t, v, \tau_n, \tau_m) + \vartheta_2(t, v, \tau_n, \tau_m) \right] \int_{0}^{T_{cor}^{m}} C_n(t - v)C_m(t - v)dt \, dv \, d\tau_n \, d\tau_m (5.18)
\]

\[
= \frac{4}{T_l} \sum_{n=2}^{N_0} \sum_{m=2, m \neq n}^{N_0} \int_{0}^{T_{cor}^{m}} C_n(t - v)C_m(t - v)dv, \quad (5.19)
\]
where \( C_n(\tau) = \int_0^{T_{mds}} \bar{g}^{(n)}(t) \tilde{g}^{(n)}(t - \tau) dt \), and \( \vartheta_1, \vartheta_2, \vartheta_3 \) and \( \vartheta_4 \) are defined in Table 5.3.

**Case 3: \( n = m \) and \( n' = m' \), but \( n \neq n' \)**

Substituting \( n \) and \( n' \) for \( m \) and \( m' \) and utilizing that \( f_{n,n}(\alpha, \beta) = 0 \) for \( |T_{d1}^{(1)} + \alpha - \beta| \geq T_{mds} \), (5.14) is reduced to (5.20) where the expectation is implicitly over \( \tau_n \) and \( \tau_{n'} \), which equals zero by using assumption (1) and (5) in section 5.1.

\[
\text{Var}\{n_0(4)\} = \sum_{n=2}^{N_u} \sum_{n'=2}^{N_u} \sum_{n'\neq n} \mathbb{E} \left\{ b_{-1}^{(n)} b_{-1}^{(n')} f_{n,n,n',n'}(T_f - T_{d1}^{(n)}, T_f, T_f - T_{d1}^{(n')}, T_f) \right. \\
+ b_{0}^{(n)} b_{0}^{(n')} f_{n,n,n',n'}(-T_{d1}^{(n)}, 0, -T_{d1}^{(n')}, 0) \left. \right\} \\
= 0. 
\] (5.21)

**Case 4: \( n = m' \) and \( m = n' \), but \( n \neq m \)**

By using assumption (1), (3), and (5) in section 5.1, and \( f_{n,m}(\alpha, \beta) = 0 \) for any \( n, m \) if \( \alpha \geq T_f \), (5.14) is reduced to

\[
\text{Var}\{n_0(4)\} = \sum_{n=2}^{N_u} \sum_{m=2}^{N_u} \sum_{m\neq n} \mathbb{E} \left\{ f_{n,m,m,n}(T_f - T_{d1}^{(n)}, T_f - T_{d1}^{(m)}, T_f - T_{d1}^{(m')}, T_f - T_{d1}^{(n)}) \right. \\
+ f_{n,m,m,n}(0, 0, 0, 0) + f_{n,m,m,n}(-T_{d1}^{(n)}, -T_{d1}^{(m)}, -T_{d1}^{(m')}, -T_{d1}^{(n)}) \left. \right\} \\
= 0,
\]

where the expectation is implicitly over \( \tau_n \) and \( \tau_m \), and the zero is the result of applying the claim in Appendix C.3.

Summarizing above four cases,

\[
\text{Var}\{n_0(4)\} = V_{self} + V_{co},
\]

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The correlator outputs of these consecutive pairs of pulses (5.4 MA performance analysis for transmitters.

The assumptions and arguments which are going to help this generalization process are

The MA performance of a TR system with one information bit conveyed by transmitter, and $V_{co}$ equal to (5.19) represents the correlation of signals from any two interfering transmitters.

5.4 MA performance analysis for $N_s > 1$

The MA performance of a TR system with one information bit conveyed by $N_s > 1$ consecutive pairs of pulses ($N_s$ frames) can be generalized from the result of $N_s = 1$ case.

The correlator outputs of these $N_s$ pairs are combined coherently to increase the received signal energy from $\int_0^{T_{corr}} [\hat{g}^{(1)}(t)]^2 dt$ to $\int_0^{T_{corr}} [\hat{g}^{(1)}(t)]^2 dt$. The eight noise/interference terms which affect the performance of detecting $b_0^{(1)}$ are now $n(j) = \sum_{n=0}^{N_s-1} n_i(j)$ for $j = 1, \ldots, 8$ with $n_i(j)$ depicted in Table 5.1, and

$$\text{Var} \left\{ \sum_{j=1}^{8} n(j) \right\} = \sum_{j=1}^{8} \mathbb{E}\{n^2(j)\} + \sum_{j=1}^{8} \sum_{i=1 \atop i \neq j}^{8} \mathbb{E}\{n(i)n(j)\}. \quad (5.22)$$

The assumptions and arguments which are going to help this generalization process are

(i) $N_s$ is assumed even and less than $N_{hs}$ and $N_{ds}$. Therefore, $\mathbb{E}\{d_i^{(n)}d_l^{(n)}\} = 0$ for $i \neq l$, and $\mathbb{E}\{d_i^{(n)}d_l^{(m)}\} = 0$ for all $i, l$ if $n \neq m$ in the computation of the noise variance.
(ii) At most two information bits from every interfering transmitter can affect $b_0^{(1)}$, and the values of these two bits are assumed the same. Two reverse bits can cancel part of the interference produced by the correlation of signals from each individual interfering transmitter. This assumption implies the interference is overestimated which gives us an upper bound of the bit error probability.

(iii) For any two random variables with the same variance, their covariance is less than or equal to their variance.

In addition, because $n_{t(ii)}(u,t)$ and $n_{t(id)}(u,t)$ are uncorrelated for all $i, l$, and $n_{t(ia)}(u,t)$ and $n_{t(la)}(u,t)$ are uncorrelated for $i \neq l$ with $a=r,d$, it can be verified that

$$
\mathbb{E}\{n^2(j)\} = N_s \text{Var}\{n_0(j)\} \text{ for } j = 1, 2, 3, 5, 6, 7, 8,
$$

and the covariance

$$
\mathbb{E}\{n(j)n(l)\} = 0 \text{ for } j, l = 1, 2, \ldots, 8 \text{ and } j \neq l.
$$

The only term left to complete the calculation of (5.22) is

$$
\mathbb{E}\{n^2(4)\} = \sum_{i=0}^{N_s-1} \mathbb{E}\{n_i^2(4)\} + \sum_{i=0}^{N_s-1} \sum_{l=0 \atop l \neq i}^{N_s-1} \mathbb{E}\{n_i(4)n_l(4)\}.
$$

The first sum in (5.23) is equal to $N_s \text{Var}\{n_0(4)\}$. With regard to the double sum, it has already been mentioned that $n_i(4)$ is composed of the correlation $x_{i,j}$ of signals in frame $i$ from each interfering transmitter $j$, and the correlation $y_{i,j,k}$ of signals from any two
interfering transmitters $j$ and $k$ in frame $i$. Let $x_i = \sum_{j=2}^{N_u} x_{i,j}$ and $y_i = \sum_{j,k=2}^{N_u} y_{i,j,k}$, for $i \neq l$.

$$\mathbb{E}\{n_i(4)n_l(4)\} = \mathbb{E}\{(x_i + y_i)(x_l + y_l)\} = \mathbb{E}\{x_i x_l\} + \mathbb{E}\{x_i y_l\} + \mathbb{E}\{x_l y_i\} + \mathbb{E}\{y_i y_l\}. \quad (5.24)$$

The direct sequence components still exist in $y_{i,j,k}$ so that $\mathbb{E}\{x_i y_l\} = \mathbb{E}\{x_l y_i\} = \mathbb{E}\{y_i y_l\} = 0$ by using assumption (3) in section 5.1. But the direct sequence components in $x_{i,j}$ might be removed due to the correlation of signals from the same transmitter which could make $\mathbb{E}\{x_i x_l\} \neq 0$. For user $j$, $x_{i,j}$ could be zero if its signals do not fall in both $I_{it}$ and $I_{id}$. In order to make $\mathbb{E}\{x_{i,j} x_{l,j}\} \neq 0$, signals from user $j$ have to be in $I_{it}$ and $I_{id}$ as well as $I_{tr}$ and $I_{ld}$. Given that $x_{i,j} \neq 0$, it follows that $x_{l,j} \neq 0$ with probability $\frac{1}{N_{ij}h}$, which is decided by the hopping sequence of transmitter $j$. By considering this probability, argument (iii), and $\mathbb{E}\{x_i^2\} = V_{self}$, (5.23) results in $\text{Var}\{n(4)\} \leq \Psi'$ with

$$\Psi' = \sum_{n=2}^{N_u} \frac{N_{n}^{(n)} N_s + N_s(N_s - 1)}{T_{f} N_{n}^{(n)}} \int_{-T_{corr}}^{T_{corr}} \int_{-T_{corr}}^{T_{corr} + T_n} \tilde{g}^{(n)}(t) \tilde{g}^{(n)}(t + T_d^{(1)} - T_d^{(n)}) dt \, dT_n \right)^2 + \frac{4 N_s}{T_f^2} \sum_{n=2}^{N_u} \sum_{m=2}^{N_u} \int_{0}^{T_{corr}} \int_{0}^{T_{corr}} C_n(t - v) C_m(t - v) dt \, dv. \quad (5.25)$$

Summarizing the results of above arguments, the upper bound of the $\text{Var}\{D_s\}$ conditioned on channel realizations of all transmitters is

$$\text{Var}\{D_s\} \leq \Psi = 2 N_s \left[ \text{Var}\{n_0(1)\} + \text{Var}\{n_0(2)\} + \text{Var}\{n_0(5)\} \right] + \Psi' + N_s \text{Var}\{n_0(8)\}, \quad (5.26)$$
where \( \text{Var}\{n_0(1)\}, \text{Var}\{n_0(2)\}, \text{Var}\{n_0(5)\}, \Psi' \) and \( \text{Var}\{n_0(8)\} \) are in (5.8), (5.10), (5.9), (5.25) and (5.11), respectively. Equation (5.26) is also suitable for \( N_s = 1 \) but with \( \text{Var}\{D_s\} = \Psi \). The upper bound of the conditional BEP of a squared weighted cross-correlation receiver in a multiple access multipath environment is

\[
P_{\text{bit}} \leq Q \left( \frac{N_s \int_{0}^{T_{\text{corr}}} [\hat{g}^{(1)}(t)]^2 dt}{\sqrt{\Psi}} \right). \tag{5.27}
\]

### 5.5 Numerical Examples and Discussions

The numerical examples of the BEP in a multipath environment described by IEEE 802.15.3a model CM1 have been calculated using (5.27). The channel realizations of each user with 60 ns channel delay spread are assumed independent from each other. A single path’s received pulse is depicted in (2.2), frame times are always chosen to be integer multiples of the single received pulse duration 0.7 ns, and the receiver bandwidth \( B_w = 4\text{GHz} \) which contains 97% signal power. The integration interval \( (T_{\text{corr}}) \), which equals 18 ns, is the optimal value for the single user case with these channel realizations, and may not be optimal for the multiple access situation.

Both theoretical and simulation results are shown for parameters with different values. Figure 5.4-5.8 are results with perfect power control, e.g., \( \int_{-\infty}^{\infty} [g_i^{(n)}(t)]^2 dt \) are the same for all \( i, n \). For \( T_f = 180.6 \text{ ns} \) and \( N_s = 10 \), as the number of users increases, additional signal energy needed to achieve the same BEP increases, and the performance floor due to the MAI also appears in Figure 5.4. On the other hand, the MAI Gaussian assumption is more accurate for large \( N_u \). For \( N_u = 50 \), theoretical and simulation results match well.
Since the theoretical result is an upper bound for \( N_s > 1 \), this perfect match indicates the tightness of this bound. Even if the Gaussian MAI assumption is not accurate when \( N_u \) is small, the theoretical analysis can still predict the BEP well when the MAI does not dominate the performance which is shown in the \( N_u = 30, 40 \) curves.

