MLSE for an Unknown Channel—Part II: Tracking Performance

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Abstract—The channel-tracking mode performance of the front-end (FE) processors which were developed in a companion paper [1] is determined through approximate analysis and computer simulation. A general model for the FE processing, which encompasses these representative FE’s, is assumed. Previous analysis techniques are extended to include the effects of FE processing. The specific system analyzed consists of a Rayleigh-fading, diffuse multipath channel with several data pulse shapes considered. An adaptive maximum likelihood sequence estimation (MLSE) algorithm based on the per-survivor processing (PSP) technique is analyzed and compared to an algorithm based on correct symbol feedback. The results show that significant performance degradation is suffered when suboptimal FE processing is used. The limitations of the results and the models used are discussed.

I. INTRODUCTION

A FRAMEWORK for the design of recursive receivers to approximate unknown channel maximum likelihood sequence estimation (MLSE) was developed in a companion paper [1]. An important aspect of this framework is the conversion from the continuous time (CT) received signal to a discrete time (DT) version suitable for algorithm implementation in digital hardware or software. As pointed out in [1], this front-end (FE) processing stage has often been overlooked or neglected. The “(Tc spaced) fractionally spaced whitened matched filter (FS-WMF)” front-end was derived in [1] and shown to be optimal for practical purposes. Specifically, it provides a set of sufficient statistics for the joint-ML estimation of a digital sequence and an unknown intersymbol interference (ISI) channel when the corrupted signal is observed in additive white Gaussian noise (AWGN) and the physical channel is assumed to be a tapped-delay-line (TDL). The spacing of this TDL, referred to as the “resolution time” Tc, must be less than \(1/(2B)\), where \(B\) is the approximate bandwidth of the overall channel, in order to accurately approximate the CT physical channel. Several suboptimal FE processors were developed in [1]. These FE’s are representative of those suggested in the literature and may also be viewed as techniques to achieve a complexity reduction relative to the FS-WMF.

The primary objective of this paper is to assess the effects of FE processing on the tracking mode performance. In addition to the optimal (Tc spaced) FS-WMF, we consider the “undersampled whitened matched filter (US-WMF)” and the “decimated whitened matched filter (D-WMF).” The relationship between these FE’s and the sampled FE, which is discussed in [1], implies that the effects of employing variations of the sampled FE can be determined by translating the results obtained in this paper. Another objective of this paper is to provide quantitative and qualitative results which will be useful as guidelines when designing a receiver algorithm for mobile communication systems.

To obtain these objectives, we consider a specific postprocessor which has been shown to be effective and may be viewed as one design alternative arising from the framework established in [1]. Specifically, the Viterbi algorithm (VA) is adopted as a suboptimal tree-search algorithm with metrics based on the least mean square (LMS) algorithm. The resulting algorithm is a version of the previously introduced “per-survivor processing (PSP)” algorithm [2], generalized to vector signals. This LMS-PSP post-processor has been shown to achieve tracking mode performance which is superior to that of nonadaptive MLSE and “Conventional Adaptive MLSE,” [3], [4], where the latter terminology implies that a single channel estimate is maintained and updated based on tentative decisions fed back from the VA [5], [2]. The VA decoding depth for providing tentative decisions to the channel estimator is a critical parameter for Conventional Adaptive MLSE. The design trade-off is that a large delay is required for reliable decision feedback, while a small delay is desirable in order to track channel dynamics. The PSP technique eliminates this trade-off by providing parallel, zero-delay decision feedback for per-path channel estimation. As a result, PSP provides superior performance and robustness.

There are two distinct modes of operation for a joint sequence and channel estimator: acquisition and tracking. The acquisition mode is when the channel is completely unknown. The tracking mode is when the channel estimates maintained by the algorithm are close to the actual channel. The tracking mode, which is most interesting when the channel is time-varying, is relevant since most packet mobile radio formats include a training period to initialize the channel estimate, while mobility and fading provide a time-varying ISI channel [6]. An important characteristic of tracking mode performance,
TABLE I
SUMMARY OF NOTATION FROM PART I

<table>
<thead>
<tr>
<th>Description</th>
<th>Part I Eq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>${a_i}$</td>
<td>data symbols; i.i.d., uniform over $M$-ary alphabet $\mathcal{A}$</td>
</tr>
<tr>
<td>$N_s$</td>
<td>number of samples per symbol duration at FE output</td>
</tr>
<tr>
<td>${f_{m,n}}_{m=0}^{L-1}$</td>
<td>unknown $(N_s \times 1)$ component-vector channel</td>
</tr>
<tr>
<td>${z_i}$</td>
<td>$(N_s \times 1)$ received vectors</td>
</tr>
<tr>
<td>${w_i}$</td>
<td>$(N_s \times 1)$ Gaussian noise vectors: $\mathbb{E}{w_iw_i^H} = N_0 I$</td>
</tr>
<tr>
<td>$\alpha_i^H$</td>
<td>$\begin{bmatrix} a_{i-L+1} &amp; a_{i-L+2} &amp; \cdots &amp; a_i \end{bmatrix}^H$</td>
</tr>
<tr>
<td>$A_k$</td>
<td>Data matrix: $\alpha_k \mid \alpha_{k-1} \mid \cdots \alpha_0$</td>
</tr>
<tr>
<td>$H_{f, z, w}$</td>
<td>vector of ${f_i}$, ${z_i}$, ${w_i}$, respectively</td>
</tr>
</tbody>
</table>

in direct contrast to the acquisition mode, is that the symbol error rate is not a function of time—i.e., the algorithm is in steady state. Although the FE processing does affect the performance in the acquisition mode, other factors may be more significant. For example, two different hypothesized sequence/channel pairs may give rise to the same noise-free signal. This is predicted by the metric derived in [1] (i.e., different data matrices may have the same associated projection matrix—see (I-48) with $\rho = 1$). Differential encoding and decoding can be used to resolve simple ambiguities, but our unpublished results suggest that more complicated cases exist and give rise to convergence to false minima of the metric. Thus, to demonstrate the importance of FE processing we concentrate on the tracking mode because i) it has no such complications, ii) an important application exists, and iii) previous investigations into the tracking mode performance have neglected the effects of FE processing.

