TCM For Frequency-Selective, Interleaved Fading Channels Using Joint Diversity Combining

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Abstract—The severity of frequency-selective fading channels necessitates the combining of multiple diversity sources to achieve acceptable performance. Traditional techniques often perform the combining of different sources of diversity separately, resulting in significant performance degradation. Recently, algorithms for the joint optimal combining were reformulated in a form suitable for practical implementation. In this paper we investigate the applicability of these algorithms for the specific problem of interleaved TCM systems in frequency-selective fading channels with or without external diversity. It is demonstrated that soft-decision equalization techniques are necessary and sufficient for the application of TCM techniques over such channels. In addition, it is shown that the design trade-offs associated with the resulting TCM techniques are significantly different than those associated with memoryless channels.

I. INTRODUCTION

Currently, there is great interest in narrowband time-division multiple access (TDMA) digital cellular systems for mobile radio communication. The North American (IS-54) and European (GSM) cellular phone systems as well as the Personal Communication Services (PCS) systems are representative examples of such designs [1], [2]. Transmission channels in mobile radio systems are often characterized as time-varying fading multipath channels corrupted by additive white Gaussian noise (AWGN). In order to cope with these severe channel conditions, three main diversity mechanisms are employed: (i) time diversity achieved by interleaving and error-correcting codes, (ii) frequency diversity achieved by the frequency-selective fading, slow frequency hopping, or multicarrier transmission and (iii) spatial diversity achieved via antenna arrays. The need for diversity can be appreciated from the fact that without it the error rate decays as \( (SNR)^{-1} \), where SNR is the average signal-to-noise ratio. However, with \( N \) orders of diversity the error rate decays roughly as \( (SNR)^{-N} \) [3]. Furthermore, it has been shown that as \( N \to \infty \), the performance of the system approaches that of the AWGN channel with the same average SNR [4]. Traditional diversity combining techniques often perform the combining of different sources of diversity separately. For example a Viterbi Equalizer is used to combine the frequency diversity from the channel, providing hard estimates of the coded symbols; these hard estimates are deinterleaved and passed to a second Viterbi decoder that combines the time diversity of the code. The limitations of such separate combining are evident when coded modulation is used. Specifically, TCM codes fail to provide coding gain when they operate on an interleaved frequency-selective fading channel [2]. A non-interleaved system may be used to overcome this, by allowing the use of a joint MLSE receiver, but suffers from a significant reduction of time diversity [2]. This is attributed to the fact that the outer code in conjunction with the interleaver constitute a powerful long overall code with built in time diversity.

Early attempts to implement some form of joint processing were centered around joint Decision Feedback Equalization (DFE), deinterleaving and decoding [5]. More recently, techniques that calculate A-Posteriori Probabilities (APP’s) of the coded symbols conditioned on a single column observation, which are then used by the outer Viterbi decoder, have been derived [6], [7]. These approaches are based on several soft decision algorithms developed in the literature [8], [9], [10], [11], [12], [13], [14]. Among these, the Minimum Sequence Metric (MSM) algorithm [14] which can be interpreted as a more efficient implementation of the SSA algorithm of [12], is an attractive candidate for practical implementation, both for its relatively low complexity and its negligible performance degradation with respect to the optimum scheme. The impact of the above mentioned advances on system design is dramatic: A large selection of existing coding/modulation techniques designed for the AWGN channel are applicable to the frequency-selective fading channel, when joint diversity combining is employed. In addition, a new research topic is to identify coding/modulation schemes that take full advantage of the power of these modern receivers. In this paper, only a small subset of this design space is explored. In particular, the problem of effectively implementing TCM codes in a frequency-selective fading channel is examined. A relevant issue—that does not appear to be widely appreciated—is also discussed extensively. Specifically, unlike the case of AWGN channels, in a frequency-selective fading channel a unified comparison of different systems in terms of their error probability as a function of the “bit energy to noise ratio” \( \gamma_b \) is not applicable. This is due to the fact that in the latter case, a reduction in Bandwidth (BW) has two main consequences: (i) reduction of the diversity order (number of channel taps) and (ii) increase of the channel dynamics (normalized Doppler spread). The synergistic effect of these competing mechanisms results in quite interesting trade-offs. Consideration of the issue of channel estimation, which is not addressed in this paper (refer to [15], [16] for joint channel estimation and soft decision algorithms), is expected to also affect the trade-offs for practical systems.