Figure 5.4 also shows the effects of roughly doubling \( N_u \) and \( T_f \) together. The conclusion that \( N_u \) and \( T_f \) affect the MAI equally are both observed in this figure and in equation (5.26) which shows that the MAI is proportional and inverse proportional to \( N_u \) and \( T_f \) with the same power. Figure 5.5 shows the effects of \( T_f \) and \( N_s \) on the performance with \( N_u = 50 \). For a specific bit energy, the MAI decreases even with \( T_f \) and \( N_s \) increasing at the same rate, which is not obvious in (5.26). But this conclusion does be supported by the numerical data, which reveals that the largest interference comes from the correlation of signals from two interfering transmitters. Figure 5.6 shows that increasing \( T_f \) decreases the MAI and improves the BEP with \( N_u \) and \( N_s \) fixed.

Figure 5.7 shows the effect of \( N_s \) while \( T_f \) and \( N_u \) are fixed. When \( E_b/N_0 \) is small, i.e., the receiver noise dominates the BEP, concentrating energy of one bit in a frame performs better than separating the energy into more than one frame because the effect of the noise \( \times \) noise term is substantial when the SNR per pulse is low. When \( E_b/N_0 \) increases, the MAI is the primary cause of bit errors, and transmitting the same information repeatedly can decrease the probability that enough collisions happen to cause errors.

Figure 5.8 compares different combinations of \( N_s \) and \( T_f \) with a fixed data rate, which is different from the case in Figure 5.7. For a specific bit energy, it shows that concentrating the bit energy in a pulse has better performance than distributing the energy to more
than one pulse due to the nonlinear property of the TR method with a cross-correlation receiver. This can be explained by using $\text{Var}\{n(5)\}$ as an example,

$$\frac{\text{Var}\{n(5)\}}{\left[ \mathbb{E}\{D_s\} \right]^2} = \frac{N_s N_0 T_{corr} \sum_{n=2}^{N_u} \int_{-\infty}^{\infty} [\tilde{g}(n)(\tau_n)]^2 d\tau_n}{T_f \left[ N_s \int_{0}^{T_{corr}} [\tilde{g}(1)(\tau_n)]^2 d\tau_n \right]^2}. \quad (5.28)$$

Two scenarios, in which the bit energy (also the average power) remains the same, are considered. In the first scenario, one bit is repeated $x > 1$ times with $T_f = y$ and the received pulse energy from transmitter $n$ equaling $z_n$. The ratio in (5.28) by substituting above numbers into parameters is

$$\frac{\text{Var}\{n(5)\}}{\left[ \mathbb{E}\{D_s\} \right]^2} = \frac{x N_0 T_{corr} \sum_{n=2}^{N_u} z_n}{y (x \eta z_1)^2}. \quad (5.29)$$

In the second scenario, $N_s = 1$, $T_f$ and the pulse energy from all the users are increased $x$ times to maintain the same data rate and average power. The ratio in (5.28) is now

$$\frac{\text{Var}\{n(5)\}}{\left[ \mathbb{E}\{D_s\} \right]^2} = \frac{N_0 T_{corr} \sum_{n=2}^{N_u} x z_n}{x y (x \eta z_1)^2}, \quad (5.30)$$

which is $x$ times less than the ration in (5.29). This nonlinearity also happens to $n(4)$, $n(7)$, and $n(8)$. Without violating the FCC regulation, the nonlinear receiver property tells us to concentrate the bit energy in only few pulses and extend the frame time to maintain the same average power.

Figure 5.9 shows the BEP without power control. Interfering transmitters are uniformly distributed in a ring, which is composed of circles with radii 1 meter and 4 meters. The receiver is at the center of the ring. The power of signals from each transmitter
Figure 5.4: BEP for different values of $N_u$ and $T_f$ with $N_s = 10$. GA denotes the analytical results exploiting MAI Gaussian assumption.

is proportional to $r^{-2}$ where $r$ is the distance between that particular transmitter and the receiver, and the power of signals from transmitter 1 is set to the average power of signals from interfering transmitters. For $N_u = 30, 40$ and $50$, with the average power normalized to 1, the maximum power to the minimum power ratios are $\frac{2.7396}{0.2204} = 12.432$, $\frac{2.6861}{0.2161} = 12.432$, and $\frac{2.8088}{0.2259} = 12.432$; and the standard deviations are 0.7214, 0.7403, and 0.7356. The performance of the systems without power control is apparently worse than that with perfect power control, and the Gaussian assumption of the MAI is less accurate in this non-power-control situation, which can be observed by comparing Figure 5.4 and 5.9.
Figure 5.5: BEP for \((N_s, T_f(\text{ns})) = (10, 180.6), (13, 240.1), (17, 300.3), \) and \((20, 360.5)\) with \(N_u = 50\). GA denotes the analytical results exploiting MAI Gaussian assumption.

Figure 5.6: BEP for different values of \(T_f\) with \(N_u = 50\) and \(N_s = 10\). GA denotes the analytical results exploiting MAI Gaussian assumption.
Figure 5.7: BEP for \( N_s = 1, 5, 10 \), \( T_f = 180.6\) ns, and \( N_u = 50 \). GA denotes the analytical results exploiting MAI Gaussian assumption.

Figure 5.8: BEP for a fixed data rate with different combinations of \( T_f \) and \( N_s \) with \( N_u = 50 \). GA denotes the analytical results exploiting MAI Gaussian assumption.
Figure 5.9: BEP for $N_u = 30, 40, 50$, $T_f = 180.6$ ns, $N_s = 10$ without power control. GA denotes the analytical results exploiting MAI Gaussian assumption.
Multiple Access UWB Differential TR Systems

6.1 Differential TR Modulation

A UWB DTR system uses a prior data-bearing waveform as a reference to increase the power efficiency over that of a conventional TR system. To do so, the transmitter includes an encoder which differentially encodes the information data bits before an antipodal modulation. Therefore the information is buried in the sign difference of two pulses in consecutive frames. The signal of user $n$ with DTR modulation is

$$s_{tr}^{(n)}(t) = \sum_{i=-\infty}^{\infty} q_i^{(n)} g_{\text{tr}}(t - iT_t - c_i^{(n)} T_c), \quad (6.1)$$

where $q_i^{(n)} = q_{i-1}^{(n)} b_{[i/N_s]}^{(n)}$ is the encoded bit, and $b_{[i/N_s]}^{(n)} \in \{+1, -1\}$ is the information bit transmitted in the $i^{th}$ frame of user $n$. Like the UWB TR system, each bit is transmitted in $N_s$ successive frames to achieve an adequate bit energy in the receiver. All other symbols have been defined in Chapter 2. Because no extra reference signals are imbedded in each frame, all the power is spent on transmitting data, thereby increasing the power efficiency.
Unlike the TR system, the number $N_h$ of hopping time slots in each frame is the same for all users. In addition, the frame time which is needed to prevent the interframe interference is simply that $T_f = (N_h - 1)T_c + T_p + T_{mds}$. An example of transmitted and received signals of transmitter $n$ is plotted in Figure 6.1.

### 6.2 Multiple Access Performance

The MA performance can be analyzed by using the approach in Chapter 5 except that Proposition 1 is not required (it is also not true in this DTR modulation case). Assumption (1), (2), (4) and (5) in Section 5.1 are still adopted here, and assumption (1) also indicates that $q_i^{(n)}$ and $q_j^{(m)}$ are independent for all $i, j$ if $n \neq m$. The decision is based on the hypothesis testing of $b_i^{(1)}$ and the assumption of perfect synchronization. The decision variable $D_d$ is given by

$$D_d = \sum_{i=1}^{(l+1)N_s-1} D_d(i) = \sum_{i=1}^{(l+1)N_s-1} \int_{iT_f + c_i^{(1)}T_c + T_{corr}}^{iT_f + c_i^{(1)}T_c} \hat{r}(t) \times \hat{r}(t - T_f + (c_i^{(1)} - c_{i-1}^{(1)})T_c) dt$$

where $T_{corr}$ is the correlator integration time. By utilizing the Gaussian MAI assumption, the bit error probability conditioned on channel realizations of all transmitters is

$$P_{bit} = Q\left(\frac{N_s \int_0^{T_{corr}} [\hat{g}^{(1)}(t)]^2 dt}{\sqrt{\text{Var}\{D_d\}}}\right).$$

(6.3)
with

\[
\text{Var}\{D_d\} \leq \frac{[T_f(3N_s - 2) + 2N_s(N_s - 2)]I_{\{N_s > 1\}} + 2T_fI_{\{N_s = 1\}}}{T_f^2} \sum_{n=2}^{N_s} \int_{-\infty}^{\infty} R_n^2(\tau_n)d\tau_n (6.4)
\]

\[
+ (2N_s - 1)N_0 \int_0^{T_{corr}} [g_0^{(1)}(t)]^2 dt + \frac{N_sT_{corr}N_0^2B_w}{2}
\]

\[
+ \frac{N_0T_{corr}(N_sT_f + N_s - 1)}{T_f^2} \sum_{n=2}^{N_s} \sum_{x=-(N_h-1)}^{N_h-1} \sum_{y=-(N_h-1)}^{N_h-1} (N_h - |x|)(N_h - |y|) \times
\]

\[
\left\{ \int_{-T_{corr}}^{T_{corr}} \int_{-\infty}^{\infty} \tilde{g}^{(n)}(t)\tilde{g}^{(n)}(t + (y - x)T_c)dt \right\}^2 d\tau
\]

\[
+ \frac{1}{2} \int_{-T_{corr}}^{T_{corr}} \int_{-\infty}^{\infty} \tilde{g}^{(n)}(t)\tilde{g}^{(n)}(t + T_f + (y - x)T_c)dt \right\}^2 d\tau
\]

\[
+ \frac{N_sT_f + N_s(N_s - 1)}{T_f^3} \sum_{n=2}^{N_s} \sum_{m=2}^{N_s} \sum_{m \neq n} \int_{-T_{corr}}^{T_{corr}} \int_0^{T_{corr}} C_n(t - v)C_m(t - v)dtdv,
\]

where \(I_{\{\cdot\}}\) is an indicator function which is equal to 1 if the condition in braces is true, and otherwise is 0.

Figure 6.1: An example of transmitted and received signals of transmitter \(n\) with \(b_0^{(n)} = 1, c_0^{(n)} = 0, c_1^{(n)} = 6, c_{N_h-1}^{(n)} = 3,\) and \(q_{-1}^{(n)} = 1.\) The letter D indicates data-modulated pulses.
6.3 Receiver Complexity and Performance Comparisons of TR and DTR Modulation

It is without question that $N_u$, $T_l$, $N_u$ and power control affect the MA performance the same way as in the TR modulated system. Therefore, the detailed performance curves for DTR modulation, which are similar to those in Figure 5.4-5.9 as well as the simulation results which validate the Gaussian MAI assumption, are not shown here. The discussion of the nonlinear property of a TR receiver in Section 5.5 also applies here. Figure 6.2, which has the same environment and system parameters as Figure 5.4, compares the bit error probability of UWB systems with TR and DTR modulation. For the single user case, DTR modulation outperforms TR modulation with 2.6dB gain. That is, for a fixed $E_b/N_0$, each pulse in a DTR modulated system has twice the energy of a conventional TR modulated system, which gives the DTR modulated system a 3dB advantage. The minor difference of 0.4dB results from the correlated noise in the correlator outputs in $N_s > 1$ frames. The DTR modulated system can also roughly double the number of active transmitters compared to the TR modulated system because only one pulse is transmitted in a frame, thereby reducing the probability of collision to a half.