The tracking mode performance of receivers with various FE’s and a LMS-PSP post-processor is assessed via Monte Carlo simulation and approximate analysis for a Rayleigh fading TDLC channel model. The approximate analysis represents an extension of three previous results to the vector signal model. The fading channel MLSE analysis of Sheen and Stüber [7] is modified, and a simplified error-event search algorithm is suggested. The original LMS channel estimation error analysis of Widrow et al. [8] is modified to account for FE processing. The joint channel/sequence estimation analysis of Magee and Proakis [9] is also extended and refined. While all results are stated in terms of a general FE processing structure, the symbol-spaced ISI model, which is often assumed, may be viewed as a special case.

This paper is organized as follows: The channel model and signal assumptions are stated in Section II. The approximate analysis is developed in Section III. Numerical results for this analysis and the Monte Carlo simulations are given in Section IV. Section V interprets the numerical results and discusses relevant limitations and open issues.

1"(I-XX)" will be used to refer to equation (XX) from [1].
2See Section V-B for the limits of this statement.

II. SIGNAL MODELS

It was shown in [1] that the signal models at the output of the FS-WMF, US-WMF, and D-WMF front-ends were each a component vector ISI model

\[ \tilde{z}_i = \tilde{x}_i + \tilde{w}_i = \sum_{m=0}^{L-1} a_{i-m} f_{m} + \tilde{w}_i, \]

\[ i = 0, 1, 2, \cdots k. \]

where the notation from [1] is summarized in Table I. As shown in [1], the signal model for the US-WMF contains additional, typically unexpected, ISI (i.e., $f_{m,n}$ for $m \notin Z_L$ are not necessarily zero). For the US-WMF analysis contained in this paper, the general model of (1) will be used and the effects of additional ISI will be assessed separately.

The LMS channel estimation scheme adapted here is defined by [10]

\[ \tilde{f}_k = \tilde{f}_{k-1} + \beta(\tilde{x}_k - \tilde{\alpha}_k) \tilde{f}_{k-1} \tilde{\alpha}_k \]

where $\beta \in \mathbb{R}$ is the LMS step size and the "mixed (inner) product" notation $\langle \cdot \rangle$ is defined in (I-2).4 The approximate joint-ML metric for a given path is

\[ \Lambda(\tilde{A}_k) = \Lambda_{k-1}(\tilde{A}_{k-1}) + \|\tilde{z}_k - \tilde{\alpha}_k^H \tilde{f}_{k-1}\|^2. \]

The objective of the receiver is to select the path with minimum metric. In the PSP receiver, an estimator of the form in (2) is maintained for each survivor in the VA and used for the metric computations of (3). In this PSP case, the data sequence used to update a given channel estimate is the associated survivor sequence (i.e., $\tilde{\alpha}_k = \tilde{x}_k$). We will also consider the artificial case in which only one channel estimate, based on the transmitted data sequence (i.e., $\tilde{\alpha}_k = \alpha_k$), is maintained and used for all metric computations. This case will be referred to as "correct symbol feedback (CSFB)."

The specific form of the FE processing is determined by the relation between the equivalent channel $\tilde{f}$ and the tap

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3The notation $Z_N = \{0, 1, \cdots, N - 1\}$ is used for any integer $N$.
4The mixed product notation will be used throughout but may be eliminated if each component of the data matrix is replaced by a weighted $(N_s \times N_s)$ identity matrix.
coefficients of the physical channel \( \{ c_m \}_{m=0}^{N_Lc} \). While the details of this relation for each FE are contained in [1], the equivalent channel coefficients in each case may be expressed as \( f_m = V_m e \), where \( e \) is the \((N_Lc + 1) \times 1\) vector of tap coefficients, so that

\[
f = \begin{bmatrix} f_{L-1} \\ \vdots \\ f_1 \\ f_0 \end{bmatrix} = \begin{bmatrix} V_{L-1} \\ \vdots \\ V_1 \\ V_0 \end{bmatrix} e = V e. \tag{4}
\]

The \((N_L - 1) \times (N_Lc + 1)\) matrix represents the effects of the equivalent DT pulse as defined in [1] for each FE [i.e., see (I-12) and (I-37)].

The model used for the TDL physical channel is illustrated in Fig. 1(a). This dynamic channel model assumes that the channel taps \( \{ c_m \}_{m=0}^{N_Lc} \) are mutually independent and change at the symbol rate, so that the \( m \)th channel tap at time \( k \), \( c_m(k) \), is the output of a linear time invariant filter with unit gain frequency response \( D(\nu) \) when the input is a white complex circular Gaussian sequences.\(^5\) A quasistatic assumption for the channel is assumed throughout the analysis, so that the temporal argument \( (k) \) is suppressed except when considering the LMS lag error. This model may be viewed as a convenient technique for introducing channel dynamics, but as originally developed by Bello [11], it is also a simple model for a wide-sense-stationary, uncorrelated-scatter (WSSUS) fading channel when the pulse/channel bandwidth is \( \leq 1/(2T_s) \). When interpreted as a diffuse multipath channel model, the bandwidth of \( D(\nu) \) corresponds to the maximum Doppler frequency.