The remainder of the paper is structured as follows: Section II contains a description of the transmitter, the channel model and the receiver structure. In Section III we present the different design goals together with extensive simulation results. Concluding re-
Consider the communication system depicted in Figure 1, consisting of a memoryless binary source which outputs bits with a rate $R_b$ (bits/sec). The bit stream is encoded by a rate $R_c$ convolutional code with memory length $L_c$. The code rate $R_c$ is such that each output symbol is mapped on a finite symbol alphabet $\{U_1, \cdots, U_m\}$ of size $m$, resulting in a sequence with rate $R_s = R_b / (R_c \log_2 m)$ (symbols/sec). The trellis-coded symbols are interleaved using a size $J \times K$ block interleaver, pulse-shaped, transmitted through a frequency selective fading channel, employing in general $N^{1/b}$ order diversity, and are observed in white noise. At the receiver side the sequence is match-filtered with the known channel shape and sampled at the symbol rate. These samples provide sufficient statistics for further processing, since the channel is assumed known [17]. The equivalent discrete-time model for the above scenario consists of the convolution of the coded symbols with an $(L + 1)$-tap FIR, time varying channel:

$$z_k^i = \sum_{jk} h_{k,j} u_{k-n} + n_k^i \quad i = 1, \cdots, N$$

where $h_{k,j}^i$ is the $i^{th}$ tap of the channel at time $k$, for the $i^{th}$ diversity branch, $u_k$ is the coded symbol and $n_k^i$ is a white complex Gaussian noise with $E[|n_k^i|^2] = N_0$. The channel and noise on each diversity branch are assumed to be independent and identically distributed. The source symbols and the channel taps are normalized such that average signal energy per diversity branch is $E_s/N$. A commonly used model for the dynamics of the fading process is the two-dimensional isotropic scattering model, first suggested by Clarke [18]. In the isotropic scattering model the autocorrelation function of the symbol-sampled, lowpass equivalent fading process can be expressed as

$$\phi(k) = J_b \left(2\pi \nu_d k\right)$$

where $\nu_d = f_d T_s$ is the normalized Doppler spread of the channel, and the corresponding power spectral density is

$$S(\nu) = \begin{cases} \frac{1}{\sqrt{2\pi(\nu_d)^2}} & |\nu| < \nu_d \\ 0 & \text{otherwise} \end{cases}$$

An associated frequency selective fading model which is widely assumed is the wide sense stationary, uncorrelated scatter (WSSUS) model of Bello [19]. Under the WSSUS assumption the channel taps are modeled as uncorrelated complex Gaussian processes with power spectral density given by (3). Regarding the interleaver design, the depth is chosen such that successive coded symbols, which are actually transmitted $J$ symbols apart, are independently faded, while the width of the interleaver $K$ is chosen to separate any $L_D + 1$ successive symbols as far as possible, where $L_D$ is the decoding depth of the code [2]. These design constraints are met with $J > 1/(2\nu_d)$ and $K > T L_s$ [20].

Two separate tasks are performed at the receiver. The first—referred to as inner equalization—consists of combining the diversity provided by the uncorrelated branches with the implicit diversity provided by the frequency selectivity of the channel, while the second—referred to as outer decoding—involves the combining of the information made available by the previous task, with the time diversity of the code. The recently introduced APP and MSM algorithms [14]—which are equivalent to the OSA and SSA algorithms [12] respectively, with approximately $m$ times less complexity—are used for the inner equalization. The former algorithm (OSA or APP) provides an “information packet” that is related to the APP of the coded symbols $u_{k-D}$ based on the observations $z_{1}^i, \cdots, z_{L}^i$, $i = 1, \cdots, N$, while the latter (SSA or MSM) provides the minimum metric (squared distance) of the sequence $u_1, \cdots, u_{k+1}$ for which $u_{k-D} = U_j$, $j = 1, \cdots, m$, and $D$ is an appropriately chosen delay. A formal description of these algorithms is not given here. It suffices to say at this point that they both have a similar structure: at each step, one forward and $D - L$ backward recursions are performed, for each state corresponding to the inner ISI channel.