Although a DTR modulated system outperforms a TR modulated system, we should keep in mind that any cross-correlation receiver for a TR system (expect the average cross-correlation receiver) needs a correlator with a fixed delay mechanism, but any cross-correlation receiver for a DTR system needs a correlator with a variable delay mechanism to accommodate time hopping. Hence the correlator for a DTR system with time hopping has higher complexity than that for a conventional TR system.
Figure 6.2: BEP versus $\frac{E_b}{N_0}$ for TR and DTR modulation with $T_l = 180.6$ ns, $T_{mds} = 60$ ns, $T_{corr} = 18$ ns, $N_s = 10$, and $B_w = 4$ GHz.
Chapter 7

Generalized Multiple Access UWB TR Systems

7.1 A Generalized Multiple Access TR Signal Model

The transmitted signal of one user in this multiple access binary or $M$-ary TR modulation system generally can be expressed as

$$s_{tr}(t) = \sum_{i=-\infty}^{\infty} d_{[i/N_a]} q_{[i/N_a, \text{mod}(i,N_a)}}$$

$$\times g_{tr} \left( t - \left[ \frac{i}{N_a} \right] T_l - \text{mod}(i, N_a) T_d - c_{[i/N_a]} T_c \right), \quad (7.1)$$

where

- $q_{jl}$ is the code symbol of the $l$th pulse in the $j$th frame which depend on the data being transmitted
- $\{c_j\} \in \{0,1,\ldots,N_h - 1\}$ and $\{d_j\} \in \{+1,-1\}$ are periodic pseudo-random hopping and direct sequences which help avoiding catastrophic collisions and smooth spikes in the power spectrum by increasing the period of the transmitted data,
- $\text{mod}(y,z)$ is $y$ modulo $z$, 

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\( N_a = N_d + N_r \) is the number of pulses in one frame which includes \( N_d \) data-modulated pulses and \( N_r \) reference pulses,

\( a \triangleq N_d / N_r \) is an integer,

\( \lfloor x \rfloor \) is the integer part of \( x \),

\( T_f \) is the frame duration (so the average repetition duration for each pulse is \( T_f / N_a \)),

\( T_d \) is the time separation between two adjacent pulses in one frame,

\( T_c \) is the time slot duration.

In (7.1), the user indicator is not shown because only single user performance is analyzed in this chapter. For a multiple user system, \( \{c_j\}, \{d_j\}, T_d \) and \( \{q_{j,l}\} \) are all user dependent.

Generally speaking, \( \{q_{j,l}\} \) are not necessarily binary, and represent a pulse amplitude modulation (PAM). Even for \( \{q_{j,l}\} \in \{+1, -1\} \), the system described in (7.1) can still be \( M \)-ary with \( M \geq 2 \), depending on how we correspond information bits to code symbols.

In addition, the values of \( N_a, N_r, \) and \( N_d \) can be different for each user if the system is a variable transmission rate system. Pulses in the same frame have the same time shift and are multiplied by the same direct sequence code symbol. Thus the number of pulses in one frame affects the probability of collisions. A signal example is shown in Figure 7.1.

![Figure 7.1](image-url)

**Figure 7.1:** An example of the generalized TR method with \( N_r = 2, N_d = 4, c_0 = 0, c_1 = 2, d_0 = 1, \) and \( d_1 = 1 \). The letter R indicates a reference pulse.
After going through a multipath channel, the received signal from one user with perfect synchronization is

\[ r(t) = \sum_{i=-\infty}^{\infty} d_{[i/N_a]} q_{[i/N_a], \text{mod}(i, N_a)} g \left( T - \left\lfloor \frac{i}{N_a} \right\rfloor T_f - \text{mod}(i, N_a) T_d - c_{[i/N_a]} T_c \right) + n(u, t), \]  

(7.2)

where \( g(t) \) being a received waveform is the convolution of \( g_{\text{tr}}(t) \) and the channel impulse response.

### 7.2 Binary TR Systems

#### 7.2.1 System Structure

In this binary system using the differential encoder, each bit along with the current state decide the corresponding coded symbols \( q_{l,k} \). The repetition number of each bit is an integer multiple \( (M_b) \) of \( N_r \) in order to simplify the receiver structure. Either \( a \) is assumed an integer multiple of \( M_b \) or \( M_b \) is an integer multiple of \( a \) in this section. This is not a necessary condition in the implementation, but simplifies the BEP analysis. If \( a \) and \( M_b \) are two arbitrary numbers, the error probability of each bit could be different. For \( M_b \geq a \), each bit is conveyed in \( M_b/a \) frames, otherwise, each frame contains \( a/M_b \) bits.

The definition of code symbols \( q_{l,k} \) in this binary system is

\[
q_{l,k} = \begin{cases} 
1 & \text{mod}(k, a + 1) = 0 \\
q_{l,k-1} \times b_{\left\lfloor \frac{la}{M_b} \right\rfloor + \zeta} & \text{otherwise,}
\end{cases}
\]
where
\[
\zeta = \begin{cases} 
0 & M_b \geq a \\
\left\lfloor \frac{\text{mod}(k,a+1)-1}{M_b} \right\rfloor & M_b < a.
\end{cases}
\]

Without the direct sequence, the amplitude of the pulse whose position is an integer multiple of \(a+1\) in each frame is always equal to one, and this pulse serves as a reference pulse. The direct sequence \(\{d_j\}\) converts the polarities of pulses in one frame together, so the cross-correlation receiver still applies here. Note that the minimum value of \(a\) is one, namely the maximum number of reference pulses in one frame (or bit) is equal to the number of data-modulated pulses, and half of the energy is spent on transmitting reference pulses. When \(N_r = N_d = 1\), it represents conventional TR systems. By using Figure 7.1 and \(M_b = 1\) as an example, \((q_{0,0}, q_{0,1}, q_{0,2}, q_{0,3}, q_{0,4}, q_{0,5}) = (1, b_0, b_0b_1, 1, b_0, b_0b_1)\) and \((q_{1,0}, q_{1,1}, q_{1,2}, q_{1,3}, q_{1,4}, q_{1,5}) = (1, b_2, b_2b_3, 1, b_2, b_2b_3)\).

### 7.2.2 Detection and Performance Evaluation

Because the time separation between any two adjacent reference pulses is fixed to \((a+1)T_d\), and \(N_r\) reference pulses exist in one frame, we need \(N_r - 1\) fixed delays \((a+1)T_d, 2(a+1)T_d, \ldots, (N_r-1)(a+1)T_d\) in the receiver to average all the reference waveforms in one frame. The correlator template is now the average of \(N_r\) reference waveforms, and is cleaner than one reference waveform if \(N_r > 1\). The larger the \(N_r\) is, the cleaner the template is. The receiver complexity is also higher but feasible. With a front-end bandpass filter,
decision statistics for the one-shot detection of the 0\textsuperscript{th} bit using a rectangular-weighted average cross-correlation receiver are

\[ z(l, j) = \int_{I_L}^{I_L+T_{corr}} \left[ \sum_{n=0}^{N_r-1} \tilde{r}(t-n(a+1)T_d) \right] \times \left[ \sum_{m=0}^{N_r-1} \tilde{r}(t-m(a+1)T_d - T_d) \right] dt \]

\[ = b_0 N_r^2 \eta \tilde{E}_p + n_d(l, j) + n_r(l, j) + n_n(l, j), \]

where \( 0 \leq l \leq \left\lfloor \frac{M_b}{a} \right\rfloor - 1, 0 \leq j \leq \min(M_b, a) - 1, I_L = lT_f + c_lT_c + (N_a - a + j)T_d, \)

\[ \tilde{E}_p = \int_0^{\infty} \tilde{g}^2(t) dt, \eta = \int_0^{T_{corr}} \tilde{g}^2(t) dt / \tilde{E}_p, \]

\[ n_d(l, j) = q_{l,N_a+j-a} N_r \sum_{m=0}^{N_r-1} \int_0^{T_{corr}} \tilde{g}(t) \times \tilde{n}(u, t-m(a+1)T_d - T_d + I_L) dt, \quad (7.3) \]

\[ n_r(l, j) = q_{l,N_a+j-a-1} N_r \sum_{m=0}^{N_r-1} \int_0^{T_{corr}} \tilde{g}(t) \times \tilde{n}(u, t-m(a+1)T_d + I_L) dt, \quad (7.4) \]

\[ n_n(l, j) = \sum_{m'=0}^{N_r-1} \sum_{m=0}^{N_r-1} \int_0^{T_{corr}} \tilde{n}(u, t-m(a+1)T_d + I_L) \times \tilde{n}(u, t-m'(a+1)T_d - T_d + I_L) dt. \quad (7.5) \]

The decision rule is

\[ z = \sum_{l=0}^{\left\lfloor \frac{M_b}{a} \right\rfloor - 1} \sum_{j=0}^{\min(M_b, a) - 1} z(l, j) \gtrless 0, \quad (7.6) \]

where the decision statistic \( z \) is the sum of all the correlator outputs affected by \( b_0 \), and the demodulation block diagram is plotted in Figure 7.2.

Generally speaking, \( z = \sum_l \sum_j z(l, j) \) does not have Gaussian distribution because of the noise \( \times \) noise term \( n_n(l, j) \), and the probability density function of \( z \) is difficult to calculate. Under some circumstances, the BEP using (7.6) can be evaluated theoretically by using the quadratic Gaussian form. Unfortunately, this only binary system with }
Figure 7.2: Demodulation block diagram of the signal plotted in Figure 7.1.

$N_r = 1$, which indicates the conventional TR system, can fit those conditions. It was pointed out in Chapter 4 that $z$ can be appropriately modelled as a Gaussian random variable when the noise time-bandwidth is large. The means of $n_r(l, j)$ and $n_d(l, j)$ are zero because of the white noise process $n(u, t)$, and the mean of $n_n(l, j)$ is also zero because $T_d$ is much greater than the noise correlation time. Therefore,

$$
E\{z(l, j)\} = b_0 N_0^2 \eta \tilde{E}_p, \quad \text{for all } l, j.
$$

Note that $n_d(l, j + 1) = q_{l,N_a+j+a+1} q_{l,N_a+j-a} n_r(l, j)$ because of the differential decoder in the receiver, otherwise the noises in (7.3), (7.4) and (7.5) are uncorrelated. The covariances of $z(l, j)z(l', j')$ can be computed as

$$
\text{Cov}\{z(l, j)z(l', j')\} = \begin{cases} 
N_r^3 N_0^2 \eta \tilde{E}_p + \frac{1}{2} N_r^2 B_w T_{\text{corr}} N_0^2, & l = l', j = j', \\
\frac{1}{2} q_{l,N_a+j+1-a} q_{l,N_a+j-1-a} N_r^3 N_0^2 \eta \tilde{E}_p, & l = l', j - j' = 1, \\
\frac{1}{2} q_{l,N_a+j+2-a} q_{l,N_a+j-a} N_r^3 N_0^2 \eta \tilde{E}_p, & l = l', j' - j = 1, \\
0, & \text{otherwise.}
\end{cases}
$$

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By using (7.7), (7.8), the mean and variance of $z$ are

$$
\mathbb{E}\{z\} = b_0 N_t^2 \eta \tilde{E}_p \left[ \frac{M_b}{a} \right] \min(M_b, a) = b_0 N_t^2 \eta \tilde{E}_p M_b,
$$

$$
\text{Var}\{z\} = M_b N_t^3 N_0 \eta \tilde{E}_p \left( 2 - \frac{1}{\min(M_b, a)} \right) + \frac{1}{2} M_b N_t^2 B_w T_{corr} N_0^2.
$$

Due to the symmetry of the probability densities of the receiver noise and transmitted data bits, the single user BEP performance conditioned on one channel realization and using a Gaussian assumption is

$$
P_b = Q \left( \left[ \left( 1 + \frac{1}{a} \right) \left( 2 - \frac{1}{\min(M_b, a)} \right) \left( \frac{N_0}{\eta \tilde{E}_b} \right) + \frac{B_w T_{corr} M_b}{2} \left( 1 + \frac{1}{a} \right)^2 \left( \frac{N_0}{\eta \tilde{E}_b} \right)^2 \right]^{\frac{1}{2}} \right),
$$

where $\tilde{E}_b = M_b N_t (1 + \frac{1}{a}) \tilde{E}_p$ because one bit uses $\frac{N_t M_b}{a}$ reference pulses. It is worth noting that the multiple access capability, which is not shown here, is expected to become worse with increasing $N_a$, while $M_b$ and $N_t$ are fixed, because the probability of collision increases.