III. APPROXIMATE ANALYSIS

An exact analysis of a joint-ML channel and sequence estimation algorithm is difficult. In this section, we obtain approximate results by assuming a quasistatic channel model and modifying well-known techniques to account for a general DT front-end processor. We begin the analysis by assuming that the fading channel coefficients are known at the receiver; we then generalize the result to the CSFB and PSP cases. Finally, the analysis is modified to predict error floors resulting from channel tracking error.

A. Channel Side Information

The analysis of [7], which assumes that the exact value of \( e \) is available at the receiver, is modified to account for the effects of FE processing in this section. The quasistatic channel model,\(^6\) allows the VA performance to be approximated using the well-known analysis techniques developed in [12]. Thus, the symbol-error probability \( P_s \) is upper-bounded by

\[
P_s \leq \sum_e w(e) P_C(e) P_{PW}(e) \tag{5}
\]

where \( e = a - \hat{a} = (\cdots, 0, c_k, c_{k+1} \cdots c_{k+K-L+1}, 0, 0 \cdots) \) is an error sequence corresponding to an error event of length \( K \) beginning at time \( k \). An allowable error event is one in which no more than \( L - 2 \) consecutive \( e_i \) values are zero for \( i \in \{ k, k + 1, \cdots, k + K - L + 1 \} \), and \( w(e) \) is the number of these values which are nonzero. The probability that the error event is consistent with the transmitted sequence \( a \) is denoted by \( P_C(e) \) and accounts for the fact that the sum is over error sequences and not pairs of correct sequences \( a \) and incorrect estimates \( \hat{a} \). The pairwise error probability \( P_{PW}(e) \) is the probability that the metric associated with \( a \) exceeds the metric associated with \( \hat{a} \)

\[
P_{PW}(E) = \Pr\{ \| z - A \circ f \|^2 > \| z - (A + E) \circ f \|^2, z = A \circ f + w \} \tag{6}
\]

where the transmitted data and error sequences have been represented in matrix form (i.e., \( P_{PW}(e) = P_{PW}(E) \)), and the vector signal notation has been used (i.e., see [1] and Table I).

Conditioned on the value of the physical channel, the pairwise error probability is

\[
P_{PW}(E|e) = P_{PW}(E|d^2(E)) = Q \left( \sqrt{\frac{d^2(E)}{2N_0}} \right) \tag{7}
\]

where the squared distance associated with an error event is defined as

\[
d^2(E) \triangleq \| E \circ f \|^2 = c^H R c \tag{8}
\]

and

\[
Q(x) \triangleq \int_x^\infty \frac{1}{\sqrt{2\pi}} \exp(-t^2/2) dt. \tag{9}
\]

The \((N_Lc \times N_Lc)\) nonnegative-definite matrix \( R \) is defined as

\[
R \triangleq f^H E^H E \circ f = \sum_{i=0}^{L-1} \sum_{j=0}^{L-1} R_c(i, j) V_i^H V_j \tag{10}
\]

with \( R_c(i, j) \) being the element of \( E^H E \) in row \( i \) and column \( j \).

\(^5\) We will use the terms “Gaussian” and “white Gaussian” for brevity.

\(^6\) It will become apparent that the channel should be approximately static over the length of significant error events.
The expression in (8) must be averaged over the statistics $c$, or equivalently $d^2(E)$, to obtain $P_{PW}(E)$. After expanding $c$ in terms of an orthonormal set of eigenvectors for $R$ (with corresponding nonzero eigenvalues $\{\lambda_i\}_{i=1}^{D-1}$), the moment generating function of $d^2(E)$ is

$$
\Phi_{d^2}(s) = \mathbb{E}\{\exp(sd^2(E))\} = \prod_{i=0}^{D-1} \frac{\lambda_i s}{1 - \lambda_i \sigma_s^2 s^2}.
$$

where relabeling the eigenvalues so that $\{\lambda_i\}_{i=1}^{D-1}$ are distinct, with $\lambda_i$ repeated $m_i$ times, and performing a partial fraction expansion results in

$$
\Phi_{d^2}(s) = \sum_{i=1}^{D} \sum_{j=1}^{m_i} \frac{\mu_{ij}}{(1 - \lambda_i \sigma_s^2 s^2)^j}.
$$

Inverting the transform and averaging over the statistics of $d^2(E)$ results in [7]

$$
P_{PW}(E) = \sum_{i=1}^{D} \sum_{j=1}^{m_i} \mu_{ij} \left(1 - \frac{p_i}{2}\right)^{j-1} \sum_{n=0}^{j-1} \binom{n+j-1}{n} \left(1 + \frac{p_i}{2}\right)^n,
$$

where

$$
p_i = \sqrt{\frac{\sigma_s^2 \lambda_i / (4N_0)}{\sigma_s^2 \lambda_i / (4N_0) + 1}}.
$$