As it will be evident from the results in the next Section, the performance of soft-decision algorithms is almost insensitive to the selection of APP or MSM, and it mostly depends on the selection of the delay parameter $D$. Based on the above observation, and the fact that the MSM is less complex than the APP type of algorithm, we will use the former as a representative of the class of soft-decision algorithms. The overall complexity of a receiver employing soft-decision or hard-decision equalization in cascade with Viterbi decoding can be expressed as

$$C_{hard} = 2^{L_c} + m^L$$

$$C_{soft} = 2^{L_c} + (D - L) \cdot m^L$$

where $L_c$, $L$, $m$ and $D$ are the code memory, channel memory, alphabet size and decision delay respectively. The first term in both equations is the complexity associated with the Viterbi decoder for the outer convolutional code, while the second term is related to the Viterbi equalizer and soft-decision equalizer (APP or MSM) respectively.

III. DESIGN GOALS AND RESULTS

Several potential applications of coded modulation to TDMA mobile radio systems are considered in this section. The design trade-off for this frequency-selective channel is more complex than that for an ideal AWGN channel. In particular, we assume that the delay spread and the Doppler spectrum of the physical channel are fixed. Thus, varying the channel symbol rate ($R_s$) directly affects the normalized delay spread ($L + 1$) and the normalized Doppler spread ($\nu_d$). The effects of the change in $\nu_d$ are not particularly relevant for the results described herein since we assume perfect channel state information and use a sufficiently large interleaver in all cases. However, the effect of a varying $L$ is significant and results in an enrichment of the design trade-off relative to an
AWGN channel. We assume that the pulse-shaping is held constant for all of the examples given so that the bandwidth is proportional to $R_s$ in all cases. We define the parameter $\eta = R_0/R_s$ as a measure of the spectral efficiency.

The following applications of coded modulation to interleaved, frequency-selective channels are considered:

- **Coding Gain without Bandwidth Expansion:** All systems compared in this case have the same $R_0$ and $\eta$. Coded modulation techniques are considered as a method to provide improved performance (i.e., coding gain) with the only cost being increased receiver complexity (i.e., no bandwidth expansion). We also investigate the role of explicit, uncorrelated diversity branches obtained using multiple carriers.

- **Throughput Increase/Bandwidth Reduction Trade-offs:**
  
  In an ideal AWGN channel, the ability to obtain coding gain without bandwidth expansion leads directly to the ability to increase throughput (i.e., $\eta$) by trading power in addition to complexity. This is clearly the case because the error performance is just a function of the symbol energy, which can be maintained at a constant level by increasing the power in the same proportion that $R_s$ is increased. In the frequency-selective fading channel the trade-off is complicated by the fact that increasing $R_s$ also increases $L$. This has a positive effect of increasing the frequency diversity experienced by the coded symbols, but the negative effect of increasing the complexity of the processing required to integrate this diversity. Again, in an ideal AWGN channel, the ability to reduce the required bandwidth without reducing the data throughput is a direct consequence of the abilities described above.

  However, this is also complicated by the mobile channel model. Specifically, if a coded modulation scheme is used to reduce $R_s$ by a factor of two, the normalized delay spread $L+1$ is also reduced by a factor of two, which results in less frequency diversity and a less complex equalizer.

  Extensive simulation results have been obtained for a variety of system configurations in order to quantify the above mentioned trade-offs. A detailed summary of the relevant system parameters is presented in Table 1 (for the column labeled $C_{\text{soft},b}$, $D = 2L$ was assumed). We note that for all the systems a $500 \times 50$ interleaver was used. The rate 2/3 8PSK TCM codes are the ones described in [21, pp. 120], while the rate 1/2 convolutional codes are the best AWGN codes given in [22].

<table>
<thead>
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<th>$m$</th>
<th>$R_c$</th>
<th>$L_c + 1$</th>
<th>$N$</th>
<th>$\eta$</th>
<th>$L + 1$</th>
<th>$C_{\text{hard},d}$</th>
<th>$C_{\text{soft},b}$</th>
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<td>1</td>
<td>2</td>
<td>40</td>
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<tr>
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<td>2/3</td>
<td>6</td>
<td>1</td>
<td>2</td>
<td>96</td>
<td>160</td>
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<tr>
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<td>6</td>
<td>1</td>
<td>2</td>
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<td>20</td>
<td>24</td>
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</table>