### 7.3 $M$-ary TR Systems

#### 7.3.1 System Structure

The TR signal model in (7.1) can also be applied to an $M$-ary modulated system by utilizing block codes. The transmitted codeword $v_j \triangleq [v_{j,0}, v_{j,1}, \ldots, v_{j,N_s-1}]^t$ are selected by $m = \log_2 M$ bits $b_j \triangleq [b_{jm}, b_{jm+1}, \ldots, b_{(j+1)m-1}]^t$ from the code book $\{u_0, u_1, \ldots, u_{M-1}\}$.
The code length $N_s$ is assumed an integer multiple of $N_d$, which is not a necessary but a convenient assumption. In this system each codeword can be transmitted in one or more than one frame, depending on the ratio of $N_s$ and $N_d$. The selected codeword is mapped to the modulated code symbols as follows

$$q_{l,k} = \begin{cases} 1 & \text{mod}(k, a + 1) = 0 \\ \xi \times \frac{l}{N_s/N_d} \cdot \zeta & \text{otherwise,} \end{cases} \quad (7.10)$$

where $\zeta = \text{mod} \left( l, \frac{N_s}{N_d} \right) N_d + \left\lfloor \frac{k - 1}{a+1} \right\rfloor a + \text{mod}(k, a + 1) - 1$, and

$$\xi = \begin{cases} 1 & N_r > 1 \\ q_{l,k-1} & N_r = 1. \end{cases} \quad (7.11)$$

Examples are given in Figure 7.3 and 7.4 with rectangular-weighted cross-correlation receivers. Transmitting each code symbol in the selected codeword more than one time is not considered here. Therefore transmitting more than one reference pulse and implementing the average process in each frame can complicate the receiver more compared to the system described in the previous section. For $N_r > 1$, each set, which is defined as one reference pulse and its following $a$ data pulses, is different from other sets in the same frame. Thus extra delays are required to retrieve each of the code symbols in different sets separately which is shown in Figure 7.4(b). When $N_r = 1$ and $a = N_d$, with a differential encoder in the transmitter, the receiver can retrieve all the code symbols by using the conventional or weighted cross-correlation receivers with only one delay $T_d$ which is shown in Figure 7.3(b). The performance improvement compared to the conventional TR system is gained
by increasing the power efficiency and selecting a good block code. The performance of this $N_r = 1$ case is discussed in the following section.

![Diagram](attachment:image.png)

*Figure 7.3: M-ary TR system with $N_r = 1$, $N_d = 4$, $d_0 = 1$, $c_0 = 2$, and $v_0 = [1, -1, -1, 1]$. Part (a) is the transmitted signal with letter R indicating a reference pulse, part (b) is the demodulation block diagram. The detection block in (b) could be a maximum likelihood detection, minimum distance detection, or hard detection.*

### 7.3.2 Detection and Performance Evaluation

In the detection of the transmitted codeword $v_0$, the decision statistics by using a rectangular-weighted cross-correlation receiver are $y_0 = [y_{0,0}, y_{0,1}, \ldots, y_{0,N_s-1}]^T$ where $y_{0,m} = z(\lfloor \frac{m}{N_d} \rfloor \mod (m, N_d) + 1)$, and

$$z(l, j) = \int_{t_1}^{t_1 + c_T + jT_d + T_{corr}} \tilde{r}(u, t) \tilde{r}(u, t - T_d) dt$$

$$= v_{0,lN_d+j-1} E_p + n_d(l, j) + n_e(l, j) + n_n(l, j)$$
Figure 7.4: M-ary TR system with $N_t = 2$, $N_d = 4$, $d_0 = 1$, $c_0 = 2$, and $v_0 = [1, -1, -1, 1] = [q_0, 1, q_0, 2, q_0, 4, q_0, 5]$. Part (a) is the transmitted signal with letter R indicating a reference pulse, part (b) is the demodulation block diagram. The detection block in (b) could be a maximum likelihood detection, minimum distance detection, or hard detection.

for $0 \leq l \leq \frac{N_t}{N_d} - 1$, $1 \leq j \leq N_d + 1$, and

$$
n_d(l, j) = q_{l,j} \int_0^{T_{corr}} \tilde{g}(t) \tilde{n}(u, t + lT_t + c_lT_c + (j - 1)T_d) dt,
$$

$$
n_t(l, j) = q_{l,j-1} \int_0^{T_{corr}} \tilde{g}(t) \tilde{n}(u, t + lT_t + c_lT_c + jT_d) dt,
$$

$$
n_n(l, j) = \int_0^{T_{corr}} \tilde{n}(u, t + lT_t + c_lT_c + jT_d) \times \tilde{n}(u, t + lT_t + c_lT_c + (j - 1)T_d) dt.
$$

The mean of $z(l, j)$ conditioned on the transmitted codeword is

$$
\mathbb{E}\{z(l, j) \mid v_0\} = \bar{z}(l, j) = v_0, lN_d + j - 1 \eta \bar{E}_p,
$$

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It is clear that $q_{l,j}q_{l,j-2}n_d(l,j) = n_r(l, j-1)$, otherwise the noises in (7.14), (7.15) and (7.16) are uncorrelated. So the covariance of any two statistics conditioned on the transmitted codeword is

$$\text{Cov}\{z(l,j)z(l',j')|\mathbf{v}_0\} = \begin{cases} 
N_0\eta\tilde{E}_p + \frac{1}{2}B_wT_\text{corr}N_0^2, & l = l', j = j', \\
\frac{1}{2}q_{l,j+1}q_{l,j-1}N_0\eta\tilde{E}_p, & l = l', j = j' - 1, \\
\frac{1}{2}q_{l,j}q_{l,j-2}N_0\eta\tilde{E}_p, & l = l', j = j' + 1, \\
0, & \text{otherwise},
\end{cases} \quad (7.18)$$

where $q_{l,j+1}q_{l,j-1}$ and $q_{l,j}q_{l,j-2}$ can be related to the transmitted codeword $\mathbf{v}_0$ by using (7.10) and (7.11).

Both (7.17) and (7.18) show that the mean and covariance matrix of $\mathbf{y}_0$ depend on the transmitted codeword $\mathbf{v}_0$. By defining $\mathbf{\bar{y}}_0 \triangleq \mathbb{E}\{[y_{0,0}, y_{0,1}, \ldots, y_{0,N_s-1}]^\dagger\}$, the covariance matrix of $\mathbf{y}_0$ conditioned on the transmitted codeword $\mathbf{v}_0 = \mathbf{u}_j$ is

$$M_{\mathbf{u}_j} = \mathbb{E}\{[\mathbf{y}_0 - \mathbf{\bar{y}}_0][\mathbf{y}_0 - \mathbf{\bar{y}}_0]^\dagger|\mathbf{u}_j\} \quad (7.19)$$

which can be acquired by applying (7.18). Maximum likelihood detection, minimum distance detection, or hard detection can be exploited in the digital signal processing to detect the transmitted codeword and the corresponding information bits. By assuming the noise $\times$ noise $n_n(l,j)$ to be Gaussian distributed, the likelihood function is

$$L(\mathbf{y}_0|\mathbf{u}_j) = \frac{1}{\sqrt{2\pi \det(M_{\mathbf{u}_j})}} \exp \left\{ -\frac{1}{2}(|\mathbf{y}_0 - \mathbf{\bar{y}}_0|^\dagger M_{\mathbf{u}_j}^{-1}|\mathbf{y}_0 - \mathbf{\bar{y}}_0|^\dagger) \right\}.$$
Maximum likelihood detection chooses the codeword which maximizes the likelihood function

\[ \hat{v}_0 = \arg\max_{\mathbf{u}_j} L(y_0 | \mathbf{u}_j). \]

Minimum distance detection selects the codeword whose distance to \( y_0 \) is the shortest

\[
\hat{v}_0 = \arg\min_{\mathbf{u}_j} \| y_0 - \mathbf{u}_j \| \\
= \arg\min_{\mathbf{u}_j} \| y_0 - \mathbf{u}_j \|^2 \\
= \arg\min_{\mathbf{u}_j} \| \mathbf{u}_j \|^2 - 2y_0^T \mathbf{u}_j, \quad (7.20)
\]

where \( \| \cdot \| \) denotes the norm of a vector. If codewords in the code book have the same norm, (7.20) can be reduced further to

\[
\hat{v}_0 = \arg\max_{\mathbf{u}_j} y_0^T \mathbf{u}_j,
\]

which corresponds to the maximum correlation detection. Hard detection uses one bit ADC to quantize \( \{y_{0,m}\}_{m=0}^{N_s-1} \) to +1 or -1, then correlates the quantized \( y_0 \) with eligible codewords in the code book to find the one producing the maximum correlation,

\[
\hat{v}_0 = \arg\max_{\mathbf{u}_j} \text{sgn}(y_0^T) \mathbf{u}_j,
\]

where \( \text{sgn}(y_0^T) = [\text{sgn}(y_{0,0}), \text{sgn}(y_{0,1}), \ldots, \text{sgn}(y_{0,N_s-1})]^T \) with \( \text{sgn}(x) = 1 \) for \( x \geq 0 \) and \( \text{sgn}(x) = -1 \) for \( x < 0 \).
7.4 Numerical Examples

Equation (7.9) with different values of $N_r$, $N_d$, and $M_b$ is plotted in Figure 7.5 with $B_w = 4\text{GHz}$ and $T_{\text{corr}} = 20\ \text{ns}$. For all the curves, each bit is transmitted through 18 pulses which include reference and data pulses. For a specific $N_r$ which determines the noise variance in the correlator template, the larger the $N_a$ (or $N_d$) is, the better the BEP performance is because of higher power efficiency. For a specific $N_a$, a larger $N_r$ means less bit energy is spent on data pulses, but more reference pulses can be averaged as a correlator template. A better performance in this situation indicates that the noisy template is a more serious problem than the low power efficiency in a TR system. For this 18 pulses per bit case, Figure 7.5 shows that the difference between the best and worst performance at BEP=1e-4 is 4.4dB.

The BEP of the UWB system with $M$-ary TR modulation in a single user multipath environment with $T_{\text{corr}} = 20\ \text{ns}$ and $B_w = 4\text{GHz}$ is simulated using repetition codes, orthogonal Walsh-Hadamard codes [41], and biorthogonal Walsh-Hadamard codes. For the $M$-ary modulation using repetition codes with length $J$ (which could be $M$ or $M/2$ in the simulation), $m = \log_2 M$ bits are transmitted in groups with each bit repeating $J/m$ times. Walsh-Hadamard codes with length $M$ are rows of an $M \times M$ matrix which is constructed as

$$H_M = \begin{bmatrix} H_M^{\frac{M}{2}} & H_M^{\frac{M}{2}} \\ H_M^{\frac{M}{2}} & -H_M^{\frac{M}{2}} \end{bmatrix}$$

with the initial matrix

$$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}.$$
Biorthogonal Walsh-Hadamard codes with length $\frac{M}{2}$ are rows of an $M \times \frac{M}{2}$ matrix which is constructed as

$$H_M = \begin{bmatrix} H_M^{\frac{M}{2}} \\ -H_M^{\frac{M}{2}} \end{bmatrix}.$$ 

The transmitted codeword are chosen by $m$ bits from matrix $H_M$. Results are plotted in Figure 7.6-7.8. Figure 7.6 shows the minimum distance detection, with a simpler digital signal processing structure, performs almost the same as the maximum likelihood detection. The minimum distance detection does not consider the covariance of any two correlator outputs in (7.18) which, seen from the structure of the covariance, does not affect the performance much. The hard detection, which uses only one bit to represent the correlator output, has the simplest receiver structure among these three detection methods with a 1.5dB penalty. This figure also exhibits that codes with longer length in the same category perform better because of the larger distance of two codewords.

