Motivation

- Motivation: Neutral atoms are good candidates for quantum information processing, according to long coherence time. However, single atoms are difficult to address and not easy to achieve strong coupling with cavity photons. On the other hand, ensembles of atoms could effectively enhance the coupling rate by \sqrt{N}. They can also be used as quantum memory, quantum register for the application of quantum communication. So, using ensembles of atoms to build quantum computers on atom chips may be attractive and scalable.

- Goal: Propose a scheme of all-optical holonomic quantum computer based on cavity QED for ensemle atoms. Decreases according to thermal motion of atoms, inhomogeneous distribution of laser field and cavity loss could be effectively eliminated.

Basic Model

- The relevant states in $^{87}$Rb are: clock states $|\Omega \rangle = |P = 1, m_P = 1\rangle$ and $|s\rangle = |P = 2, m_P = 1\rangle$; ancillary states $|a\rangle = |P = 1, m_P = 1\rangle$; and $|b\rangle = |P = 2, m_P = 1\rangle$ in the submanifold. A state of manifold $S_{P_1P_2}$ serves an intermediate state $|\sigma\rangle (P = 1, m_P = 0)$.

- Single energy level can be directly mapped to a three-level system in second quantization representation.

- The Hamiltonian in second quantization representation could be written as:

$$H_2(\theta) = \Omega_0 (|\Omega\rangle \langle \Omega| + |s\rangle \langle s|) + \hbar \omega_c (|\Omega\rangle \langle s| - |s\rangle \langle \Omega|)$$

- The geometric phase could be written as:

$$\phi_2 = \int \frac{d\phi_2}{\sin^2 \theta} = 0$$

One Qubit Phase Gate

- The level diagram of coupling is shown in figure above to realize the gate $R_{\phi_2}^{(1)}(\phi_2) = \exp(i\phi_2 |\Omega\rangle \langle |\Omega| + |s\rangle \langle s|)$.

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Two Qubit Controlled Phase Gate

- Coupling diagram to realize a controlled phase gate $U^{(1)}(\phi_2) = \exp(i\phi_2 |0\rangle \langle 1| 0 \rangle \langle 0|$ is shown in the left.

- When Stark effect and Zeeman effect are considered.

- We use external laser control to overcome the non-uniform coupling rate between cavity photon and atoms caused by uncertainty of atomic position and the inhomogeneous distribution of cavity mode.

- The geometric phase is $\phi_2 = \frac{\pi}{16}$.

- Fidelity of a gate with $\phi_2 = \pi/16$ is shown in the figure of the right with different coupling constants.

- The average number of photons during the process of the gate $\phi_2 = \pi/16$ when initially prepared in $|\Omega\rangle$ shown in the box above as a function of time and coupling strength $\Omega_c$.

One Qubit Rotation Gate Around Y

- The coupling diagram to realize the gate $R_{\phi_2}^{(1)}(\phi_2) = \exp(i\phi_2 |0\rangle \langle 1| 0 \rangle \langle 0|$.

- There are three steps to realize this gate. 1. Adiabatically pump the a single excitation from ground state to a register state. 2. Adiabatically control the coupling between single excitation and the cavity state. 3. Reverse Step 1.

- $|n_1\rangle$ and $|n_2\rangle$ are two R$^{87}$b states that served as register states in the process. Only single atom can be pumped to $|r\rangle$ due to dipole-dipole interaction when atoms are prepared in states $|r\rangle$ and $|s\rangle$. We choose them as the outmost Stark state when the cloud of atoms are illuminated in constant electrical field.

- The simulation of the transition is shown in the figure below.

- We need to find Rydberg states which have large energy shift but small strength of transition to be our register state and intermediate state. This turns out to be the outmost Stark eigenstate. For the manifold $n = 15$ of $^{87}$Rb, we circle the state we choose as our register state in the figure above. For our purpose, $n_1$ and $n_2$ are chosen to be 70 and 60 have large enough dipole-dipole blockade effect.

Average Number of Photons

- The figure shows the number of cavity photons during the processing of controlled $\pi/2$ gate operation for the states prepared initially in $|10\rangle_1$ (big a) and $|11\rangle_2$ (big b). The parameters are chosen as $\Omega_0 = 100 \text{MHz}, \Omega_1 = 300 \text{MHz}, \Omega_2 = 50 \text{MHz}, \Omega_3 = 10 \text{MHz}$. The number of photons has a peak value of about $10^{10}$ during 4ps.