Since the pairwise error probability $P_{PW}(E) = P_{PW}(\{\lambda\})$ depends only on the eigenvalues of $R$, which may be the same for different error events, the sum in (5) and the well known lower bound may be written as

$$
K_{LB} P_{PW}^{\text{max}} \leq P_s \leq \sum_{\{\lambda\}} K_{UB}(\{\lambda\}) P_{PW}(\{\lambda\})
$$

where the constants are defined by

$$
K_{LB} = \sum_{e \in \mathcal{E}^{\text{max}}(\{\lambda\})} P_{C}(e),
$$

$$
K_{UB}(\{\lambda\}) = \sum_{e \in \mathcal{E}(\{\lambda\})} w(e) P_{C}(e),
$$

with $\mathcal{E}(\{\lambda\})$ denoting the set of all allowable error sequences with a common set of eigenvalues. The lower bound requires knowledge of $\mathcal{E}^{\text{max}}(\{\lambda\})$, the set of allowable error sequences with the maximum associated pairwise error probability, $P_{PW}^{\text{max}}$. The sum in (15) is over all distinct sets of eigenvalues. An approximation for $P_s$ at low error rates is

$$
P_s \approx \sum_{e \in \mathcal{E}^{\text{max}}(\{\lambda\})} w(e) P_{C}(e) P_{PW}^{\text{max}} = K_A P_{PW}^{\text{max}}.
$$

If the physical channel, and hence $d^2(E)$, were deterministic, it is apparent from (7) that the value of $P_{PW}^{\text{max}}$ could be determined by finding the minimum value of $d^2(E)$. For the random channel model, determination of $P_{PW}^{\text{max}}$ and the other significant terms in the bound of (15) is more difficult. The method suggested in [7] requires computation of the eigenvalues and the associated partial fraction expansion for each error event investigated. This computation is especially costly for the fractionally-spaced FE processors under consideration. In place of this search, we suggest a simple approximate characterization of $P_{PW}(E)$; we rank error events corresponding to the average of $d^2(E)$ over the channel statistics

$$
d^2_{\text{ave}}(E) = \mathbb{E}\{d^2(E)|E\} = \sigma_s^2 \text{tr}(R).
$$

Although $P_{PW}(E)$ is not monotonic in $d^2_{\text{ave}}(E)$, in all cases investigated it was found that, for $P_s < 0.1$, the maximum pairwise error corresponded to the sequences with the minimum values of $d^2_{\text{ave}}(E)$. The obvious advantage of ranking error sequences by this average squared distance value is that a preliminary search to roughly characterize the error contribution of a very large number of error events can be conducted without computation of the eigenvalues and the partial fraction expansion. A second search over a subset of error events with small values of $d^2_{\text{ave}}(E)$ can then be conducted with the exact error contribution computed.

Any interference from unexpected ISI suffered by the US-WMF can be approximately accounted for by replacing $N_0$ by an effective value, $N_{0,\text{eff}} = N_0 + \sigma_s^2$ with

$$
\sigma_s^2 = \frac{\sigma_k^2}{N_s} \sum_{m \in \mathcal{E}_I} \mathbb{E}\{|f_m|^2\},
$$

where $\sigma_k^2 = \mathbb{E}\{|a_k|^2\}$.

B. Channel Estimation Effects when Thermal Noise Dominate

The above analysis can be modified to approximate the effects of an LMS channel estimator by incorporating channel estimation error into $N_{0,\text{eff}}$ as suggested in [9]. When thermal noise dominates the contribution from the channel estimation error, we may model the composite effect as AWGN.

For the CSFB case, the effective thermal noise level can be defined as

$$
N_{0,\text{eff}} = \frac{1}{N_s} \mathbb{E}\{|h_k - \alpha_k^H \circ \hat{f}_{k-1}|^2\} + \sigma_s^2 = N_0 + \sigma_s^2 + \sigma_{\text{est}}^2.
$$

where the mean-squared error (MSE) associated with the channel estimator is

$$
\sigma_{\text{est}}^2 = \frac{\sigma_k^2}{N_s} \mathbb{E}\{|f(k) - \hat{f}_{k-1}|^2\} = \sigma_s^2 + \sigma_{\text{est}}^2.
$$

The notation $f(k)$ is used to emphasize that the calculation will include consideration of channel dynamics, and $\hat{f}_{k-1}$ is the special case of (2) based on CSFB. The original LMS analysis of [8] separates the MSE contribution into two components: $\sigma_s^2$, which accounts for the gradient noise inherent to the stochastic-gradient-based LMS algorithm and, $\sigma_{\text{est}}^2$, which
accounts for channel dynamics. The MSE contribution from gradient noise, which does not depend on channel dynamics, is

$$
\sigma_{\text{N}}^2 = M \sigma_{\text{min}}^2
$$

(23)

where the LMS misadjustment is [10]

$$
\mathcal{M} = \frac{\sigma_{\text{N}}^2 \beta L}{2 - \sigma_{\text{N}}^2 \beta (L + 1)}
$$

(24)

and $\sigma_{\text{min}}^2$ is the MSE of the Wiener estimator, which is $N_0$ for the CSFB case.