For codewords with the same norm which is the case for repetition, Walsh-Hadamard, and biorthogonal Walsh-Hadamard codes, the BEP depends on the distributions of the cross-correlations of any two codewords, $\mathbf{u}_i^\dagger \mathbf{u}_j$, in the code book. The smaller the cross-correlations are, the more unlike the codewords are, and the better the performance is. When $E_b/N_0$ increases, the largest cross-correlation dominates the BEP. Figure 7.7 compares repetition codes to orthogonal Walsh-Hadamard codes with the same code length $(M)$ by using the minimum distance detection. When $M = 4$, the cross-correlation of any two repetition codewords is either 0 or $-4$, and that of any two Walsh-Hadamard codewords is always 0. Therefore, repetition codes perform better than Walsh-Hadamard codes. But for $M \geq 8$, the largest cross-correlation of repetition codes is always greater.
than that of Wash-Hadamard codes. Thus Wash-Hadamard codes outperform repetition codes with increasing $E_b/N_0$.

Figure 7.8 compares repetition codes to biorthogonal Walsh-Hadamard codes with the same code length ($M/2$) using minimum distance detection. For $M = 4$, biorthogonal codewords are the same as the repetition codewords, thus their performance is the same. When $M \geq 8$, the values of the cross-correlation of any biorthogonal codeword with other codewords are all zero with only one exception which is equal to $-M/2$. But we can always find two repetition codewords with cross-correlation greater than zero. Therefore, biorthogonal codes outperform repetition codes.

For a fixed $E_b/N_0$, the codewords in Figure 7.8 with code length $M/2$ perform better than the codewords with code length $M$ in Figure 7.7. This again illustrates that the noise $\times$ noise degrades BEP performance more when the pulse energy to the noise power ratio of the correlator input is smaller. However in a realistic system, how large the pulse energy can be depends on the hardware issues as well as the FCC regulation.
Figure 7.5: BEPs of the binary TR system with $B_w = 4$GHz, $T_{corr} = 20$ ns, and different values of $N_a$, $N_r$, and $M_b$. In this figure, each bit is conveyed in 18 pulses. The efficiency factor of the cross-correlator is denoted as $\eta$, so $\eta E_b$ is the energy in one bit that the cross-correlator can capture.
Figure 7.6: BEP comparisons between different detection methods for biorthogonal Walsh-Hadamard code. The efficiency factor of the cross-correlator is denoted as $\eta$, so $\eta E_b$ is the energy in one bit that the cross-correlator can capture.
Figure 7.7: BEP comparisons between orthogonal Walsh-Hadamard codes and repetition codes using minimum distance detection. The efficiency factor of the cross-correlator is denoted as $\eta$, so $\eta E_b$ is the energy in one bit that the cross-correlator can capture.
Figure 7.8: BEP comparisons between biorthogonal Walsh-Hadamard codes and repetition codes using minimum distance detection. The efficiency factor of the cross-correlator is denoted as $\eta$, so $\eta E_b$ is the energy in one bit that the cross-correlator can capture.
Chapter 8

Conclusion

This thesis focuses on UWB radios with transmitted reference methods. Optimal and suboptimal receivers based on the average likelihood ratio test and generalized likelihood ratio test without any complexity constraints are derived in both Rayleigh and lognormal environments, and GLRT optimal receiver is shown as one of the suboptimal receivers in ALRT sense. Performance results show that ALRT optimal receivers derived with Raleigh and lognormal path strength models can perform equally well in each other’s environments, and the Rayleigh suboptimal receiver 1, which has a simple receiver structure, performs close to the optimal one when the multipath component existence probability is normal to high. In a low path arrival probability environment, the performance of both Rayleigh and lognormal suboptimal receiver 1 becomes closer to and even worse than that of the Rayleigh and lognormal suboptimal receiver 2 as $E_b/N_0$ increases.

The bit error probability of conventional and average cross-correlation receivers are discussed in detail with the help of the orthogonal functions expansion and central limit theorem, and two weighted cross-correlation receivers, which are an improvement of the
conventional cross-correlation receiver with a complexity constraint, are proposed and analyzed in detail. These *ad hoc* cross-correlation receivers are now contrasts to the theoretical optimal and suboptimal receivers, and their structures and performance are compared.

The cross-correlation receivers perform worse than ALRT optimal and suboptimal receivers, and the BEPs of the conventional and weighted cross-correlation receivers degrade as $N_s$ increases for a fixed $E_b/N_0$. The Rayleigh suboptimal receiver 2, by expanding the number of correlator templates, can be equivalent to the average cross-correlation receiver. Central limit theorem can help evaluate the BEP of cross-correlation receivers well by approximating the noise × noise term Gaussian distributed when the noise time × bandwidth product is large.

A differential transmitted reference method which has higher power efficiency than the conventional TR method is proposed. The multiple access capability of UWB radios with TR and DTR modulation in multipath environments using a rectangular-weighted cross-correlation receiver are studied. With the Gaussian MAI assumption, the BEP and a tight upper bound are obtained for $N_s = 1$ and $N_s > 1$, respectively. The Gaussian MAI assumption can relieve the burden of theoretical analysis, and make a fair estimation of the BEP for a power control TR and DTR system in the range of interest. Without any power control, the Gaussian assumption is less precise under the same system parameters, and the BEP also degrades. Compared to TR modulated systems, DTR modulated systems with a more complex correlator in the receiver can double the user numbers or reduced the required bit energy for a specific BEP.
Compared to conventional cross-correlation receivers, average cross-correlation receivers improve the BEP by cleaning the correlator template using average process. But this average process can complicate the receiver a lot with the conventional TR modulation. This thesis also proposes a novel TR signal model for a multiple access UWB system which makes the average process feasible within a restrictive complexity increase, and can be applied to both binary and $M$-ary modulation. For the binary system, the BEP performance and receiver complexity can be traded by choosing different system parameters. For the $M$-ary system, block codes other than repetition codes are exploited. Results show both orthogonal and biorthogonal codes outperform repetition codes when the size of the code book is greater than or equal to 8. And larger the size of the code book is, better the performance is.

The transmitted reference method, with the benefit of simplifying the receiver structure, still has problems to be solved and analyzed. One is how to choose a proper time separation between the reference and data-modulated pulses. Large time separation prevents the inter-pulse interference, but reduces the achievable data rate and increases the difficulty of implementing the delay in the cross-correlation receiver. If the delay is implemented by using a transmission line, the longer the line is, the larger the signal decay and receiver size are. If the delay is implemented by using an all-pass filter, if the filter can have a linear group delay and phase delay over several giga-hertz is an issue. Small time separation causes inter-pulse interference, and how much this interference degrades the bit error probabilities or how to reduce this interference should be investigated.
Appendix A

Detailed Calculations for Theoretical Optimal and Suboptimal Single User Receivers

A.1 Log-Likelihood Function Evaluation with Rayleigh Path Strength Models

The nuisance parameter $\alpha_k$ in (3.8) is integrated first by inserting (2.12) and (2.13) into $f(\alpha_k)$. The integral (A.2) is derived by applying formula 3.462.5 in [35] to (A.1)

\[
L_k(x) = \ln \left\{ \int_{-\infty}^{\infty} f(p_k) \left[ \int_0^{\infty} \frac{a\alpha_k}{\sigma_k^2} \exp(p_k\alpha_k x) \right. \right. \\
- \alpha_k^2 \left( \frac{2N_s}{N_0} + \frac{1}{2\sigma_k^2} \right) \left. \right] d\alpha_k + (1 + a) \right\} dp_k \\
= \ln \left\{ \int_{-\infty}^{\infty} \left[ \frac{a}{\sigma_k^2} \left( \frac{1}{4N_s/N_0 + 1/\sigma_k^2} + \frac{p_k x}{4N_s/N_0 + 1/\sigma_k^2} \right) \right. \right. \\
\times \sqrt{\frac{\pi}{2N_s/N_0 + 1/\sigma_k^2}} \exp \left( \frac{x^2}{8N_s/N_0 + 2/\sigma_k^2} \right) \left. \right] \left. \right\} f(p_k) dp_k \\
\times Q \left( \frac{-p_k x}{2} \sqrt{\frac{2N_s}{N_0 + 1/2\sigma_k^2}} + (1 - a) \right) \\
\times f(p_k) dp_k \right\}.
\]
By defining $SNR_k = \frac{4N_k\sigma_k^2}{N_0}$,

\begin{align*}
L_k(x) &= \ln \left\{ \int_{-\infty}^{\infty} \left[ \frac{a}{1 + SNR_k} + \frac{ap_kx}{1 + SNR_k} \sqrt{\frac{2\pi\sigma_k^2}{1 + SNR_k}} \right] \times Q \left( -p_kx \sqrt{\frac{\sigma_k^2}{1 + SNR_k}} \right) \exp \left( \frac{\sigma_k^2 x^2}{2 + 2SNR_k} \right) + (1 - a) \right\} f(p_k) dp_k \right\}.
\end{align*}

(A.3)

In the following integration over $p_k$ using (2.11), (A.4) is simplified to (A.5) because $Q(-x) = 1 - Q(x)$ and $x[1 - 2Q(x\sqrt{w_R(k)})] \geq 0$

\begin{align*}
L_k(x) &= \ln \left\{ \int_{-\infty}^{\infty} \left[ \frac{aw_R(k)}{\sigma_k^2} + \frac{ap_kx}{\sigma_k^2} \sqrt{\frac{2\pi w_R^3(k)}{\sigma_k^2}} \exp \left( \frac{x^2 w_R(k)}{2} \right) \right] \times Q \left( -p_kx \sqrt{w_R(k)} \right) + (1 - a) \right\} \left\{ \frac{1}{2} \delta_D(p_k - 1) + \frac{1}{2} \delta_D(p_k + 1) \right\} dp_k \right\} \\
&= \ln \left\{ \left[ \frac{aw_R(k)}{\sigma_k^2} + \frac{ax}{2\sigma_k^2} \sqrt{2\pi w_R^3(k)} \exp \left( \frac{x^2 w_R(k)}{2} \right) \right] \times Q \left( -x \sqrt{w_R(k)} \right) - Q \left( x \sqrt{w_R(k)} \right) \right\} + (1 - a) \right\} \right\} \\
&= \ln \left\{ \left[ \frac{aw_R(k)}{\sigma_k^2} + \frac{ax}{2\sigma_k^2} \sqrt{2\pi w_R^3(k)} \exp \left( \frac{x^2 w_R(k)}{2} \right) \right] \times [1 - 2Q \left( \sqrt{w_R(k)x^2} \right)] + (1 - a) \right\}.
\end{align*}

(A.4)

(A.5)
A.2 Log-Likelihood Function Approximations with Rayleigh Path Strength Models

By substituting $C(k)$ for $x$ in $L_k(x)$ in (3.12),

$$L_k(C(k)) = \frac{w_R(k)C^2(k)}{2} + \ln \left\{ \exp \left( -\frac{w_R(k)C^2(k)}{2} \right) ight. \right. \right.$$

$$+ \left. \left. \sqrt{\frac{\pi w_R(k)C^2(k)}{2}} \left( 1 - 2Q \left( \frac{\sqrt{w_R(k)C^2(k)}}{w_R(k)C^2(k)} \right) \right) \right. \right. \right.$$

$$+ \left. \left. \left( \frac{1-a}{a} \right) \cdot (1 + SNR_k) \exp \left( -\frac{w_R(k)C^2(k)}{2} \right) \right\} \right. \right.$$  \hspace{1cm} (A.6)

When the value of $w_R(k)C^2(k)$ is large and $a$ is close to 1,

$$1 - 2Q \left( \frac{\sqrt{w_R(k)C^2(k)}}{w_R(k)C^2(k)} \right) \approx 1,$$

$$\exp \left( -\frac{w_R(k)C^2(k)}{2} \right) \approx 0,$$

$$\frac{1-a}{a} \approx 0.$$  \hspace{1cm}

With $1 + SNR_k$ being bounded, by substituting these approximations into (A.6),

$$L_k(C(k)) \approx \frac{w_R(k)C^2(k)}{2} + \ln \left\{ \sqrt{\frac{\pi w_R(k)C^2(k)}{2}} \right\} \right. \right.$$  \hspace{1cm} (A.6)

$$\approx \frac{w_R(k)C^2(k)}{2}. $$
A.3 Auxiliaries for the Performance Evaluation of Rayleigh Suboptimal Receivers

