Joint Spatial Division and Multiplexing (JSDM) is an approach to multiuser MIMO downlink that exploits the structure of channel correlation in order to allow for a large number of antennas at the BS while requiring reduced-dimensional Channel State Information at the Transmitter (CSIT).

**Channel Model**

![Diagram of a UT at AoA θ with a scattering ring of radius r generating a two-sided AS ∆ with respect to the BS](image)

- BS has M antennas and serves K users.
- \( y = H^t x + z \) \( (1) \)
- \( H = [h_1 \ldots h_K] \) is the concatenation of user channels, \( x = V d \) is the transmit signal vector, with \( V \) the precoder and \( d \) the vector of data symbols.
- Channel of user \( k \) given by \( h_k = CN(0, R_k) \), \( R_k = U_k \Lambda_k U_k^H \) being the channel covariance of rank \( r_k \).
- Equivalently, \( h_k = U_k \Lambda_k^{\frac{1}{2}} w_k \), where \( w_k \sim CN(0, I) \).
- A UT with AoA \( \theta \) and angular spread \( \Delta \) has \( |R_{\Delta \theta}|_{m,p} = \frac{1}{2} \pi \Delta \lambda e^{j\pi(\alpha_1 - \alpha_2)} dx \)
- \( k(\alpha) = \frac{1}{2\pi(\cos(\alpha), \sin(\alpha))^T} \) is the wave vector for a planar wave impinging with AoA \( \alpha \), \( \lambda \) the carrier wavelength, and \( w_k, w_p \in \mathbb{R}^2 \) are vectors indicating the position of BS antennas.

**JSDM Basics**

- K UTs are partitioned into G different groups based on the similarity of their covariance matrices, each containing \( s = \frac{M}{G} \) users.
- We assume all users in a group \( g \) have the same covariance matrix \( R_g \) with rank \( r_g \).
- JSDM uses a two stage precoder \( V = BP \).
- \( B \in \mathbb{C}^{M \times r} \) is the pre-beamformer, independent of the instantaneous channel realization and \( P \in \mathbb{C}^{r \times K} \) is a precoding matrix depending on the effective channel \( H = B^* H \) of reduced dimensions. In our work, we use two choices for \( P \), i.e., regularized-ZFBF and ZFBF.
- \( B \) can be designed jointly for all groups (joint group processing (JGP)) or separately for each group \( g \) (per group processing (PGP)).
- In PGP, the received signal by users in group \( g \) is \( y_g = H_g^t P_g d_g + \sum_{g' \neq g} H_g^t B_{g'} P_{g'} d_{g'} + z_g \) where \( H_g = B_{g}^* H_{g} \) and \( B_{g} \in \mathbb{C}^{M \times r} \) with \( B = [B_1 B_2 \ldots B_G] \).
- In JGP, We choose \( B_{g} = U_{g} \). This is called **eigen beamforming**.

- In PGP, the pre-beamforming process creates virtual sectors, similar to spatial sectorization in current cellular standards.
- Channel covariance \( R_g \) changes slowly compared to the instantaneous channel matrix. So, \( R_g \) can be estimated based on a suitable subspace estimation and tracking algorithm, exploiting the downlink training phase.

**Achieving capacity with JSDM**

**Theorem 1** Let the channel covariances of the \( G \) groups are such that \( U = [U_1 \ldots U_G] \) is tall unitary (i.e., \( U^t U = I \)). For this scenario, JSDM achieves the same sum capacity of the corresponding MU-MIMO downlink channel (1) with full CSIT.

- Note that choosing \( B = U_1 U_2 \ldots U_G \) achieves capacity, and gives a set of decoupled MU-MIMO downlink channels.
- It is beneficial to partition users into groups based on the similarity of their eigenspaces and then scheduling across groups satisfying the tall unitary condition.
- If this is not possible, block diagonalization (or approximate block diagonalization) is used to design \( B_g \) such that \( H_g^t B_g \approx 0, \forall g' \neq g \).
- Approximate block diagonalization is important because a majority of the non-zero eigen values of \( R_g \) are very small, and hence, going for exact block diagonalization may degrade the achievable rates.

**Performance Analysis**

![Comparison of sum spectral efficiency (bit/s/Hz) vs. SNR (dB) for JSDM](image)

- Plots are obtained by using an analytical tool based on random matrix theory, called **deterministic equivalents** avoiding lengthy Monte Carlo simulations.
- **Uniform Circular Array with** \( M = 100, G = 6, K = 24 \), with 4 users per group.
- JSDM with PGP performs well compared to full CSIT case.
- Imperfect CSIT (resulting from estimation of channels by users via downlink pilots and ideal feedback) reduces the achievable rates by 70 percent.

**Uniform Linear Arrays**

- For a uniformly linear array, the channel covariance \( R \) for a UT at AoA \( \theta \) and angular spread \( \Delta \) is
  \[
  R_{\Delta \theta} = \frac{1}{2\pi} \int \Lambda(\theta) e^{-j2\pi D(m-p)\sin(\theta)} dm
  \]
- For this special case, \( R \) is Toeplitz (with rank \( r \)) and can be approximated by a circulant matrix \( C \), and the approximation holds as follows.
- The set of eigenvalues \( \{\lambda_m(R)\}, \{\lambda_m(C)\} \) are asymptotically equally distributed, i.e., for any continuous function \( f(x) \) defined over \( [\alpha_1, \alpha_2] \), we have
  \[
  \lim_{M \to \infty} \frac{1}{M} \sum_{m=1}^{M} f(\lambda_m(R)) = \lim_{M \to \infty} \frac{1}{M} \sum_{m=1}^{M} f(\lambda_m(C))
  \]
- The tall unitary matrix of the channel covariance eigenvectors, i.e., \( U \), can be approximated with a submatrix \( F_g \) of the DFT matrix \( F \), formed by a selection of \( S \) columns of \( F \) in the following sense:
  \[
  \lim_{M \to \infty} \frac{1}{M} U U^H - F_{g} F_{g}^H = 0
  \]

A good approximation of the actual rank \( r \) for large but finite \( M \) is given by \( r \approx \rho M \), where \( \rho \) is given as above.

**Theorem 3** Groups \( g \) and \( g' \) with angle of arrival \( \theta_g \) and \( \theta_{g'} \) and common angular spread \( \Delta \) have orthogonal eigenspaces if their AoA intervals \( [\theta_g - \Delta, \theta_g + \Delta] \) and \( [\theta_{g'} - \Delta, \theta_{g'} + \Delta] \) are disjoint.

![Sum spectral efficiency (bit/s/Hz) vs. SNR (dB) for JSDM for DFT pre-beamforming and PGP](image)

- When the BS has a large antenna array, an efficient way consists of selecting groups of users with almost identical AoA intervals, and scheduling \( G \) groups with non-overlapping AoAs.
- JSDM is attractive because only a coarse knowledge (AoA interval) is required, rather than an estimate of the sample covariance matrix.