The lag error analysis contained in [8] is easily modified to account for the FE processing. This is summarized in Fig. 1(b). The transfer function between the components of $f(k)$ and the corresponding component of the channel error vector due to lag $\Delta f(k)$ is

$$
G(\nu) = \frac{\exp(-j2\pi \nu) - 1}{1 - (1 - \sigma_{\text{N}}^2 \beta) \exp(-j2\pi \nu)}
$$

(25)

so that MSE contribution from channel dynamics is

$$
\sigma_{\text{lag}}^2 = \frac{1}{N_s} \mathbb{E}\{||\mathbf{a}_k^H \Delta f(k)||^2\}
$$

(26)

$$
= \frac{\sigma_{\text{lag}}^2}{N_s} \text{tr}(\mathbf{V} \mathbf{V}^H) \int_{-1/2}^{1/2} |D(\nu)G(\nu)|^2 d\nu.
$$

(27)

It follows that for CSFB the effective noise level is

$$
N_{0,\text{eff}} = (1 + \mathcal{M})N_0 + \sigma_{\text{lag}}^2 + \sigma_{\text{SI}}^2.
$$

(CSFB)

(28)

Extension of the above analysis to the PSP-based algorithm will be only a rough approximation because multiple channel estimates are maintained. However, since the survivors typically merge at some point back in the trellis history, it can be argued that the analysis is applicable with the assumption that the single $\hat{f}_{k-1}$ is based on the sequence which is eventually selected by the VA. With this assumption, the relation in (21) can still be used for a PSP algorithm with the modification

$$
\sigma_{\text{min}}^2 = \frac{1}{N_s} \mathbb{E}\{||\mathbf{x}_k - \alpha_k^H \circ \hat{f}_{k-1}||^2\}
$$

(29a)

$$
= \frac{1}{N_s} \mathbb{E}\{||\mathbf{x}_k - \alpha_k^H \circ \hat{f}_{k-1}||^2\}

+ (\alpha_k - \hat{\alpha}_k)^H \circ \hat{f}_{k-1} \circ \hat{f}_{k-1} \circ \hat{f}_{k-1}^H

\approx (N_{0,\text{eff}} - \sigma_{\text{SI}}^2) + \mathbb{E}\{d^2(\mathbf{E})\}/N_s
$$

(29b)

where the cross term has been neglected and $\mathbb{E}\{\cdot\}_W$ denotes evaluation at the Wiener solution. It has been assumed that the performance of the PSP algorithm under consideration is near that of the Wiener solution, and that the statistics of the data and channel estimates are approximately the same as those of the true parameters. As suggested in [9], the average power of the error sequence convolved with the channel is approximated by

$$
\mathbb{E}\{d^2(\mathbf{E})\} \approx \sum_{\mathbf{e} \in \mathcal{E}^{\text{max}}(\{\lambda\})} d_{\text{ave}}^2(\mathbf{E}) P_{PW}(\{\lambda\})
$$

(30)

so that combining (23), (29), and (30) yields

$$
N_{0,\text{eff}} = \frac{1}{1 - \mathcal{M}} \left[ N_0 + \sigma_{\text{lag}}^2 \right]

+ \frac{\mathcal{M}}{N_s} \sum_{\mathbf{e} \in \mathcal{E}^{\text{max}}(\{\lambda\})} d_{\text{ave}}^2(\mathbf{E}) P_{PW}(\{\lambda\})

+ \sigma_{\text{SI}}^2.
$$

(PSP)

(31)

Since $P_{PW}(\{\lambda\})$ will be a function of $N_{0,\text{eff}}$, (31) can be solved iteratively starting from $\mathbb{E}\{d^2(\mathbf{E})\} = 0$.8

The minimum error of a joint channel and sequence estimator is taken in [9] to be $\sigma_{\text{min}}^2 = N_0 + \mathbb{E}\{d^2(\mathbf{E})\}/N_s$. Since $\mathbb{E}\{d^2(\mathbf{E})\}$ is a relatively small term for low error rates, this seems to indicate that CSFB and PSP algorithms will perform virtually the same—a result not predicted by (28)/(31) and not observed in the simulation results.

C. Error Floor Prediction

Since the LMS lag error is colored, it is not surprising that the white noise approximation of Section III-B breaks down when thermal noise is insignificant. Simulations suggest that the white noise assumption is invalid when $\sigma_{\text{lag}}^2$ is approximately $N_0$ or greater. The simple model in Fig. 1 is used to predict the error floor in this section.

For the CSFB case, the effective additive estimation noise with thermal noise absent is

$$
w_k = \alpha_k^H \circ \Delta f(k).
$$

(32)

Assuming that the zero-mean lag error is independent of the data sequence, the conditional correlation of this effective noise is

$$
\mathbb{E}\{w_{k+m} w_k^H | A_{k+m}\} = \alpha_{k+m}^H \circ R_{\Delta f}(m) \circ \alpha_k
$$

(33)

where

$$
R_{\Delta f}(m) = \mathbb{E}\{\Delta f(k + m) | \Delta f(k)\} = \sigma^2 \mathbf{V} \mathbf{V}^H R_{\text{DG}}(m)
$$

(34)

and $R_{\text{DG}}(m)$ is the inverse DT Fourier Transform of $|D(\nu)G(\nu)|^2$. Averaging over the data yields

$$
\mathbb{E}\{w_{k+m} w_k^H\} = \sigma^2 \sigma^2 R_{\text{DG}}(m) \sum_{j=0}^{L-1-m} \mathbf{V}_{j+m} \mathbf{V}_{j+m}^H, m \in \mathbb{Z}_L.
$$

(35)

Noise component vectors separated by $L$ or more symbol intervals are uncorrelated.