Claim 1

\[
\int_{-j\infty}^{j\infty} G(z)dz = 2\pi j \sum_{k=1}^{K/2} \text{Res}_{z=-z_k} G(z).
\]

Proof: In Figure A.1, the line from \((0, -jR)\) to \((0, jR)\) plus \(C_R\) which comes back to \((0, -jR)\) compose of a positively oriented simple closed contour including all negative poles of \(G(z)\) in it. It is directly from the Cauchy’s residue theorem that

\[
\int_{-jR}^{jR} G(z)dz + \int_{C_R} G(z)dz = 2\pi j \sum_{k=1}^{K/2} \text{Res}_{z=-z_k} G(z).
\]

Next, we show that \(\int_{C_R} G(z)dz\) tends to 0 as \(R\) tends to \(\infty\). Let \(z = z_R + jz_I \in C_R\), it is obvious that \(|z| = R, \ |z_R| \leq R,\) and \(z_R \leq 0\). The absolute value of \(G(z)\) in (3.31) is

\[
|G(z)| = |\exp(-z\theta)| \prod_{k=1}^{K/2} \left| \frac{\exp\left\{ \frac{z\xi_k|d_{2k-1}^2}{1-z\xi_k} \right\}}{\xi_k^2 \left| z - \frac{\xi_k}{1 - z\xi_k} \right| \left| z + \frac{\xi_k}{1 - z\xi_k} \right|} \right| \leq |\exp(-z\theta)| \prod_{k=1}^{K/2} \left| \frac{\exp\left\{ \frac{z\xi_k|d_{2k-1}^2}{1-z\xi_k} \right\}}{\xi_k^2 \left( R - \frac{1}{\xi_k} \right)^2} \right|,
\]

and the last inequality results from \(|z - \frac{\xi_k}{1 - z\xi_k}| \geq ||z| - \frac{1}{\xi_k}| = R - \frac{1}{\xi_k}| = |z + \frac{\xi_k}{1 - z\xi_k}| \geq ||z| - \frac{1}{\xi_k}|\).

For each \(k\),

\[
\left| \exp\left\{ \frac{z\xi_k|d_{2k-1}^2}{1-z\xi_k} \right\} \right| = \left| \exp\left\{ \frac{-|z - z\xi_k||d_{2k-1}^2 + |d_{2k-1}^2|}{1 - z\xi_k} \right\} \right| = \exp\left\{ -|d_{2k-1}^2| \right\} \left| \exp\left\{ \frac{|d_{2k-1}^2|}{1 - (z\xi_k + jz_I)\xi_k} \right\} \right| = \exp\left\{ -|d_{2k-1}^2| \right\} \left| \exp\left\{ \frac{|d_{2k-1}^2(1 - z_R\xi_k)}{(1 - z_R\xi_k)^2 + (z_I)^2} \right\} \right| \quad (A.7)
\]
by using the fact that $|\exp\{ju\}| = 1$ for any real number $u$. In addition, $1 - 2z_R\xi_k \leq 1 + 2R\xi_k$ because $z_R \leq 0$ and $|z_R| \leq R$ which results in

\[(1 - z_R\xi_k)^2 + (z_I\xi_k)^2 = 1 - 2z_R\xi_k + (z_R\xi_k)^2 + (z_I\xi_k)^2 \geq 1 - 2z_R\xi_k + R^2\xi_k^2.\]

Therefore, (A.7) is reduced to

\[\left|\exp\left\{\frac{z\xi_k|d_{2k-1}|^2}{1 - z\xi_k}\right\}\right| \leq \exp\left\{ -|d_{2k-1}|^2 + \frac{|d_{2k-1}|^2(1 + R\xi_k)}{R^2\xi_k^2}\right\}.\]

Beside, for $\theta \in (-\infty, 0]$,

\[|\exp(-z\theta)| = |\exp\{-z_R + jz_I\theta\}| = \exp(-z_R\theta) \leq 1.\]

Therefore,

\[\left|\int_{C_R} G(z)dz\right| \leq \int_{C_R} |G(z)|dz = \pi R|G(z)| \leq \pi R \prod_{k=1}^{K/2} \exp\left\{ -|d_{2k-1}|^2 + \frac{|d_{2k-1}|^2(1 + R\xi_k)}{R^2\xi_k^2}\right\} \xi_k^2 \left(R - \frac{1}{\xi_k}\right)^2 \to 0 \quad \text{as } R \text{ tends to } \infty.\]
A.4 Evaluation of the Moments of $D_c$

Given the transmitted bit $b_0$ and the filtered channel realization $\tilde{g}(t)$, the mean of $D_c$ is computed as

\[
m = \mathbb{E}\{D_c|b_0, \tilde{g}(t)\} = b_0 N_s \tilde{E}_p + \mathbb{E}\{\tilde{N}\}
\]

\[
= b_0 N_s \tilde{E}_p + \sum_{j=0}^{N_s-1} \int_{jT_d + T_d}^{jT_d + T_d + T_mds} \mathbb{E}\{\tilde{n}(u, t-T_d)\tilde{n}(u, t)\} dt
\]

\[
= b_0 N_s \tilde{E}_p + \sum_{j=0}^{N_s-1} \int_{jT_d + T_d}^{jT_d + T_d + T_mds} R_{\tilde{n}}(T_d) dt
\]

\[
= b_0 N_s \tilde{E}_p
\]
where \( R_\tilde{n}(\tau) = \mathbb{E}\{\tilde{n}(u,t)\tilde{n}(u,t-\tau)\} \) is the correlation function of the filtered Gaussian noise \( \tilde{n}(u,t) \). In the derivation of \( m \), \( R_\tilde{n}(T_d) \approx 0 \) because \( T_d \gg \frac{1}{B_w} \). The variance of \( D_c \) given \( \tilde{g}(t) \) is

\[
\sigma^2 = \text{Var}\{D_c|\tilde{g}(t)\} = \mathbb{E}\{[b_0\tilde{N}_d + \tilde{N}_r + \tilde{N}]^2\} = \mathbb{E}\{\tilde{N}_d^2\} + \mathbb{E}\{\tilde{N}_r^2\} + \mathbb{E}\{\tilde{N}^2\},
\]

and the third equality comes from that \( \tilde{N}_d, \tilde{N}_r, \) and \( \tilde{N} \) are uncorrelated. The variance of \( \tilde{N}_d \) is

\[
\mathbb{E}\{\tilde{N}_d^2\} = \sum_{j=0}^{N_s-1} \sum_{i=0}^{N_s-1} \int_0^{T_{\text{mds}}} \int_0^{T_{\text{mds}}} \mathbb{E}\{\tilde{n}(u,t + jT_t)\tilde{n}(u,v + iT_t)\} \tilde{g}(t)\tilde{g}(v)dt dv
\]

\[
= \sum_{j=0}^{N_s-1} \sum_{i=0}^{N_s-1} \int_0^{T_{\text{mds}}} \int_0^{T_{\text{mds}}} R_\tilde{n}(t - v + (j - i)T_t) \tilde{g}(t)\tilde{g}(v)dt dv.
\]

Considering the range of \( t \) and \( v \), \( t - v - (j - i)T_t \geq T_d \) for \( j \neq i \). So \( R_\tilde{n}(t - v - (j - i)T_t) \) has significant value only if \( j = i \). Therefore

\[
\mathbb{E}\{\tilde{N}_d^2\} \approx \sum_{j=0}^{N_s-1} \int_0^{T_{\text{mds}}} \int_0^{T_{\text{mds}}} R_\tilde{n}(t - v) \tilde{g}(t)\tilde{g}(v)dt dv
\]

\[
\approx \frac{N_s}{2} \int_0^{T_{\text{mds}}} \tilde{g}^2(t) dt = \frac{N_sN_0\tilde{E}_p}{2}.
\]

The first approximation is simplified to the second one because the inverse of the noise bandwidth is extremely small compared to the duration of \( \tilde{g}(t) \), so \( R_\tilde{n}(t - v) \) can be
approximated by \( \frac{N_0}{2} \delta_D(t - v) \). The variance of \( \tilde{N}_r \) can be computed similarly, and the result is \( \mathbb{E}\{\tilde{N}_r^2\} = \mathbb{E}\{\tilde{N}_d^2\} \). The computation of \( \mathbb{E}\{\tilde{N}^2\} \) is

\[
\mathbb{E}\{\tilde{N}^2\} = \sum_{j=0}^{N_s-1} \sum_{i=0}^{N_s-1} \int_{jT_t + T_d + T_{\text{mds}}}^{(j+1)T_t + T_d + T_{\text{mds}}} \int_{iT_t + T_d}^{(i+1)T_t + T_d} \mathbb{E}\{\tilde{n}(u, t - T_d)\tilde{n}(u, t)\} \times \tilde{n}(u, v - T_d)\tilde{n}(u, v)dv dt
\]

\[
= \sum_{j=0}^{N_s-1} \sum_{i=0}^{N_s-1} \int_{jT_t + T_d + T_{\text{mds}}}^{(j+1)T_t + T_d + T_{\text{mds}}} \int_{iT_t + T_d}^{(i+1)T_t + T_d} \left[ R_n^2(T_d) + R_n^2(t - v) \right] + R_n(t - v - T_d)R_n(t - v + T_d) dv dt
\]

\[
= \sum_{j=0}^{N_s-1} \int_{jT_t + T_d + T_{\text{mds}}}^{(j+1)T_t + T_d + T_{\text{mds}}} \int_{jT_t + T_d}^{(j+1)T_t + T_d} R_n^2(t - v) dv dt
\]

Because \( R_n^2(T_d) \cong 0 \), \( R_n(t - v - T_d)R_n(t - v + T_d) \cong 0 \) \( (R_n(t - v - T_d) \) has significant value when \( t = v + T_d \), and \( R_n(t - v + T_d) \) has significant value when \( t = v - T_d \), and \( R_n^2(t - v) \neq 0 \) only for \( i = j \). Using change of variables by letting \( x = t - v \) and \( y = t + v \),

\[
\mathbb{E}\{\tilde{N}^2\} = \frac{1}{2} \sum_{j=0}^{N_s-1} \int_{-T_{\text{mds}}}^{T_{\text{mds}}} \int_{2jT_t + 2T_d + 2T_{\text{mds}}}^{2jT_t + 2T_d + 2T_{\text{mds}} - 2|y|} dy R_n^2(x) dx
\]

\[
= N_s T_{\text{mds}} \int_{-T_{\text{mds}}}^{T_{\text{mds}}} R_n^2(x) dx
\]

\[
\cong N_s T_{\text{mds}} \int_{-\infty}^{\infty} R_n^2(x) dx
\]

\[
= N_s T_{\text{mds}} \int_{-\infty}^{\infty} |S_n(f)|^2 df = \frac{N_s T_{\text{mds}} N_0^2 B_w}{2}.
\]

In the derivation, Parsaval theorem is applied, and \( S_n(f) \) is the power spectral density of the filtered Gaussian noise with two-sided bandwidth \( 2B_w \) and magnitude \( \frac{N_0}{2} \). Therefore,

\[
\sigma^2 \cong N_s N_0 \hat{E}_p + \frac{N_s T_{\text{mds}} N_0^2 B_w}{2}.
\]

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Appendix B

Detailed Discussions of Rectangular-weighted Cross-correlation Receivers

B.1 Integration time analysis

The integration time of the correlator $T_{\text{corr}}$ affects the BEP which can be seen in (4.23). Conditioned on a channel realization, the efficiency factor $\eta$ increases as $T_{\text{corr}}$ increases, therefore $\exp(-\eta E_b / 2N_0)$ decreases but $(\eta E_b / 2N_0)^n$ increases. In addition, the number of terms in the summation also increases as $T_{\text{corr}}$ increases. The effect of $T_{\text{corr}}$ can also be seen in (4.24). Thus the BEP has its minimum at some value of $T_{\text{corr}}$, and starts to raise as $T_{\text{corr}}$ diverges from this value.