**TABLE 1**

**SUMMARY OF SYSTEM PARAMETERS**

A. **Coding Gain without Bandwidth Expansion**

As a first example of systems that provide the same throughput $R_0$ and occupy the same BW, the performance of system S2 using the APP and MSM algorithms for different delays $D$ is presented. For comparison, the performance of uncoded QPSK is also considered. The results are shown in Figure 2. The previously noted result that coding gain cannot be obtained using hard decision equalization is confirmed [2]. Comparing the different soft-decision algorithms, it becomes apparent that MSM behaves almost identically to APP. Moreover, a delay $D = 2L$ proves to be—for all practical purposes—adequate to close the performance gap between the two extremes (i.e. $D = L$ and $D = \infty$). Based on this observation, the MSM algorithm with $D = 2L$ will only be considered for the rest of the simulation results. Comparing hard versus soft-decision algorithms, we observe that the combination of the MSM algorithm and Viterbi decoding, enables us to effectively combine the frequency diversity from the channel with the time diversity of the code/interleaver, and trade complexity ($160$ state updates) for coding gain. This is evident from the slopes of the performance curves corresponding to the soft decision algorithm which suggest a diversity order of more than 6; a result that strengthens our initial choice of AWGN codes.

The addition of explicit diversity further illustrates the superiority of the joint diversity combining method. The particular implementation of the explicit diversity can be either in the form of a repetition code, or by using $N$ carriers sufficiently separated, carrying the same information. Both implementations result in the same BW for the compared systems S5 and S6. The error probability curves are depicted in Figure 3. The superiority of the QPSK system when hard decision decoding is used is on the order of 2dB for an error probability of $10^{-5}$; the 8PSK system almost fails to provide any practically acceptable performance. We stress here that the combining of the explicit uncorrelated diversity and the implicit diversity of the frequency-selectivity is done jointly, in the optimal manner as dictated by Maximum Likelihood (ML) theory. The scenario changes dramatically when soft decisions are used; it is now possible to utilize the higher order constellation TCM code, and with the additional use of the explicit frequency diversity, to obtain better performance with respect to the QPSK system by approximately 2dB at $P_b = 10^{-4}$, for roughly the same complexity (note that the $N = 2$ system requires one RF stage for each
B. Throughput Increase/Bandwidth Reduction trade-offs

In Section III-A, systems of varying complexity, but fixed BW and throughput, were considered. Although comparing two systems under same throughput and BW requirements seems to be the most fair comparison, in most practical systems, the BW is already allocated and what is required is a system that provides increased throughput by trading SNR and/or complexity. Alternatively, we would like to know how BW reduction can be achieved using higher order constellations and joint diversity combining. Finally, it would be desirable to assess the additional SNR loss encountered when reducing the BW in a frequency-selective fading channel, due to the loss of frequency diversity. All the above concepts are illustrated in Figure 4. The results in Figure 4 indicate that it is almost impossible to trade SNR and complexity for increased throughput or reduced BW when hard decision decoding is used. In particular, the 8PSK/TCM coded systems, which provide twice the throughput or half the BW of the QPSK coded systems, operate with probabilities of bit error greater than 10^-2 for the entire practical range of SNR. On the other hand, using soft-decision decoding at the first stage of the receiver, enables us to either (i) double the throughput in the same BW by operating at 2.5dB higher Er/N0, or (ii) reduce BW by half for the same throughput in the expense of 4dB (system S1 versus S5). By comparing the same system under different BW requirements (e.g. S1 versus S3 and S4 versus S5) the disagreement with the AWGN case can be quantified. In particular the systems S3 and S5 benefit by 1.75dB and 0.75dB respectively from the higher diversity channel, which means that when BW (i.e. data rate) is doubled, an SNR loss of only 3 - 1.75 = 1.25 and 3 - 0.75 = 2.25dB respectively is experienced for the same performance level. This is contrasted to the AWGN case, where the corresponding loss would be 3dB in all cases.

IV. CONCLUSIONS

There are three conclusions that can be drawn from the results reported in Section III. First, using a TCM scheme with a constellation size greater than four on a frequency-selective, interleaved fading channel is not possible using a VA-based equalizer making hard decisions on the coded symbols. Second, the use of TCM on such channels is enabled by the use of soft decision processing on the coded symbols. In fact, for the particular examples presented, the simple MSM algorithm with D = 2L provided this enabling feature. Third, while the use of joint combining of all diversity sources allows one to use TCM for this mobile channel, the details of the design trade-offs depend intimately on the intended application. The result of this last point is that it is not possible to translate a single performance curve for all applications, as may be done in the case of an ideal AWGN channel.

The results presented in this paper lead directly to many interesting open problems. For example, the trade-off between channel estimation error effects, modulation, coding, and joint diversity combining are unclear. Finally, we point out that there is no simple way to predict the performance gains associated with joint diversity combining which can be expected for a given coding and modulation combination.

REFERENCES