Approximating the lag noise as Gaussian with correlation given in (35) allows (6) to be evaluated as

$$
P_{PW}(\mathbf{E}|\mathbf{c}) = Q\left(\frac{\mathbf{c}^H \mathbf{R}_{\text{lag}} \mathbf{c}}{\sqrt{2\mathbf{c}^H \mathbf{R}_{\text{lag}} \mathbf{c}}}ight)
$$

(36)

where

$$
\mathbf{R}_{\text{lag}} = \mathbf{V} \mathbf{V}^H \circ \mathbf{V} \mathbf{V}^H \circ \mathbf{R}_m \circ \mathbf{E} \circ \mathbf{V}.
$$

(37)

8Iterative solution of this form was suggested in [13] for the analysis of PSP with a deterministic channel.
An \((N_s \times N_s)\) block partitioning of the \(R_{WW}\), the effective noise vector correlation matrix, is defined by (35).

Obtaining a closed form expression for \(P_{FW}(E)\) by averaging (36) over the white Gaussian statistics of \(c\) does not appear to be possible. The need to compute this expectation numerically essentially limits the utility of (36) to determining approximate lower bounds for the error floor.

Error floor analysis for the PSP case is complicated by the fact that the \(E[d^2(E)]\) term in (31) will be nonzero even in the absence of thermal noise. However, for reasonably low error floors (below \(10^{-2}\)) this term is insignificant compared to the lag error. It follows that a good approximation to the error floor for the PSP algorithm can be obtained by multiplying the lag correlation matrix in (37) by \(1/(1-M)\), as suggested by (28)(31).

### IV. Numerical Results

Numerical evaluation of the bounds and approximations obtained in Section III are compared with Monte Carlo simulation results in this section. The specific channel model used was an \(N_s = 6\) TDL channel with \(L_c = 2\), with each of the 13 taps having equal power. The Doppler filter was a second order DT Butterworth filter with normalized cutoff frequency \(\nu_d\). Two levels of dynamics were considered, “high dynamics” \((\nu_d = 7.7 \times 10^{-4})^9\) and “extreme dynamics” \((\nu_d = 2.8 \times 10^{-3})\). QPSK modulation was used with \(\sigma_s^2 = 1\) and the LMS step size was fixed at \(\beta = 0.1\). Three data pulses were considered, a rectangular (rect) pulse, a low-pass filtered rectangular (LPF-rect) pulse, and a raised cosine (rcos) pulse. The rect pulse is constant for one symbol period. The LPF-rect pulse is defined by first passing the rect pulse through an ideal low pass filter which eliminates frequencies above \(1/T\), then windowing for \(t \in [0,2T)\). The rcos pulse had unit roll-off parameter [4] and was time windowed to duration \(2T\).

The equivalent pulse matrix \(V\) was computed numerically for each FE as defined in [1]. The performance expressions were computed and Monte Carlo simulations were conducted for the CSFB- and PSP-based algorithms using each of the three pulses.

Evaluation of the performance bounds and approximations were conducted by searching all allowable error sequences up to length 9 and retaining the 600 sequences with the smallest \(d_{\text{ave}}^{2}(E)\) values. The eigenvalues and partial fraction expansion computations were performed for each of these 600 sequences. In all cases, the weight of the dominant error event was one, so that the approximation in (18) and the lower bound in (15) were equivalent.

The Monte Carlo simulations were conducted using a representative mobile radio packet format. The length of the \(4^{L-1}\)-state trellis for the VA was taken to be \(7(L-1)\). The channel estimates were initialized to the true channel values plus a white Gaussian vector with each component having variance \((MN_0 + \sigma_s^2)/L\). The VA was initialized in the correct state. This models the effects of an initial channel estimate obtained from a short training sequence and also corresponds to the tracking mode assumed in the analysis. Approximately the first 60 data symbols were decoded—for the LPF-rect and rcos pulse (\(L = 4\)) the exact number was 63, while simulations using the rect pulse (\(L = 3\)) were 56 symbols long. Tail effects were neglected by only making decisions on symbols, \(a_k\), for which the received symbol had been observed up to time \(k+7(L-1)\). This approach is similar to adding tail bits to the packet to properly terminate the VA [5]. For each simulation point, packets were generated and decoded until at least 2000 symbol errors were observed.

The numerical results are summarized by plotting the symbol error probability \(P_e\) versus the available signal-to-noise ratio (SNR) per symbol, defined by

\[
\frac{E}{N_0} = \frac{\sigma_s^2 \sigma_e^2 R_{ee}(0)(N, L_c + 1)}{N_0}
\]  

(38)

PSP simulation results are summarized in Fig. 2 for the rect pulse and high dynamics, and Fig. 3 for the rcos pulse and extreme dynamics. The results for the LPF-rect and the rcos pulses were qualitatively the same. These results are discussed further in Section V-A.

The performance difference between the practical PSP approach and the artificial CSFB receiver with extreme dynamics was found to be approximately 2 dB in \(E/N_0\) for the rcos pulse.
and 1 dB for the rect pulse. A much smaller difference was observed at the high dynamics level. Note that the smaller degradation for the rect pulse than for the rcos pulse is predicted by the relations in (28)/(31) in light of the fact that the misadjustment for the rect pulse ($L = 3$) is less than that for the rcos pulse ($L = 4$). The original form of the analysis in [9] would predict essentially no difference in the PSP and CSFB performance.

The effect of increased channel dynamics is illustrated in Fig. 4, where performance curves from simulations as well as the AWGN ($N_{0,\text{eff}}$) analysis of Section III-B are plotted for the PSP case. Qualitatively similar results for the CSFB case are omitted for the sake of compactness. As expected, this analysis breaks down when the LMS lag error dominates the thermal noise. Specifically, the AWGN analysis provides an optimistic estimate of the error floor.