Due to the simple receiver constraint and that it is difficult to implement an adaptive algorithm using analog devices, the value of $T_{\text{corr}}$ is fixed once the receiver is implemented. The best choice of $T_{\text{corr}}$ is to minimize the average BEP which is immediately seen a difficult
task from (4.24) or (4.23) because frequency selective UWB channels with random path arrival times make finding the distribution of $\frac{\eta E_p}{2N_0}$ difficult. By defining the decision SNR

$$f_s(T_{\text{corr}}) = \frac{N_s(\eta E_p)^2}{N_0\eta E_p + \frac{B_w T_{\text{corr}}}{2} N_0^2},$$

which is the ratio of the signal energy to the noise power in the decision statistic of a bit, another choice to optimize the integration time is to maximize $f_s(T_{\text{corr}})$. This criterion is equivalent to minimizing $N_s f_s^{-1}(T_{\text{corr}})$

$$N_s f_s^{-1}(T_{\text{corr}}) = \frac{N_0}{\eta E_p} + \frac{B_w T_{\text{corr}}}{2} \left( \frac{N_0}{\eta E_p} \right)^2,$$  \hspace{1cm} (B.1)

which indicates that the optimal integration time based on this criterion depends on the energy per pulse $E_p$ instead of energy per bit $E_b$. Minimizing $N_s f_s^{-1}(T_{\text{corr}})$ is still difficult to manage theoretically. By exploiting the average power profile of the received signal which is assumed exponential decays here without loss of generality [42], then

$$\mathbb{E}\{g^2(t)\} = \Omega a \exp(-at)$$

where $\Omega = \mathbb{E}\{E_p\}$ and $\frac{1}{a}$ is the power decay time constant. Under the exponential power decay profile assumption,

$$\mathbb{E}\{\eta E_p\} = \Omega [1 - \exp(-aT_{\text{corr}})].$$  \hspace{1cm} (B.2)
In the following three subsections, we are going to replace $\eta E_p$ in $f_s(T_{\text{corr}})$ by $\mathbb{E}\{\eta E_p\}$, and investigate the effects of choosing different values of $T_{\text{corr}}$. Note that $f_s(T_{\text{corr}})$ with this substitution does not equal the average decision SNR over channel statistics, and the observations we obtain will be justified in the next section by evaluating the average decision SNR and average BEP numerically.

### B.1.1 Minimal integration time

A special case in which the noise power is extremely large is considered. In this condition, $\left( \frac{N_0}{\eta E_p} \right)^2 \gg \frac{N_0}{\eta E_p}$, and the quantity we want to minimize is approximate

$$N_s f_s^{-1}(T_{\text{corr}}) \approx B_w T_{\text{corr}}^2 \left( \frac{N_0}{\eta E_p} \right)^2.$$  \hspace{1cm} (B.3)

By replacing $\eta E_p$ with $\mathbb{E}\{\eta E_p\}$, we now want to minimize

$$\frac{B_w T_{\text{corr}}}{2} \times \left( \frac{N_0}{\Omega} \right)^2 \times \left[ \frac{1}{1 - \exp(-a T_{\text{corr}})} \right]^2,$$  \hspace{1cm} (B.4)

which is a convex function of $T_{\text{corr}} \in (0, \infty)$, and has an unique minimum. After differentiating (B.4) with respect to $T_{\text{corr}}$ and equating it 0, the equation which determines the optimal value of $T_{\text{corr}}$ is

$$\ln(1 + 2a T_{\text{corr}}) = a T_{\text{corr}},$$  \hspace{1cm} (B.5)
which does not depend on the receiver bandwidth $B_w$ and $\Omega/N_0$. In this extremely high noise power case, the value of $T_{corr}$ only depends on the power decay time constant $\frac{1}{a}$. The solution of (B.5) is $aT_{corr} = 1.2564$, and

$$
T_{corr} = \frac{1.2564}{a} = 1.2564 \times \text{time constant.}
$$

This high noise power case represents the minimum value of $T_{corr}$ for a conventional correlation receiver, and $T_{corr}$ in a general situation should be larger than this value.

For another special case that the received signal power is extremely high, it is not really meaningful because the integration time then should be as long as possible, i.e., the channel delay spread.

### B.1.2 Optimal integration time

For the normal $E_p/N_0$ case, $\eta E_p$ in (B.1) is replaced by (B.2), and the quantity to be minimized is

$$
N_0 f_\eta^{-1}(T_{corr}) = \frac{N_0}{\Omega} \times \frac{B_w T_{corr}}{2[1 - \exp(-a T_{corr})]} \left[ \frac{N_0}{\Omega} \times \frac{1}{1 - \exp(-a T_{corr})} \right]^2.
$$

(B.7)

The right hand side of (B.7) is differentiated with respect to $T_{corr}$ to achieve

$$
\left( \frac{2\Omega}{B_w N_0} + 1 + 2a T_{corr} \right) \exp(-a T_{corr}) - \frac{2\Omega}{B_w N_0} \exp(-2a T_{corr}) - 1 = 0.
$$

(B.8)

Equation (B.8) shows that the optimal integration time depends on $B_w$, $1/a$, and $\Omega/N_0$.

Given $B_w$ and $\Omega/N_0$, the value of $aT_{corr}$ which makes (B.8) sustained can be computed
numerically. For a specific BEP, the required pulse energy increases and the optimal integration time decreases as $B_w$ increases because of the increasing incoming noise.

**B.1.3 Performance degradation versus excess or lack of integration**

The optimal integration time changes according to application environments, but the value adopted by the receiver is difficult to change once the correlator is implemented. How much the performance degrades because of the excessive or short integration should be considered before choosing the proper value. In (B.1) with $\eta E_p$ replaced by $\mathbb{E}\{\eta E_p\}$, $N_s f_s^{-1}(T_{corr})$ includes two portions

\begin{align*}
 g_s(T_{corr}) &= \frac{N_0}{\Omega} \times \frac{1}{1 - \exp(-a T_{corr})}, \quad \text{(B.9)} \\
 h_s(T_{corr}) &= \frac{B_w T_{corr}}{2} \left[ \frac{N_0}{\Omega} \times \frac{1}{1 - \exp(-a T_{corr})} \right]^2. \quad \text{(B.10)}
\end{align*}

Equation (B.9) indicates that $g_s(T_{corr})$ decreases as $T_{corr}$ increases for $T_{corr} \in [0, \infty)$. Equation (B.10) shows that $h_s(T_{corr})$ also decreases as $T_{corr}$ increases for $T_{corr} \in [0, t_B)$ with some value $t_B$, then starts to increase as $T_{corr}$ increases for $T_{corr} \geq t_B$. The value of $t_B$ is determined by $B_W$, $N_0/\Omega$ and $1/a$, and the optimal $T_{corr}$ is greater than or equal to $t_B$. Another observation from (B.9) and (B.10) is that $g_s(T_{corr})$ for $T_{corr} \in (0, \infty)$ as well as $h_s(T_{corr})$ for $T_{corr} \in (0, t_B)$ decrease roughly exponentially as $T_{corr}$ increases, and $h_s(T_{corr})$ for $T_{corr} \in [t_B, \infty)$ increases roughly linearly. Therefore, under integration degrades BEP performance more than over integration.
This section uses a channel model in (B.11) to analyze the average BEP and average decision SNR versus different integration time numerically to verify the analysis and observations in Section B.1. The model is

\[ h(t) = \sum_{l=0}^{L} \alpha_l \delta(t - T_l), \]  

(B.11)

where \( \alpha_l \) and \( T_l \) are the amplitude and arrival time of the \( l\)th path. The magnitude of \( \alpha_l \) has lognormal distribution, and the polarity of it can be +1 or −1 with equal probability. In addition, \( \alpha_l \) and \( \alpha_j \) are independent for \( l \neq j \). The energy of a single transmitted pulse is normalized to 1, and \( \mathbb{E}\{\alpha_l^2\} = c \exp(-aT_l) \) with some constant \( c \) such that \( \sum_l \mathbb{E}\{\alpha_l^2\} = E_p \).

The channel delay spread \( T_{mds} \) is defined as the interval containing 99% of the energy in the average received waveform. The probability that a path arrives at time \( T_l \) has poisson distribution with the path arrival rate \( \lambda \). The receiver bandwidth is equal to 4GHz, and 100 channel realizations are generated to get the numerically average BEP and decision SNR.

The parameters used in this numerical analysis are listed in Table B.1. The resolution of searching the optimal \( T_{corr} \) is equal to 1ns.

<table>
<thead>
<tr>
<th>( \frac{1}{a} ) (ns)</th>
<th>c1</th>
<th>c2</th>
<th>c3</th>
<th>c4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda ) (1/ns)</td>
<td>3.5</td>
<td>3.5</td>
<td>3.5</td>
<td>3.5</td>
</tr>
<tr>
<td>( \eta ) : mean</td>
<td>0.85908</td>
<td>0.83218</td>
<td>0.82051</td>
<td>0.81577</td>
</tr>
<tr>
<td>( \eta ) : std</td>
<td>0.0432</td>
<td>0.041828</td>
<td>0.039349</td>
<td>0.038193</td>
</tr>
</tbody>
</table>

Table B.1: Channel parameters and the efficiency factor \( \eta \).
Figure B.1 shows the average decision SNR and BEP for the extremely large noise power case. Crosses in the figure, which mark the positions of the optimal integration time \( T_{\text{corr}}^{\text{opt}} \) for each channel model, indicate that \( T_{\text{corr}}^{\text{opt}} \)'s acquired by using these two criteria are the same and fit the results predicted by (B.6). This figure also shows that for a fixed \( N_s, B_w \) and \( E_f/N_0 \), the value of \( T_{\text{corr}}^{\text{opt}} \) increases as \( \frac{1}{\alpha} \) increases but with worse performance because the incoming noise power also increases. This figure verifies that excessive integration harms the performance less than short integration.

Figure B.2-B.4 show \( T_{\text{corr}}^{\text{opt}} \)'s acquired through minimizing the average BEP, maximizing the average decision SNR, and fineing the solution of (B.8), as well as the corresponding performance. In Figure B.2, the values of \( T_{\text{corr}}^{\text{opt}} \) obtained through different criteria are close at small \( E_f/N_0 \), but could be different at large \( E_f/N_0 \). Minimizing the average BEP produces larger \( T_{\text{corr}}^{\text{opt}} \) than maximizing the average decision SNR, and the value of \( T_{\text{corr}}^{\text{opt}} \) increases as \( E_f/N_0 \) increases. The value of \( T_{\text{corr}}^{\text{opt}} \) obtained by solving (B.8) is the largest one among the three because the received waveform energy acquired by integrating an exponential function can be overestimated. Even divergence resulted from different criteria is demonstrated, Figure B.3 and B.4 display that the influence of this divergence on both the average BEP and the average decision SNR is small, which allows us to acquire \( T_{\text{corr}}^{\text{opt}} \) easily through solving (B.8) or maximizing the average decision SNR instead of minimizing the average BEP. Figure B.3 also shows that compared to integrating over the channel delay spread, the correlator adopting the optimal integration time can have approximate 2dB gain at BEP=1e-4. As \( E_f/N_0 \) increases, \( T_{\text{corr}}^{\text{opt}} \) approaches \( T_{\text{mds}} \). Table B.1 includes the mean and the standard deviation of the efficiency factor \( \eta \) over the 100 channel realizations with the optimal integration time for the average BEP=1e-4 and
Figure B.1: Average BEP and average decision SNR for $E_p/N_0 = -23$dB ($E_b/N_0 = -10$dB) with $N_s = 10$.

$N_s = 10$. The mean value of $\eta$ decreases as $\frac{1}{a}$ increases, and small standard deviations show that the value of $\eta$ for every channel realization is close to each other.
Figure B.2: The optimal integration time obtained by minimizing the average BEP (labelled bep), maximizing the average decision SNR (labelled snr), and the solution of (B.8) (labelled exponential) with $N_s = 10$.

Figure B.3: The average BEP with optimal integration time obtained by minimizing the average BEP (labelled bep), maximizing the average decision SNR (labelled snr), and the solution of (B.8) (labelled exponential) with $N_s = 10$. 

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Figure B.4: The average decision SNR with optimal integration time obtained by minimizing the average BEP (labelled bep), maximizing the average decision SNR (labelled snr), and the solution of (B.8) (labelled exponential) with $N_s = 10$. 
Appendix C

Detailed Calculations for the MA Analysis with Conventional TR Modulation

C.1 Explicit Expression of Noise/Interference Variables

<table>
<thead>
<tr>
<th>Equation</th>
<th>Expression</th>
</tr>
</thead>
</table>
| \( n_0(1) \) | \[
\sum_{n=2}^{N} d_0^{(n)} b_0^{(n)} \left[ d_{-1}^{(n)} R_{1n}(\tau_n - T_I - (c_0^{(1)} - c_{-1}^{(n)}) T_c + T_{d_{-1}^{(n)}}) + d_0^{(n)} b_{-1}^{(n)} R_{1n}(\tau_n - T_I - (c_0^{(1)} - c_{-1}^{(n)}) T_c + T_{d_{-1}^{(n)}}) + d_0^{(n)} R_{1n}(\tau_n - (c_0^{(1)} - c_{-1}^{(n)}) T_c) + d_0^{(n)} b_0^{(n)} R_{1n}(\tau_n - (c_0^{(1)} - c_{-1}^{(n)}) T_c + T_{d_{-1}^{(n)}}) \right] \]
| \( n_0(2) \) | \[
\sum_{n=2}^{N} d_0^{(n)} b_0^{(n)} N_1(0, c_0^{(1)} T_c) \]
| \( n_0(3) \) | \[
\sum_{n=2}^{N} \sum_{m=2}^{N} \left\{ d_{-1}^{(n,m)} \left[ f_{m,n}(T_I - c_{-1}^{(n)} T_c - T_{d_{-1}^{(n)}}) + b_{-1}^{(n,m)} f_{m,n}(T_I - c_{-1}^{(n)} T_c - T_{d_{-1}^{(n)}}) + d_0^{(n)} b_{-1}^{(n,m)} R_{1n}(\tau_n - T_I - (c_0^{(1)} - c_{-1}^{(n)}) T_c + T_{d_{-1}^{(n)}}) + d_0^{(n)} b_{0}^{(n,m)} R_{1n}(\tau_n - T_I - (c_0^{(1)} - c_{-1}^{(n)}) T_c + T_{d_{-1}^{(n)}}) + d_0^{(n)} R_{1n}(\tau_n - (c_0^{(1)} - c_{-1}^{(n)}) T_c) + d_0^{(n)} b_0^{(n,m)} R_{1n}(\tau_n - (c_0^{(1)} - c_{-1}^{(n)}) T_c + T_{d_{-1}^{(n)}}) \right] \right\} \]
| \( n_0(4) \) | \[
\sum_{n=2}^{N} \sum_{m=2}^{N} \left\{ \left[ f_{m,n}(T_I - c_{-1}^{(n)} T_c - T_{d_{-1}^{(n)}}) + b_{-1}^{(n,m)} f_{m,n}(T_I - c_{-1}^{(n)} T_c - T_{d_{-1}^{(n)}}) + d_0^{(n)} b_{-1}^{(n,m)} R_{1n}(\tau_n - T_I - (c_0^{(1)} - c_{-1}^{(n)}) T_c + T_{d_{-1}^{(n)}}) + d_0^{(n)} b_{0}^{(n,m)} R_{1n}(\tau_n - T_I - (c_0^{(1)} - c_{-1}^{(n)}) T_c + T_{d_{-1}^{(n)}}) + d_0^{(n)} R_{1n}(\tau_n - (c_0^{(1)} - c_{-1}^{(n)}) T_c) + d_0^{(n)} b_0^{(n,m)} R_{1n}(\tau_n - (c_0^{(1)} - c_{-1}^{(n)}) T_c + T_{d_{-1}^{(n)}}) \right] \right\} \]

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Three steps are applied to manipulate $\mathbb{V} \text{ar}\{n_0(1)\}$. First, by using assumption (1) and (3) in section 5.1 which indicate the expectation values of $n_0(1)$ and the cross products in $n_0^2(1)$ are zero, the variance of $n_0(1)$ can be reduced to (C.1). Second, by using a change of variables which sets $x$ to $c_0^{(n)} - c_{-1}^{(n)}$ or $c_0^{(n)} - c_0^{(n)}$ and $y$ to $c_0^{(n)} + c_{-1}^{(n)}$ or $c_0^{(n)} + c_0^{(n)}$, as well as the equality $\int_a^b f(t)dt = \int_a^c f(t)dt + \int_a^c f(t)dt$, (C.1) is further simplified to (C.2).

$$\mathbb{V} \text{ar}\{n_0(1)\} = \sum_{n=2}^{N_a} \sum_{c_0^{(n)}=0}^{N^{(1)}_{h}-1} \frac{1}{N^{(1)}_{h} N^{(n)}_{h} T_f} \left\{ \begin{array}{l} N^{(n)}_{h} - 1 \left[ \int_{-\tau_f - (c_0^{(n)} - c_{-1}^{(n)}) T_f}^{\tau_f - (c_0^{(n)} - c_{-1}^{(n)}) T_f} R^2_{1n}(\tau_n)d\tau_n + \int_{-\tau_f - (c_0^{(n)} - c_{-1}^{(n)}) T_f + T_d^{(n)}}^{\tau_f - (c_0^{(n)} - c_{-1}^{(n)}) T_f + T_d^{(n)}} R^2_{1n}(\tau_n)d\tau_n \right] \\
+ \sum_{c_0^{(n)}=0}^{N^{(n)}_{h} - 1} \left[ \int_{-\tau_f - (c_0^{(n)} - c_0^{(n)}) T_f}^{\tau_f - (c_0^{(n)} - c_0^{(n)}) T_f} R^2_{1n}(\tau_n)d\tau_n + \int_{-\tau_f - (c_0^{(n)} - c_0^{(n)}) T_f + T_d^{(n)}}^{\tau_f - (c_0^{(n)} - c_0^{(n)}) T_f + T_d^{(n)}} R^2_{1n}(\tau_n)d\tau_n \right] \right\} \\
= \sum_{n=2}^{N_a} \sum_{x=-(N^{(n)}_{h})-1}^{N^{(1)}_{h}-1} \sum_{y=1}^{a_2} \frac{f_{-\tau_f-x\tau_f}^{T_f-x\tau_f} R^2_{1n}(\tau_n)d\tau_n}{2 N^{(1)}_{h} N^{(n)}_{h} T_f} + \frac{f_{-\tau_f-x\tau_f}^{T_f-x\tau_f} R^2_{1n}(\tau_n)d\tau_n}{2 N^{(1)}_{h} N^{(n)}_{h} T_f}, \\
= \frac{2}{T_f} \sum_{n=2}^{N_a} \int_{-\infty}^{\infty} R^2_{1n}(\tau_n)d\tau_n, \\
\right.$$
where \( a_1 = \max(-x, x) \) and \( a_2 = \min[2(N_h^{(1)} - 1) - x, 2(N_h^{(n)} - 1) + x] \). Equation (C.3) comes from the fact that the integration limits of the two integrals in (C.2) cover the whole region in which \( R_{1n}(\tau_n) \neq 0 \) for all possible \( x \) and \( y \).

### C.3 Derivation of \( \text{Var}\{n_0(4)\} \)

**Claim 2** \( \mathbb{E}\{f_{n,m,n}(\alpha, \beta, \beta, \alpha)\} = 0 \) for any \( n, m \) with the expectation being implicitly over \( \tau_n \) and \( \tau_m \).

**Proof:** By using (5.4) with \( c_0^{(1)} = 1 \) and interchange integrals, \( \mathbb{E}\{f_{n,m,n}(\alpha, \beta, \beta, \alpha)\} \) can be written explicitly as

\[
\mathbb{E}\left\{ \int_0^{T_{\text{corr}}} \int_0^{T_{\text{corr}}} \left[ \int_0^{T_i} \tilde{g}^{(n)}(t + T_d^{(1)} - \tau_n + \alpha)\tilde{g}^{(n)}(\nu - \tau_n + \alpha)d\tau_n \right] \\
\times \left[ \int_0^{T_i} \tilde{g}^{(m)}(t - \tau_m + \beta)\tilde{g}^{(m)}(\nu + T_d^{(1)} - \tau_m + \beta)d\tau_m \right] d\tau \right\}.
\]

For the term in the first pair of brackets not equal to zero, it needs that

\[
0 < |t - \nu + T_d^{(1)}| < T_{\text{mds}}.
\]

(C.4)

For the term in the second pair of brackets not equal to zero, it needs that

\[
0 < |t - \nu - T_d^{(1)}| < T_{\text{mds}}.
\]

(C.5)

But (C.4) and (C.5) are mutually exclusive because \( T_d^{(1)} \geq T_{\text{mds}} \). This completes the proof. \( \blacksquare \)
C.4 Derivation of $\text{Var}\{n_0(5)\}$

By using the same manipulation techniques shown in Appendix C.2, white noise assumption, and interchange integrals, the variance of $n_0(5)$ is computed in follows

$$\text{Var}\{n_0(5)\}$$

\[= \sum_{n=2}^{N_a} \sum_{c_0^{(1)}=0}^{N_h^{(1)}-1} \sum_{c_0^{(n)}=0}^{N_h^{(n)}-1} \frac{N_0}{2N_h^{(1)}N_h^{(n)}T_f} \int_0^{T_f} \left\{ \int_{c_0^{(1)}-c_0^{(n)}}^{c_0^{(1)}-c_0^{(n)}} \left[ \tilde{g}^{(n)}(t + \tau_n) \right]^2 dt \right\} d\tau_n \]

\[+ \sum_{n=2}^{N_a} \sum_{c_0^{(1)}=0}^{N_h^{(1)}-1} \sum_{c_0^{(n)}=0}^{N_h^{(n)}-1} \frac{N_0}{2N_h^{(1)}N_h^{(n)}T_f} \int_0^{T_f} \left\{ \int_{c_0^{(1)}-c_0^{(n)}}^{c_0^{(1)}-c_0^{(n)}} \left[ \tilde{g}^{(n)}(t + \tau_n) \right]^2 dt \right\} d\tau_n \]

\[= \sum_{n=2}^{N_a} \sum_{x=-N_h^{(n)}-1}^{N_h^{(1)}-1} \min\left[2(N_h^{(1)}-1)-x,2(N_h^{(n)}-1)+x\right] \frac{N_0}{4N_h^{(1)}N_h^{(n)}T_f} \]

\[\left\{ \int_{xT_f+T_d^{(1)}}^{xT_f+T_d^{(4)}} \left[ \tilde{g}^{(n)}(\tau_n) \right]^2 d\tau_n dt + \int_{-T_f+t}^{T_f+t} \left[ \tilde{g}^{(n)}(\tau_n) \right]^2 d\tau_n dt \right\}, \]

which conducts to

$$\text{Var}\{n_0(5)\} = \frac{N_0T_{\text{corr}}}{T_f} \sum_{n=2}^{N_a} \int_{-\infty}^{\infty} \left[ \tilde{g}^{(n)}(\tau_n) \right]^2 d\tau_n$$

because the integration limits of $\tau_n$ covers $[0, T_{\text{mds}}]$ in which $\tilde{g}^{(n)}(\tau_n) \neq 0$ for all possible $x$, $y$, and $t$. 

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