The error floor predicted by the AWGN analysis for the rcos pulse with $N_s = 6$ was particularly optimistic. This is due to the fact that the rcos pulse is effectively bandlimited below $1/T$ so that the LMS lag error is strongly correlated. This is illustrated in Fig. 5, where, along with the AWGN analysis, the error floor predicted by the colored noise lag analysis of Section III-C is included. This value was computed by numerically integrating (36) over the white Gaussian statistics of $e$ using the Monte Carlo technique [14] and a standard approximation for $Q(\cdot)$ [15, (7.1.26)]. Two additional simulation curves are also shown in Fig. 5. The “injected thermal noise” curve is the result of a simulation using the correct channel information, with the thermal noise level set to $N_{0,\text{eff}}$ as defined in (28). The “injected channel noise” was obtained by setting

$$\hat{f}_k = f(k) + \epsilon_k$$

(39)

where $\{\epsilon_k\}$ is white Gaussian sequence of vectors, each with covariance matrix $[(1 + M) N_0 + \sigma_k^2]/L$, so that the effective noise level is as in (28). From these artificial simulations we may conclude that the assumption which breaks down near the error floor is the white lag-noise assumption, as opposed to the assumption that channel estimation error may be modeled as additional additive noise.

V. FACTORS AFFECTING PERFORMANCE

In this section, we qualitatively characterize the effects of suboptimal FE processing for the model and algorithm investigated in Sections II–IV. Some of the issues which may complicate matters in practice are then discussed with an emphasis on application in packet mobile radio systems.

A. The Effects of Suboptimal FE Processing

Based on the results in Sections III–IV, at least four distinct types of performance degradation resulting from suboptimal FE processing can be identified:

- **SNR Degradation:** An approximate measure of the SNR degradation suffered is the ratio of $\text{tr}(VV^H)$ for a given FE processor to the same quantity for the optimal FS-WMF. Neglecting edge effects, the SNR loss for the D-WMF is $N_s/N_c$; a value which is intuitively expected given the sampled FE approximation to the D-WMF discussed in [1]. The SNR degradation for the US-WMF is more sensitive to the spectral properties of the pulse, but is typically much less severe than that suffered by the D-WMF. An SNR degradation is manifested by a shift in the $P_e$ curve without a change in slope. From the numerical results, it is apparent that this is the primary degradation suffered by the D-WMF processor.

- **Resolution Reduction:** The ability of the data pulse to resolve the physical channel taps is characterized by $D$, the number of nonzero eigenvalues of $R$. From the analysis of Section III, it is clear that $D$ is also the diversity order obtained by the receiver. The US-WMF processor is the most susceptible to a reduction in resolution. This is observed in the case of the rect pulse, which has the largest bandwidth of three pulses and can resolve all 13 channel taps to some extent. The eigenvalues for the error event with minimum $d_{\text{min}}(E)$ are shown in Fig. 6 for different FE processors. This illustrates the resolution reduction as well as the SNR degradation. A reduction in resolution is manifested as a reduced slope in the $P_e$ versus $E/N_0$ curves.
1) Channel Models and Post-Processor Structure: A primary consideration is the level of detail assumed about the channel model and the degree to which it is exploited. For example, a careful review of [1] shows that the quasistatic channel assumption is used twice. For the derivation of the FS-WMF, the channel taps need only be assumed constant over the integration time of the pulse-matched filter, $L_0T$. This is a valid assumption for the level of dynamics considered in Section IV, but as the normalized Doppler $v_d$ increases toward 0.5, this assumption will eventually break down and the FS-WMF will not be optimal. The derivation of the joint-ML metric recursion in [1] hinged on the assumption that the channel was static over the entire observation interval; the introduction of the forgetting factor in the metric recursion may be viewed as an ad hoc modification to avoid including the channel dynamics in the model. With a simple extension of the concepts introduced in [16]–[18], an optimal metric recursion based on the channel model of Fig. 1, which includes both the dynamics and statistics of $c(t)$, would result in per-data-path Kalman filtering (i.e., minimum MSE estimation of the Gauss-Markov channel). Since the channel model is more fully exploited, it is expected that a PSP algorithm based on this metric would outperform the LMS-PSP post-processor considered in this paper. It is more difficult to predict the effects of a mismatch between the actual channel conditions and the assumptions made to derive the metrics. In view of the complexity and variability of the channel conditions for modern mobile communication systems, the robustness of receiver structures to mismatches between the assumed and actual channel conditions may be the most significant issue in selecting a design.

The general framework established in [1] suggests that, in addition to the metric used, the tree-search algorithm selected is an important element of post-processor design. How to design a tree-search algorithm in order to maximize performance/complexity ratio is a difficult open problem. A complete receiver design would consider the effectiveness and complexity of the FE processing, the metric (channel estimation), and the tree-search algorithm. Integration of the data/sequence estimator with the error correction coding and interleaving schemes may also prove to be valuable.

2) System Parameters: In addition to the channel model, the parameters selected in the simulations of Section IV also affect the performance. The packet length $P$ is relevant only if it is increased enough so that the tracking mode assumption breaks down. As $P$ increases, the performance is best described as “mixed-mode”—i.e., a mixture of tracking and acquisition. This is because a deep fade during the packet, which may cause the algorithm to lose lock of the channel, is more likely to occur as $P$ increases. Analysis of the more realistic mixed-mode performance would require characterization of both the tracking and acquisition modes, as well as the probability of transitioning between these modes. Although the transition from the tracking mode to mixed-mode is gradual, the simulation results reported in Section IV are for the tracking mode. In fact, doubling the packet length ($P = 10$).
120) yields results which are still within the tracking mode performance region. In the mixed-mode performance regime, it is difficult to obtain reproducible error rate estimates via simulation at very low values of \( P_e \).

The LMS step size \( \beta \) is another important parameter in the results of Section IV. Since the primary objective was to demonstrate the effects of FE processing, no extensive effort was made to determine the optimal value of \( \beta \) for each case; rather, the value \( \beta = 0.1 \), which was known to provide reasonably good performance at both levels of dynamics, was used throughout. However, the analysis of Section III clearly predicts the relevance of \( \beta \). For the PSP case, (31) indicates that there is a trade-off analogous to that associated with known data LMS estimation. Specifically, (24) implies that increasing \( \beta \) increases the misadjustment and hence \( 1/(1 - M) \), while, since \( D(\nu) \) is generally low-pass and \( G(\nu) \) is high-pass, (25)–(27) imply that increasing \( \beta \) decreases the lag-error. The effect of varying \( \beta \) is illustrated in Fig. 7 for a the rect pulse with a \( N_s = 2 \) D-WMF front-end and extreme dynamics.

In theory, one could use the approximate analysis of Section III to optimize the choice of \( \beta \) for a specific \( D(\nu) \), \( L \), and SNR. When the lag error is relatively insignificant (i.e., not near the error floor), (31) suggests that \( \beta \) should be selected small.

A guideline for the case when thermal noise dominates is to compare the factor \( 1/(1 - M) \) for various \( \beta \) as suggested by (31). For example, this predicts a 2.1 dB degradation in SNR for \( \beta = 0.2 \) relative to \( \beta = 0.1 \), but only another 0.5 dB improvement for reducing the step size to \( \beta = 0.05 \). This agrees well with simulation results for high dynamics, which have been omitted for the sake of compactness. As with other factors, the effect of varying \( \beta \) is magnified in the error floor since a small SNR loss may translate to an order of magnitude in the value of the error floor. When the thermal noise level is insignificant and the error rate is small, (31) suggests that the quantity \( \sigma_{\delta e}^2/(1 - M) \) be minimized.

A quantity proportional to this is plotted versus \( \beta \) in Fig. 8; this would lead to a selection of \( \beta = 0.2 \) to minimize the error floor. Note that for the case under consideration (i.e., \( L = 3 \) and \( \sigma_{\delta e}^2 = 1 \)), (31) breaks down for \( \beta \geq 0.3 \). However, for the values of \( \beta = 0.05, 0.1, 0.2 \) shown in Fig. 7 under extreme dynamics, the relative values of the error floor are consistent with what would be predicted by Fig. 8. This is in spite of the fact that the AWGN channel estimation error assumption breaks down in this region presumably because the FE is fixed (e.g., fixed lag error correlation) so that the relative value of \( \sigma_{\delta e}^2 \) is useful. Note that (28) does not seem to predict this trade-off for the CSFB case—i.e., as \( N_0 \to 0 \), the effective noise level is apparently minimized by continuing to increase \( \beta \).

This is limited in practice by the complete breakdown of the assumptions in [8] for the LMS algorithm as \( \beta \) becomes large, just as the \( \beta = 0.3, 0.4 \) curves in Fig. 7 show that the error floor eventually increases as \( \beta \) increases in the PSP case.

VI. CONCLUSION

The simulation results obtained in this paper illustrate several important points. The most significant in relation to [1] is that FE processing is important—i.e., the advantages of a superior post-processing algorithm may be negated by suboptimal FE processing. The TDL channel model with the associated notion of a resolution time was also verified. Specifically, for the rcos pulse, assuming that the \( T/6 \)-spaced physical channel is a \( T/2 \)-spaced TDL (i.e., building a US-WMF with \( N_s = 2 \)) results in essentially no loss in performance while reducing the complexity by a factor of 3. This is due to the fact that the bandwidth of the rcos pulse is approximately \( 1/T \), so that this pulse can resolve no more than two taps per symbol period.

The approximate analysis represents the synthesis, extension, and refinement of the work in [7], [8], and [9]. This led to performance predictions more consistent with CSFB and PSP simulation results. While the limitations of analysis, especially in predicting the error floor, were pointed out, the results are useful because they predict the factors effecting the tracking mode performance (e.g., the various degradations suffered by suboptimal FE processing) and therefore provide guidelines for system design (e.g., the selection of the LMS step size).

While [1] introduced a framework for the investigation of unknown channel MLSE, it has been demonstrated in this paper that the associated receiver design task is nontrivial. For example, a conceptual partition between the FE and post-processor was developed in [1] (i.e., for a fixed FE, various post-processors may be compared), but the results presented in this paper suggest that a design which attempts to maximize performance for a given complexity should consider both stages. Conducting an actual receiver design is complicated by the issues discussed in Section V-B, foremost among these are the accuracy of the assumed channel model and the robustness of the receiver design.

Finally, we point out the importance of the operating environment in selecting a receiver design. The best FE for a fixed \( N_s < N_e \) often depends on both the operating SNR and the level of dynamics. As demonstrated, the best choice of \( \beta \) for this design is also sensitive to the level of dynamics. This raises the issue of whether one should optimize the design for the worst case channel conditions, which may occur rarely,
or for good channel conditions, when low error rates are achievable.

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Keith M. Chugg (S’88-M’95), for photograph and biography, see p. 846 of the July 1996 issue of this TRANSACTIONS.

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